Chapter 23

Electric Potential

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Lectures by Wayne Anderson

Goals for Chapter 23

- To calculate the **electric potential energy** of a **group of charges**
- To know the **significance** of electric potential
- To calculate the **electric potential** due to a collection of charges
- To use **equipotential surfaces** to understand electric potential
- To **calculate** the **electric field using the electric potential**

Introduction

- How is electric potential related to welding?
- Electric potential energy is an integral part of our technological society.
- What is the difference between electric potential and electric potential energy?
- How is electric potential energy related to charge and the electric field?

Gravitational potential energy in a uniform field

- The behavior of a **point charge in a uniform electric field** is analogous to the motion of a **baseball in a uniform gravitational field**.

1) **Work done by a force**

$$W_{a\to b} = \int_a^b \mathbf{F}\bullet d\mathbf{l} = \int_a^b F\cos\theta\, dl$$

2) **Work done by a conservative force**:

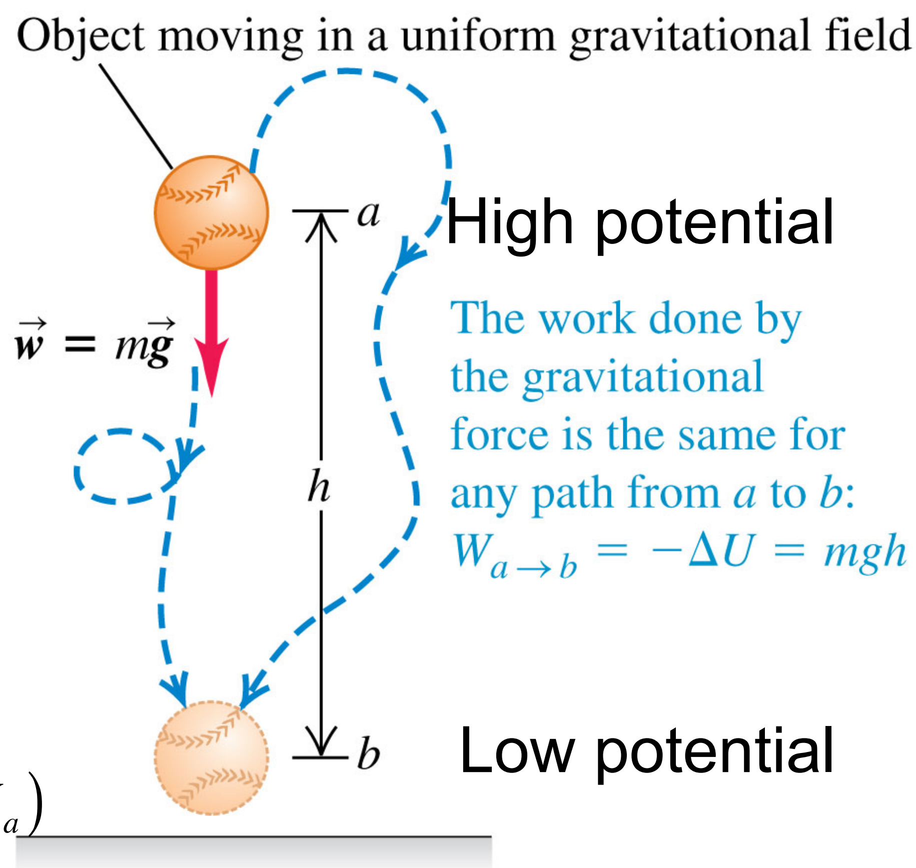
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

is independent of the path

3) **Work - energy theorem**: Change in kinetic energy during a displacement is equal to the total work done on the particle **if all forces are conservative.**

$$K_a + U_a = K_b + U_b \rightarrow K_b - K_a = -(U_b - U_a)$$

$$-(U_b - U_a) = -\Delta U = W_{a\to b}$$



Electric potential energy in a uniform field

- Analogy between **point charge in a uniform electric field** and **baseball in a uniform gravitational field**.

