1.1) $S = \frac{1}{2} E \times H^{*}$ in our definition E & H are not time dependent

Poynting theorem: surface integral of the poynting vector S over a closed surface s equals the power leaving the enclosed surface or

$$-\oint_s S \cdot ds = \frac{\partial}{\partial t}\int (u_e + u_m)\, dV + \int p_o\, dV = \int \nabla \cdot S'\, dV$$

Where $S' = E \times H$ the poynting vector can be considered as directional energy flux density or rate of energy transfer per unit area W/m^2

Now for time-harmonic (sinosoidal) EM fields the avarage power flow over time is more useful

$$S(t) = e \times h = Re(E e^{j\omega t}) \times Re(H e^{j\omega t})$$

$$= \frac{1}{2}\left(E e^{j\omega t} + E^{*} e^{-j\omega t}\right) \times \frac{1}{2}\left(H e^{j\omega t} + H^{*} e^{-j\omega t}\right)$$

$$= \frac{1}{4}\left(E \times H^{*} + H \times E^{*} + E \times H e^{2j\omega t} + E^{*} \times H^{*} e^{-2j\omega t}\right)$$

$$= \frac{1}{4}\left(E \times H^{*} + (E \times H^{*})^{*} + E \times H e^{2j\omega t} + (E \times H e^{2j\omega t})^{*}\right)$$

$$= \frac{1}{2} Re(E \times H^{*}) + \frac{1}{2} Re\left(E \times H e^{2j\omega t}\right)$$

Average over time gives

$$\bar{S} = \frac{1}{T}\int_0^T S(t)\, dt = \frac{1}{T}\int_0^T \left[\frac{1}{2} Re(E \times H^{*}) + \frac{1}{2} \cancel{Re(E \times H e^{2j\omega t})}^{\,0}\right] dt$$

T → Period

has $\cos 2\omega t$ time average over a T is zero

$$\boxed{\bar{S} = \frac{1}{2} Re(E \times H^{*})}$$

The average of a harmonic function^2 = $\frac{1}{2}$

1.2 $\quad e = E e^{j\omega t} e^{-jk.r}$

$h = H e^{j\omega t} e^{-jk.r}$

$k.r = k_x x + k_y y + k_z z$

$$\underbrace{\nabla}_{\text{operator}} \underbrace{e^{-jk.r}}_{\text{function}} = \frac{\partial}{\partial x} e^{-j(k_x x + k_y y + k_z z)} \hat{a}_x + \frac{\partial}{\partial y} e^{-j(k_x x + k_y y + k_z z)} \hat{a}_y + \frac{\partial}{\partial z} e^{-j(k_x x + k_y y + k_z z)} \hat{a}_z$$

$$= -jk_x e^{-jk.r} \hat{a}_x - jk_y e^{-jk.r} \hat{a}_y - jk_z e^{-jk.r} \hat{a}_z$$

$$= -j(k_x \hat{a}_x + k_y \hat{a}_y + k_z a_z) e^{-jk.r}$$

$$\nabla e^{-jk.r} = -j\vec{k}\, e^{-jk.r}$$

operator acts on the function $\quad$ operator is replaced by a value acting on the function

So act of gradiant (∇) is replaced by $-j\vec{k}$

The same way we can show that act of divergance ($\nabla.$) is replaced by $-j\vec{k}.$ and curl ($\nabla\times$) is replaced by $-j\vec{k}\times$

so $\quad \nabla \to j\vec{k}$

$\nabla. \to -j\vec{k}.$

$\nabla\times \to -j\vec{k}\times$

Then $\nabla \times h = \epsilon_0 \frac{\partial e}{\partial t} \Rightarrow \nabla \times He^{j\omega t} e^{-jk.r} = \epsilon_0 \frac{\partial}{\partial t} E e^{j\omega t} e^{-jk.r}$

