HW1 phys 168 | Eradat | SJSU

1 a) λ in m, mm, μm, nm, ... depending on the size of λ we change the unit.

b) $\lambda = \frac{\lambda_0}{n}$ wavelength inside the material

when you look up n pay attention that it is frequency dependent so calculate the $\underline{n}$ for the frequency range of interest.

c) $\nu = \frac{c}{\lambda}$

Pay attention $d\nu = d\left(\frac{c}{\lambda}\right) = -c\frac{d\lambda}{\lambda^2} = -\nu\frac{d\lambda}{\lambda}$

$$\boxed{\frac{d\nu}{\nu} = -\frac{d\lambda}{\lambda}}$$ for calculating the intervals

d) $E = h\nu$ $\qquad E = hc/\lambda$

$dE = h\,d\nu$ $\qquad dE = -\frac{hc\,d\lambda}{\lambda^2}$

$1\,eV = 1.602 \times 10^{-19}\,J$

$1\,W = 1\,J/s$

e) Same methods $\qquad \lambda_{red} = 630\,nm$

$\lambda_{green} = 530\,nm$

$\lambda_{blue} = 475\,nm$

HW1 phys 168

Problem 3

Find T for which $B_{21} = A_{21}$ for $\lambda = 500\,nm$

$$R = \frac{\frac{dN_2}{dt}\big|_{sp}}{\frac{dN_2}{dt}\big|_{st}} = \frac{A_{21}}{B_{21}\,\rho_{\nu_0}} \Rightarrow \rho_{\nu_0} = \frac{A_{21}}{B_{21} R}$$

Planck's Formula $\rightarrow$ $\rho(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12} g_1}{B_{21} g_2}(e^{h\nu/kT} - 1)}$

to match $R = e^{h\nu/kT} - 1$

to have $R = 1$ we need $e^{\frac{hc}{\lambda kT}} = 2$

$$\frac{hc}{\lambda k T} = \ln 2 \Rightarrow T = \frac{hc}{\lambda k \ln 2}$$

$\lambda = 500\,nm \Rightarrow \boxed{T = 41562\,K}$

Problem 4

$I = 200\, W/m^2 = 200 \frac{J}{s \cdot m^2}$

# of photons per sec per unit area = F (Photon flux)

$$F = \frac{I}{h\nu} = \frac{I\lambda}{hc}$$

$\lambda = 500\,nm \Rightarrow F = 5\times10^{20} \frac{photons}{m^2 \cdot s}$

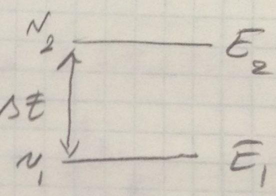
$\lambda = 100\,\mu m \Rightarrow F = 1\times10^{23} \frac{photons}{m^2 \cdot s}$

longer λ smaller freq. smaller energy/photon

so the number of photons are higher.

HW1 phys 168

2 $\Delta E = E_2 - E_1$



$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{\Delta E}{kT}}$$

Assuming $g_2 = g_1$ same degeneracy for two levels

ΔE(eV) \ T(K)	100	300	1000
rotational 10^{-4}	$\frac{N_2}{N_1} = 0.9885$	0.9962	0.9988
vibrational 5×10^{-2}	3×10^{-3}	1.45×10^{-1}	5.6×10^{-1}
electronic excitation 3	5×10^{-164}	8×10^{-49}	8×10^{-16}

small ΔE high probability

large ΔE

For large ΔE the probability of atoms being exited is very low even at very high Ts such as 1000 k.

HW4 phys 168

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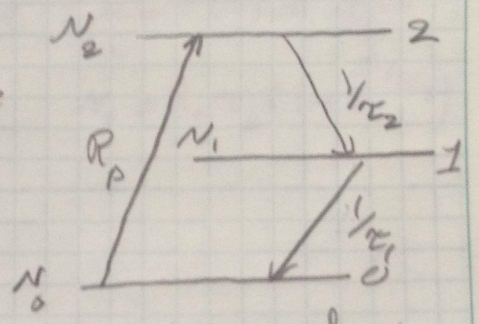
HW1 Phys 168

5) Rate equations of a three-level system.