1) **Work done by** the electric **force**

$$W_{a\to b} = \int_a^b \mathbf{F}\cdot d\mathbf{l} = F_e d = q_0 E d$$

2) **Work done by** the electric **force**

$F_y = -q_0 E$ on going from y_a to y_b:

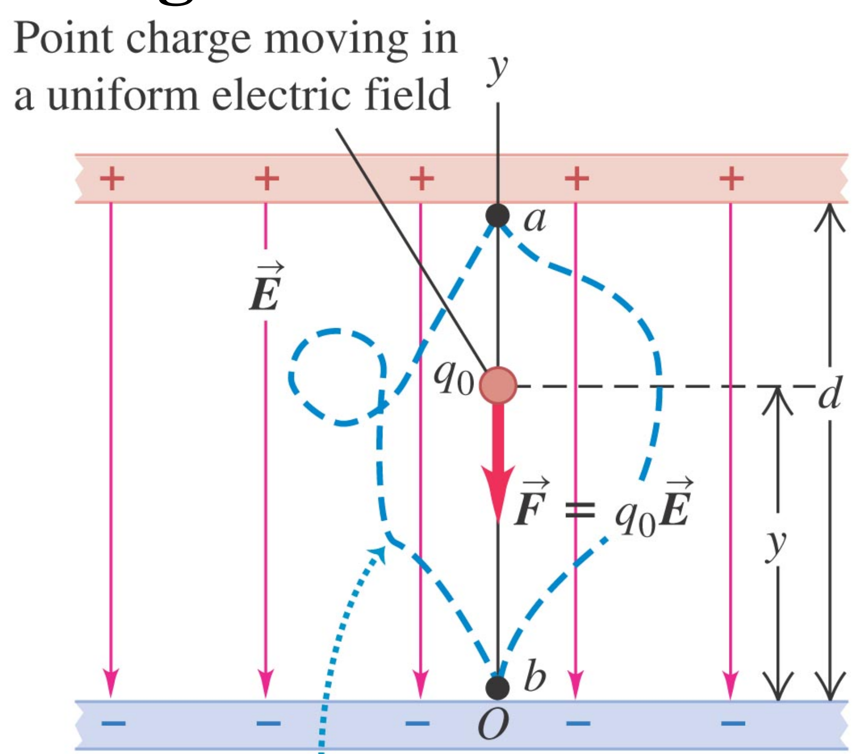
$$W_{a\to b} = q_0 E\left(y_a - y_b\right) = U_a - U_b = -\Delta U$$

is independent of the path

3) **Work - energy theorem**: Change in kinetic energy during a displacement is equal to the total work done on the particle **if all forces are conservative.**