$-jk \times H e^{j\omega t} \cancel{e^{-jk.r}} = \epsilon_0 j\omega E e^{j\omega t} \cancel{e^{-jk.r}}$

$$\boxed{k \times H = -\epsilon_0 \omega E}$$

$\nabla \times e = -\mu_0 \frac{\partial h}{\partial t} \Rightarrow -jk \times E = -\mu_0 j\omega H$

$\nabla . e = 0 \Rightarrow \boxed{-j\vec{k}.E = 0} \quad \boxed{k \times E = \mu_0 \omega H} \quad \nabla.h = 0 \quad \boxed{-jk.H = 0}$

1.3) $K \times H = -\omega D$
$K \times E = \omega B$ } Algabraic forms of the maxwell equations in a dielectric
$B = \mu_0 (H + M)$
$D = \epsilon_0 E + P$

Assume $M = 0$ No restrictions on D & E

a) show $K \cdot D = 0$

P is not along E so D is not along D

We start with $\rightarrow K \times H = -\omega D$

Dot product with $K \rightarrow K \cdot (K \times H) = -K \cdot (\omega D)$

$$\underbrace{K \cdot (K \times H)}_{\text{perpendicurar to } K}$$

$$0 = -\omega (K \cdot D)$$

so $\boxed{K \cdot D = 0}$

b) K points in the direction of $D \times B$

$D = -\frac{1}{\omega} K \times H$

$B = \frac{1}{\omega} K \times E$

$D \times B = -\frac{1}{\omega^2} \overset{A}{(K \times H)} \times (\overset{B}{K} \times \overset{C}{E})$ use BAC-CAB

use identity $\underset{A}{(A \times B)} \times (\underset{B}{C} \times \underset{C}{D}) = C[(A \times B) \cdot D] - D[(A \times B) \cdot C]$

$D \times B = -\frac{1}{\omega^2} \{ K[(K \times H) \cdot E] - E[(K \times H) \cdot K] \}$

$= -\frac{1}{\omega^2} \{ K(K \cdot (H \times E)) - E(\underbrace{K \cdot (H \times K)}_{\perp \text{ to } K \; 0}) \}$

$D \times B = +\frac{1}{\omega^2} \{ K(K \cdot (E \times H)) \}$ this term is a scalar

So $D \times B$ is along the $\vec{K}$

c) Show amplitude of k is $k^2 = \omega^2 \mu_0 \frac{D\cdot D}{E\cdot D}$

We start from: $\overset{A}{K} \times \overset{B}{K} \times \overset{C}{E} = \omega\mu_0 (K \times H)$

multiply by D: $D \cdot [(k(k\cdot E) - E(k\cdot k)) = -\omega^2 \mu_0 D]$

$$D\cdot(\cancel{k(k\cdot E)}^{\,0}) - D\cdot E\, k^2 = -\omega^2 \mu_0\, D\cdot D$$

But we have proved in part a $D\cdot k = 0$

$$-D\cdot E\, k^2 = -\omega^2 \mu_0\, D\cdot D$$

$$\boxed{k^2 = \omega^2 \mu_0 \frac{D\cdot D}{D\cdot E}}$$

d) Show that direction of $S = \frac{1}{2} E \times H^*$ can be other than k

If S & k were in the same direction $\vec{S} \times \vec{K} = 0$

so we cross k with S

$$K \times S = \frac{1}{2} \underbrace{K \times E}_{\omega B} \times H^* = \underbrace{\frac{1}{2}\left[E(k\cdot H^*) - H^*(k\cdot E)\right]}_{RHS}$$

$$= \frac{1}{2} \omega B \times H^*$$

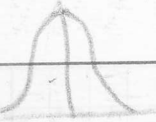
$$= \frac{1}{2} \omega \mu_0 H \times H^* \neq 0$$

Also from the RHS $k\cdot H = 0$

But $K\cdot E \neq 0$

We showed that $K\cdot D = 0$ but D is not parallel to E.