R_p Pump rate

No depletion of $N_0 \Rightarrow N_0 = Cte$



i) $$\frac{dN_2}{dt} = +R_p - \frac{N_2}{\tau_2} \quad (1)$$

$$\frac{dN_1}{dt} = -\frac{N_1}{\tau_1} + \frac{N_2}{\tau_2} \quad (2)$$

* where N_1 & N_2 are functions of t

ii) Solving the differential equations

$$\frac{dN_2}{dt} + \frac{N_2}{\tau_2} = R_p \quad (1)$$

Boundary conditions $t=0$ $N_2=0$

A solution of the form $N_2 = Ae^{ct} + B$ (3)

$\frac{dN_2}{dt} = Ace^{ct}$ into (1) $\quad Ace^{ct} + \frac{Ae^{ct}+B}{\tau_2} = R_p$ (4)

this has to be true for all times so we group the exp & linear terms together in (4)

$$\begin{cases} Ace^{ct} + \frac{A}{\tau_2}e^{ct} = 0 \Rightarrow \boxed{c = -\frac{1}{\tau_2}} \\ \frac{B}{\tau_2} = R_p \Rightarrow \boxed{B = R_p\tau_2} \end{cases}$$

$N_2 = Ae^{-t/\tau_2} + R_p\tau_2$ using the $\begin{cases} t=0 \\ N_2=0 \end{cases}$ init. cond.

$A = -R_p\tau_2$

$$\boxed{N_2 = R_p\tau_2(1 - e^{-t/\tau_2})}$$

Solving the (2)

$$\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = \frac{N_2}{\tau_2}$$

we set $\frac{N_2}{\tau_2} = f(t) = \frac{N_2(t)}{\tau_2}$

$N_2 = R_p \tau_2 (1 - e^{-t/\tau_2})$

$$e^{t/\tau_1}\left[\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = \frac{f(t)}{\tau_2}\right]$$

$$\frac{d}{dt}\left(N_1 e^{t/\tau_1}\right) = \frac{N_2(t)}{\tau_2} e^{t/\tau_1}$$

$$N_1(t) e^{t/\tau_1} = \int_0^t \frac{N_2(t')}{\tau_2} e^{t'/\tau_1} dt'$$

$$N_1(t) = \frac{e^{-t/\tau_1}}{\tau_2} R_p \tau_2 \int_0^t \underbrace{(1 - e^{-t'/\tau_2})}_{u} \underbrace{e^{t'/\tau_1} dt'}_{dv}$$

We can use integration by part $\int u\,dv = uv - \int v\,du$

$$u = 1 - e^{-t'/\tau_2} \Rightarrow du = \frac{1}{\tau_2} e^{-t'/\tau_2} dt'$$

$$dv = e^{t'/\tau_1} dt' \Rightarrow v = \tau_1 e^{t'/\tau_1}$$

$$N_1(t) = R_p e^{-t/\tau_1}\left[\tau_1 e^{t'/\tau_1}(1 - e^{-t'/\tau_2})\Big|_0^t - \int_0^t \frac{\tau_1}{\tau_2} e^{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)t'} dt'\right]$$

$$N_1(t) = R_p \tau_1 (1 - e^{-t/\tau_2}) - R_p \frac{\tau_1}{\tau_2} e^{-\frac{t}{\tau_1}} \left(\frac{\tau_1 \tau_2}{\tau_2 - \tau_1}\right) e^{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)t'}\Big|_0^t$$

$$N_1(t) = R_p \tau_1 (1 - e^{-t/\tau_2}) + R_p \frac{\tau_1^2 e^{-t/\tau_1}}{\tau_2 - \tau_1}\left(1 - e^{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)t}\right)$$

$$N_1(t) = R_p \tau_1 \left(1 - e^{-t/\tau_2} + \frac{\tau_1}{\tau_2 - \tau_1} e^{-t/\tau_1} - \frac{\tau_1}{\tau_2 - \tau_1} e^{-\frac{t}{\tau_2}}\right)$$

$$R_p \tau_1 \left(1 + \frac{\tau_1}{\tau_2 - \tau_1} e^{-t/\tau_1} + \frac{-\tau_2 + \tau_1 - \tau_1}{\tau_2 - \tau_1} e^{-t/\tau_2}\right)$$

$$\boxed{N_1(t) = R_p \tau_1 \left(1 + \frac{\tau_1}{\tau_2 - \tau_1} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}\right)}$$