$$K_a + U_a = K_b + U_b \rightarrow K_b - K_a = -\left(U_b - U_a\right)$$

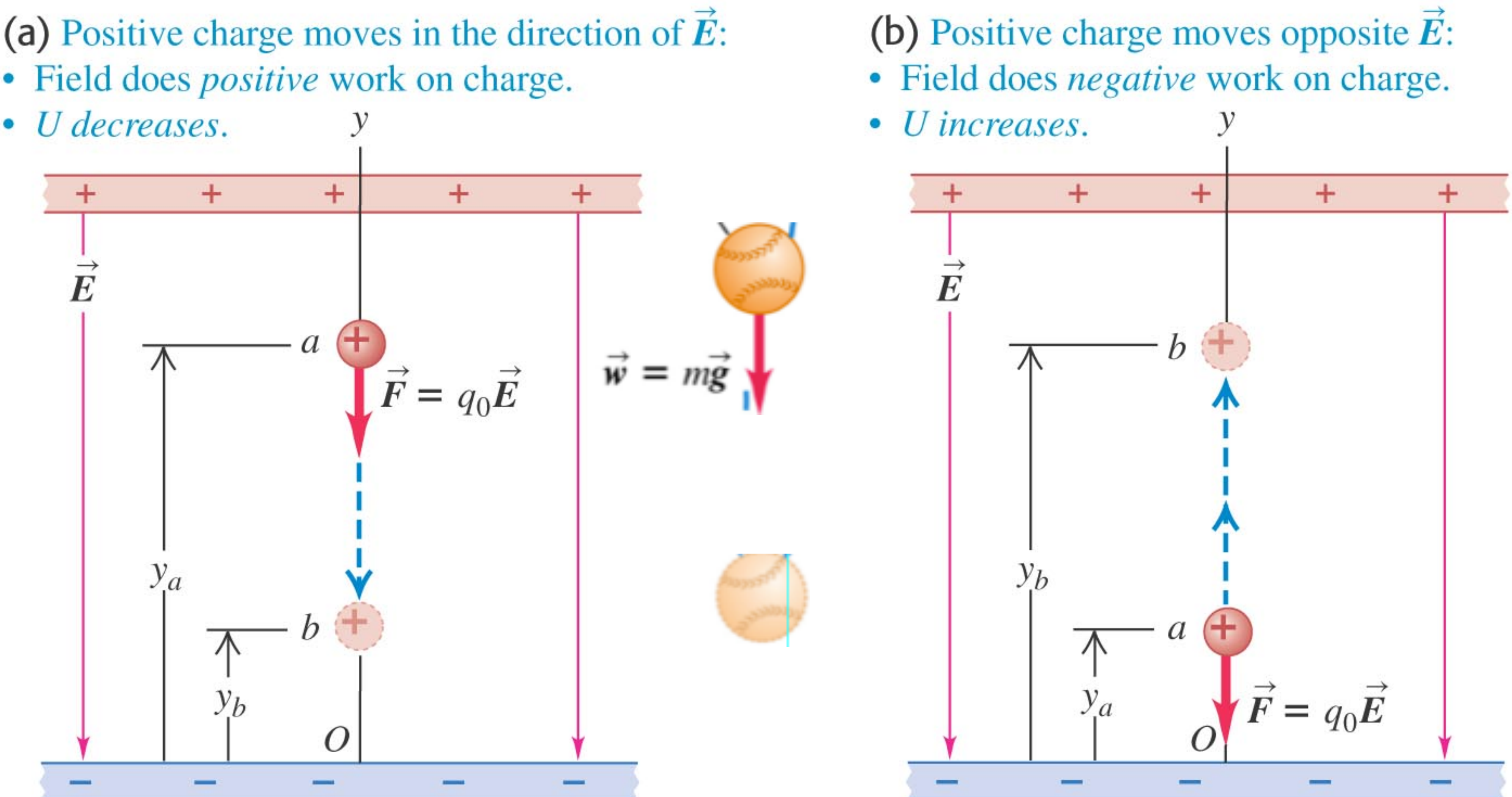
$$-\left(U_b - U_a\right) = -\Delta U = W_{a\to b}$$



The work done by the electric force is the same for any path from a to b: $W_{a\to b} = -\Delta U = q_0 E d$