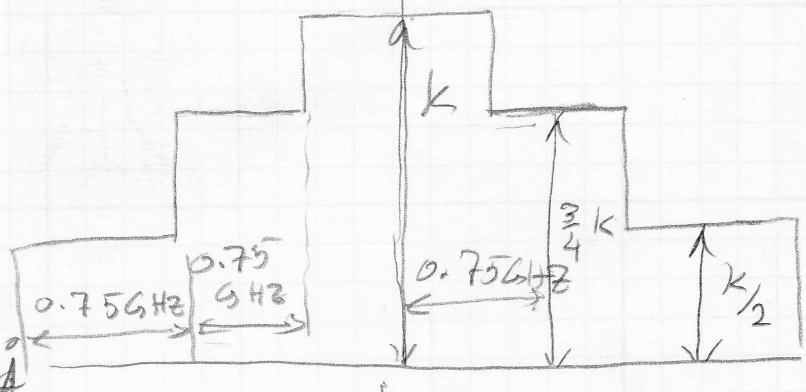
∴ So $\vec{S}$ is not parallel to k in general



7.12

$A_{21} = 6.56 \times 10^{6}\ sec^{-1}$

$\lambda = 6328\ A^{\circ}$



a) $g(\nu_0)$

Value of $g(\nu_0)$ will be the k at $\lambda = 6328\ A^{\circ}$

$\lambda = 6328\ A^{\circ}$

From $\int_{-\infty}^{\infty} g(\nu)\, d\nu = 1$

$\nu_0 = \frac{c}{\lambda_0}$

We write: $1 = k(0.75\,GHz) + 2\left(\frac{3}{4}k\right)(0.75\,GHz) + 2\left(\frac{k}{2}\right)(0.75\,GHz)$

$1 = (0.75\,GHz)k\left(1 + \frac{3}{2} + 1\right) = 0.75\,GHz\ \frac{7}{2}k$

$g(\nu_0) = \boxed{k = 3.8 \times 10^{-10}\ Sec}$

b) Stimulated emission Cross Section

$\sigma = A_{21}\frac{\lambda_0^2}{8\pi} g(\nu_0) = \left(6.56 \times 10^{-6}\frac{1}{sec}\right)\frac{(6328 \times 10^{-10})^2 m^2}{(8)(3.14)}(3.8 \times 10^{-10})$

$\boxed{\sigma = 4.04 \times 10^{-12}\ cm^2}$

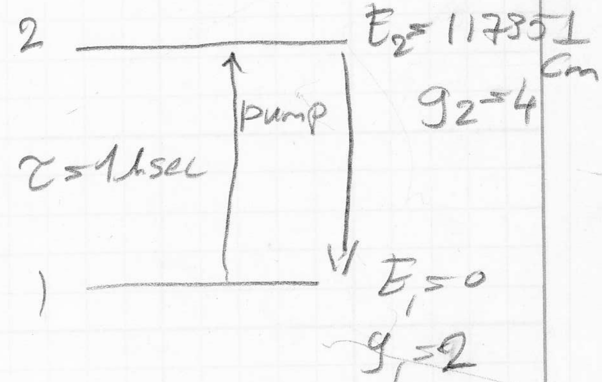
c) $g(\nu)\,d\nu$ is the probability of spontaneous emission into $d\nu$ at ν.
$g(\nu)$ is the relative strength of stimulated emission or absorption in the interval of ν & $\nu + d\nu$ by atoms

7.13) $A_{21} = \frac{1}{\tau}$

N total density

$\sigma = 10^{-14}\ cm^2$

pump causes absorption & stimulated emission



a) rate equations

$$\frac{dN_2}{dt} = \frac{\sigma I_p}{h\nu}\left[\frac{g_2}{g_1}N_1 - N_2\right] - \frac{N_2}{\tau}$$