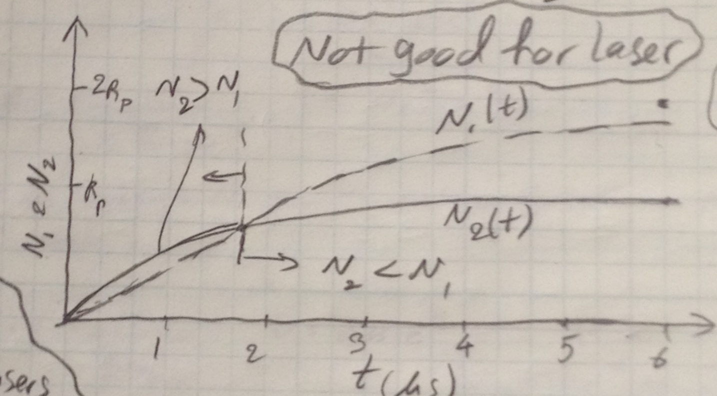
$$\begin{cases} N_1(t) = R_p\tau_1\left(1 + \frac{\tau_1}{\tau_2-\tau_1} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2-\tau_1} e^{-t/\tau_2}\right) \\ N_2(t) = R_p\tau_2\left(1 - e^{-t/\tau_2}\right) \end{cases}$$

iii) Population densities at $\tau_1 = 2\mu s$ & $\tau_2 = 1\,\mu s$

$N_1(t) = 2R_p(1 - 2e^{-t/2} + e^{-t})$ $\qquad N_1(0) = 0$ & $N_1(\infty) = 2R_p$

$N_2(t) = R_p(1 - e^{-t})$ $\qquad N_2(0) = 0$ & $N_2(\infty) = R_p$

Not good for laser



$\tau_2 < \tau_1$

we loose the population inversion after $2\mu s$

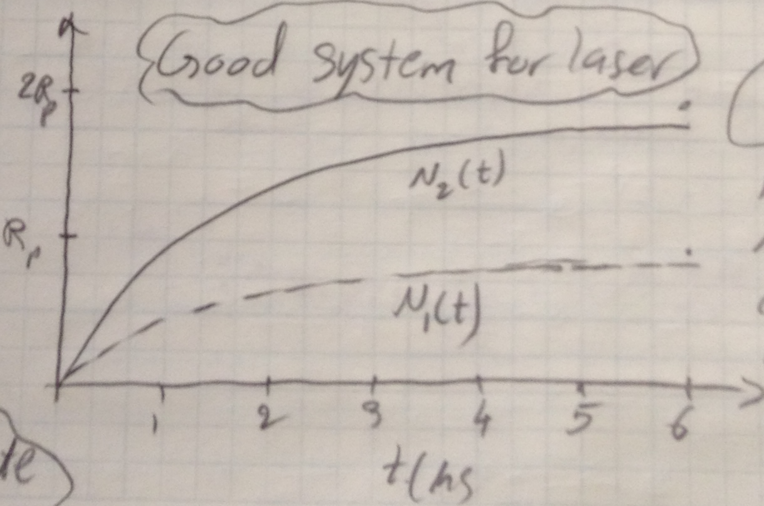
maybe ok for pulsed lasers

for the case $\tau_1 = 1\mu s$ & $\tau_2 = 2\mu_s$

$N_1(t) = R_p(1 + e^{-t} - 2e^{-t/2})$ $\qquad N_1(0) = 0$ & $N_1(\infty) = R_p$

$N_2(t) = 2R_p(1 - e^{-t/2})$ $\qquad N_2(0) = 0$ & $N_2(\infty) = 2R_P$

Good system for laser



$\tau_2 > \tau_1$

Population inversion condition is maintained in steady-state regime

good for steady state