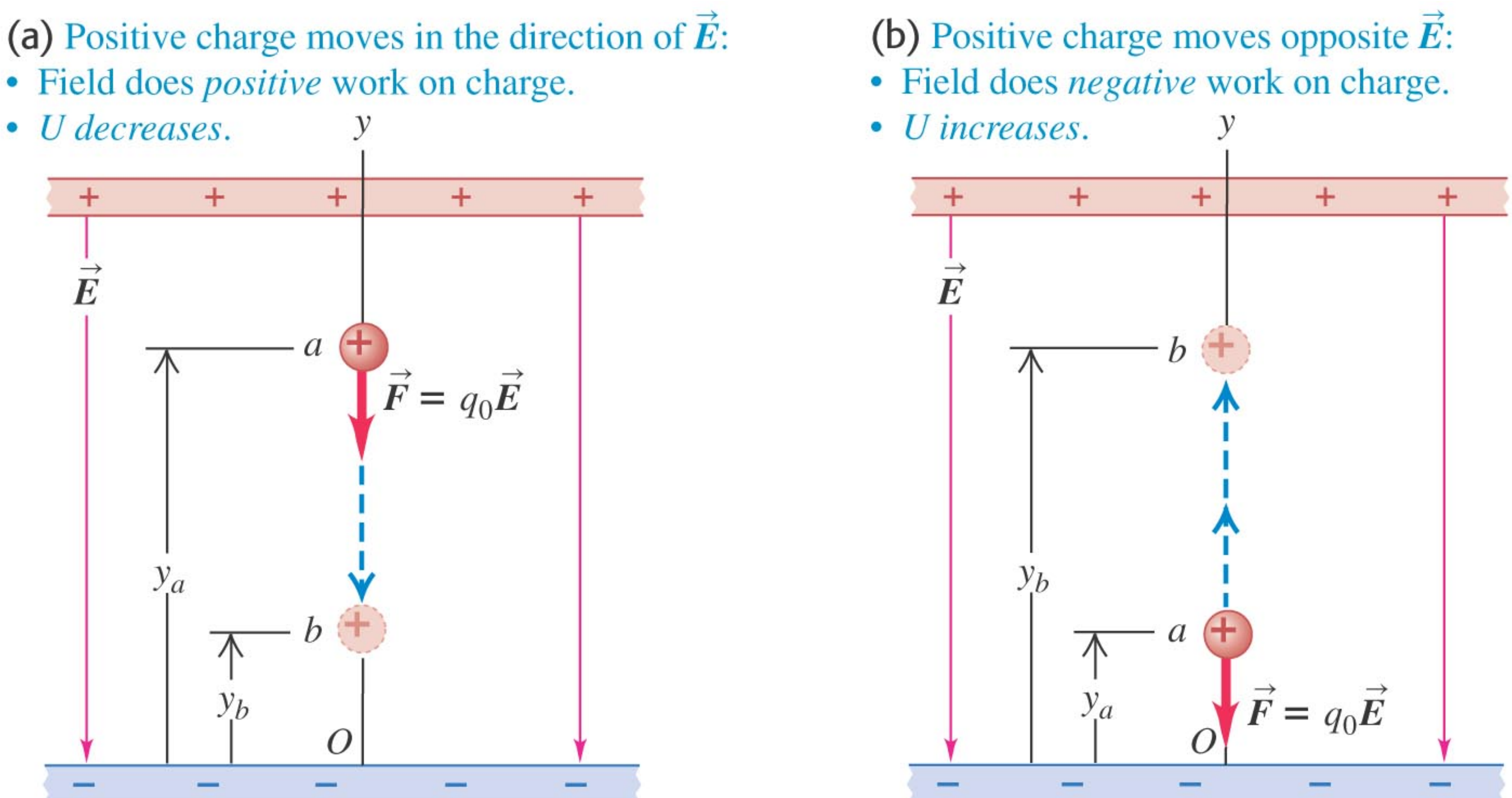
A positive charge moving in a uniform field

- When potential energy of a system of charges increase? When it decreases?



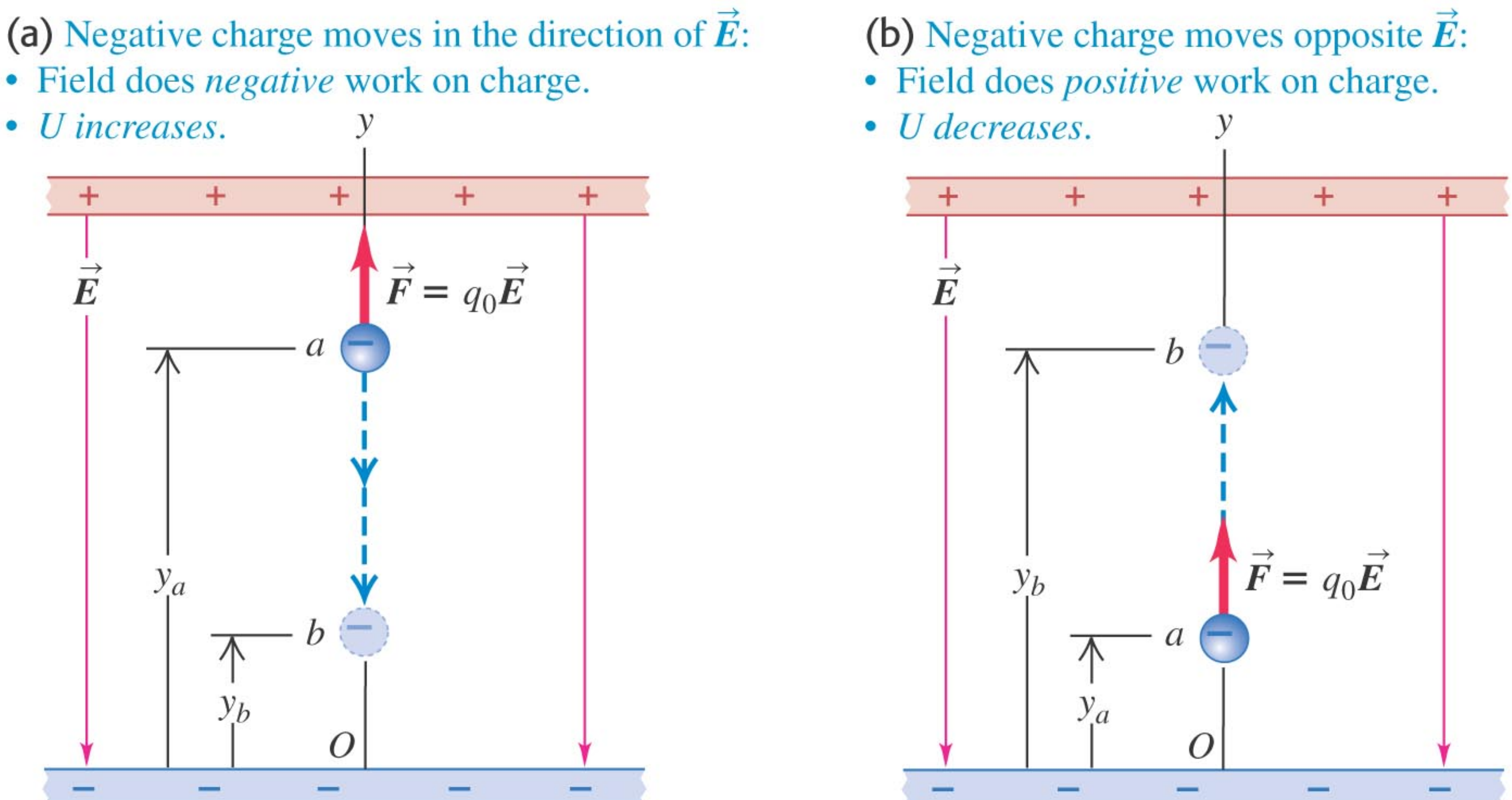
A positive charge moving in a uniform field