Since $N_1 + N_2 = N$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

b) If $I_p \to \infty$ the equation becomes physically meaningless unless the term in brackets is zero, then $\frac{g_2}{g_1}N_1 - N_2 = 0 \Rightarrow \boxed{\frac{g_2}{g_1} = \frac{N_2}{N_1}}$

c) $I_p = ?$ for the case $\frac{N_2}{N_1} = \frac{1}{2}$

For the steady-state case $\frac{dN_2}{dt} = 0$

$$\frac{\sigma I_p}{h\nu}\left[\frac{g_2}{g_1}N_1 - N_2\right] - \frac{N_2}{\tau} = 0 \Rightarrow \frac{\sigma I_p}{h\nu}\frac{g_2}{g_1}N_1 = N_2\left(\frac{\sigma I_p}{h\nu} - \frac{1}{\tau}\right)$$

$$\frac{N_2}{N_1} = \frac{\frac{\sigma I_p}{h\nu}\frac{g_2}{g_1}}{\frac{\sigma I_p}{h\nu} - \frac{1}{\tau}} = \frac{\frac{\sigma\tau}{h\nu} I_p \frac{g_2}{g_1}}{\frac{\sigma\tau}{h\nu} I_p - 1} = \frac{N_2}{N_1} = \frac{1}{2}$$

Place σ & τ & g_2 & g_1 to get $\boxed{I_p = 5.82\ W/cm^2}$

d) $kT = 208 \frac{1}{cm}$

$I_p = 0$

$\left.\frac{N_2}{N_1}\right|_{\text{steady state}} = ?$

Using the Boltzmon factor

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\Delta E/kT} = \frac{4}{2} e^{-\frac{11735\,cm^{-1}}{208\,cm^{-1}}}$$

$$\boxed{\frac{N_2}{N_1} = 6.25 \times 10^{-25}}$$

N_2 is almost zero so no excitation without pumping.

Intensity of the sun light & He-Ne laser on the retina

$I_{SI} = 1\,kW/m^2$ $\quad P_{He\text{-}Ne} = 1\,mW$

$I_{SR} = ?$

$\lambda_{He\text{-}Ne} = 632.8\,nm$

$D_p = 2\,mm$

$D_{He\text{-}Ne} = 2\,mm$

$f_e = 22.5\,mm$

$\Omega_S = 0.5°$ $\quad D_F = \dfrac{4 f \lambda}{\pi D_o}$

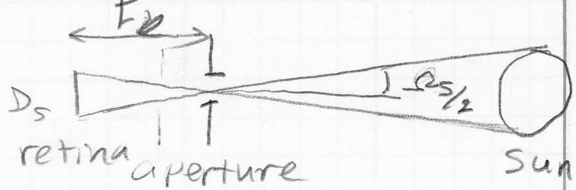


Image of sun on the retina due to pupil acting as an aperture:

$$\tan\frac{\Omega_S}{2} = \frac{D_S/2}{f_e} \Rightarrow D_S = 2 f_e \tan\left(\frac{\Omega_S}{2}\right) = 0.2\,mm$$

Power reaching the retina $= P = \pi\left(\frac{D_P}{2}\right)^2 I_S$

$$I_S = \frac{P}{\pi\left(\frac{D_S}{2}\right)^2} \Rightarrow I_S = \frac{4P}{\pi (D_S)^2} = \frac{4 \times 3.14 \times 10^{-3}\,W}{(3.14)(0.2\times 10^{-3})^2}$$

$$\boxed{I_S = 10^5\,W/m^2}$$

In case of HeNe laser

Diameter of the laser spot on the retina:

$$D_L = \frac{4 f_e \lambda_{HeNe}}{\pi D_p} = \frac{(4)(22.5\times 10^{-3}\,m)(632.8\times 10^{-9}\,m)}{\pi\,(2\times 10^{-3}\,m)}$$

$$\boxed{D_L = 9\times 10^{-6}\,m}$$

$$I_L = \frac{4 P_L}{\pi D_L^2} = \frac{(4)(1\times 10^{-3}\,W)}{(3.14)(9\times 10^{-6}\,m)}$$

$$\boxed{I_L = 1.6\times 10^7\,W/m^2}$$

160 time the intensity of direct sun light onto the retina. Be Carefuul working in the lab!