- If the **positive charge moves in the direction of the field**, the potential energy ***decreases***.
  If the **positive charge moves opposite the field**, the potential energy ***increases***.



A negative charge moving in a uniform field

- If the **negative charge moves in the direction of the field**, the potential energy ***increases***,
  If the **negative charge moves opposite the field**, the potential energy ***decreases***.



Complicated? Think about investment and potential

- If we have to force the motion then the potential energy increases. **We are investing in the system acquires the potential to pay us back**.
- If the system forces the motion, **it is paying us so the potential to pay decreases**.
- Test yourself: draw the pictures of the last two pages upside down and see if you can still get it right

Electric potential energy of two point charges

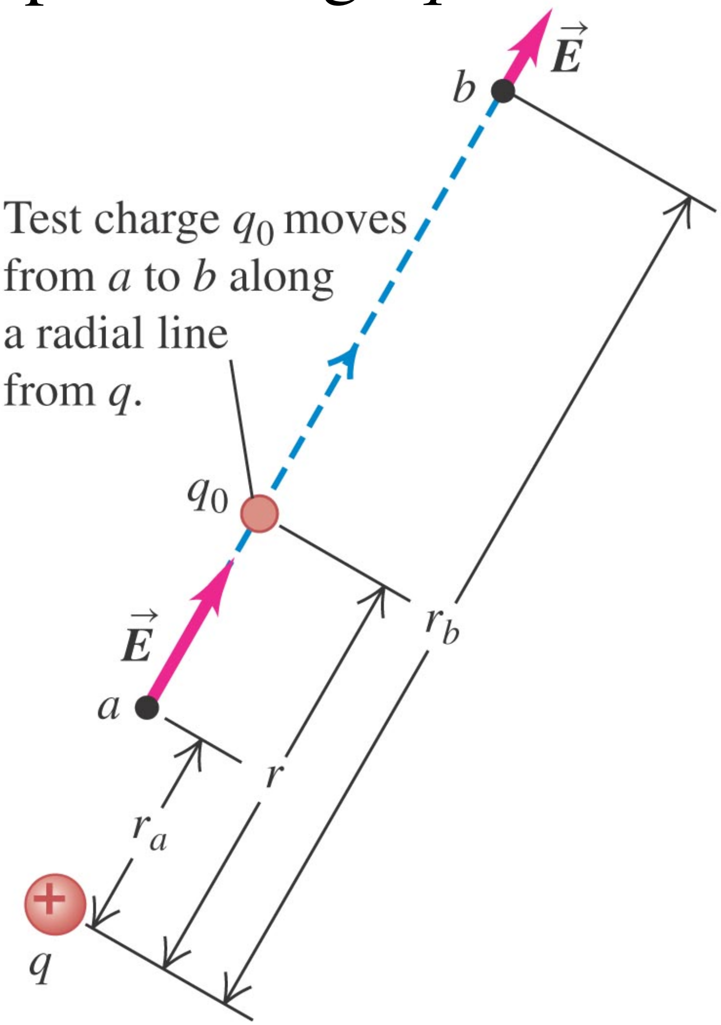
Calculating the work done on a test chrge q_0 moving in the electric field of a single stationary point charge q

$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} \quad \text{(radial force)}$$

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\, dr$$

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Work ony depends on the end points so the path of integration does not matter.



Electric potential energy of two point charges

Calculating the work done on a test chrge q_0 moving in the electric field of a single stationary point charge q

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\mathbf{r} \ \ (\text{radial force})$$

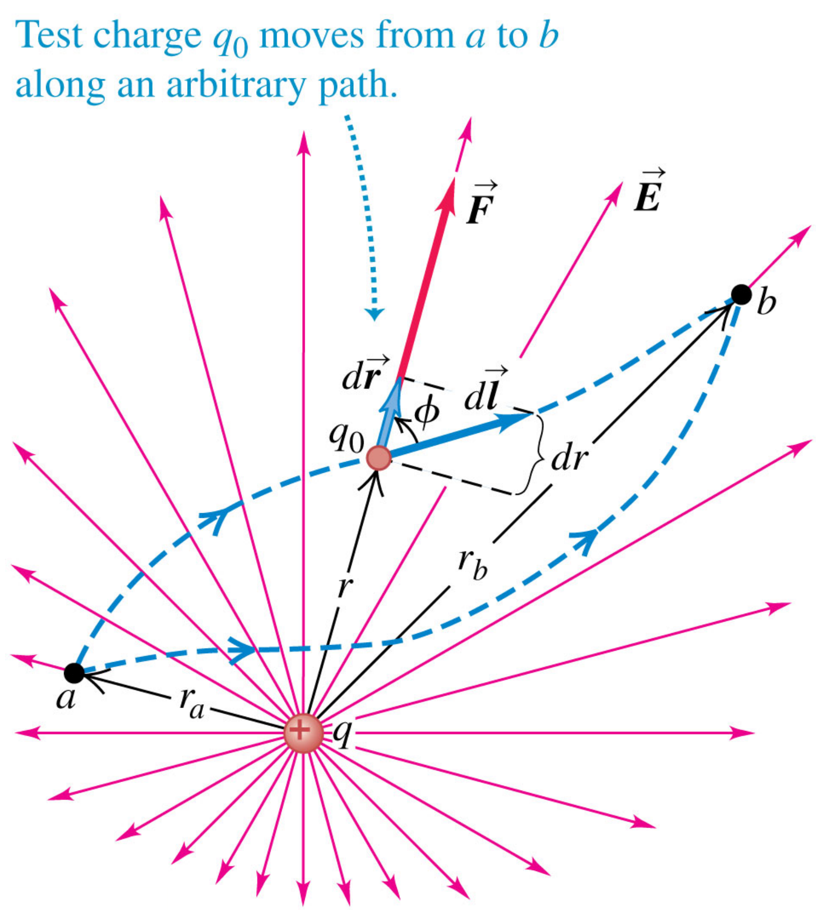
$$W_{a\to b} = \int_{r_a}^{r_b} \mathbf{F}_r \bullet d\mathbf{l} = \int_{r_a}^{r_b} F\cos\phi\, dl$$

$$W_{a\to b} = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cos\phi\, dl$$

$$\cos\phi dl = dr$$

$$W_{a\to b} = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Work ony depends on the end points so the path of integration does not matter.

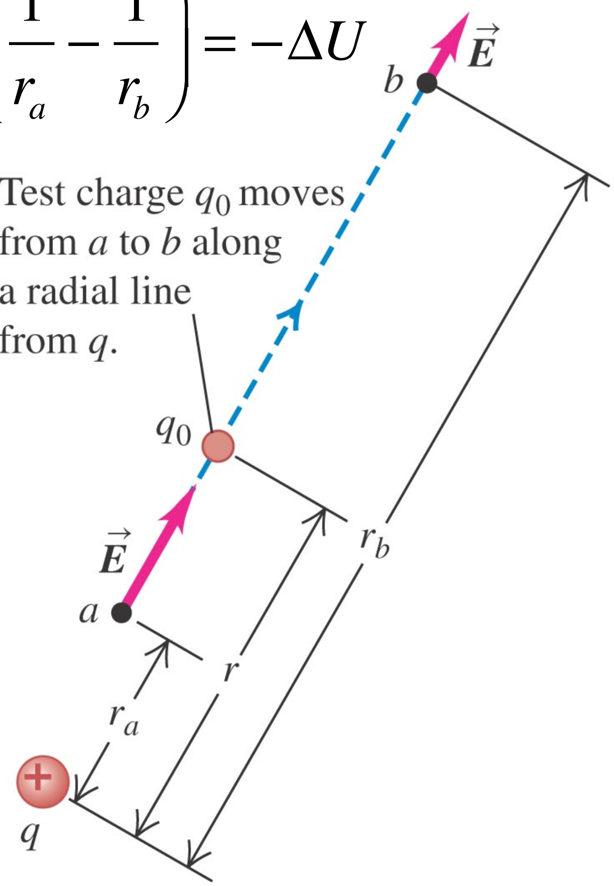


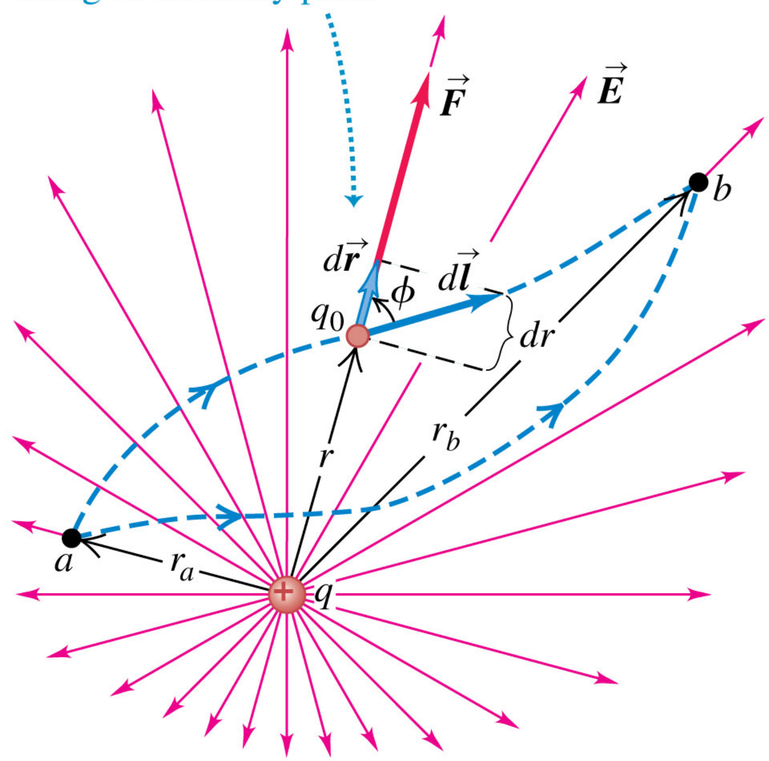
Electric potential energy of two point charges

- The electric potential is the same whether q_0 moves in a radial line (left figure) or along an arbitrary path (right figure).

Because the fore doing the work is a **conservative force**

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = -\Delta U$$





Graphs of the potential energy

Electric potential energy of a point charge q_0 at a distance r from another point charge q.

Energy required to bring the q_0 from infinity to a distnce r around the q

$$W_{a\to b} = \underbrace{\frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_a}}_{\substack{\text{Potential energy} \\ \text{when } q_0 \text{ is at} \\ \text{point a distance } r_a \\ \text{from the charge q}}} - \underbrace{\frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_b}}_{\substack{\text{Potential energy} \\ \text{when } q_0 \text{ is at} \\ \text{point b distance } r_b \\ \text{from the charge q}}} = U_a - U_b$$

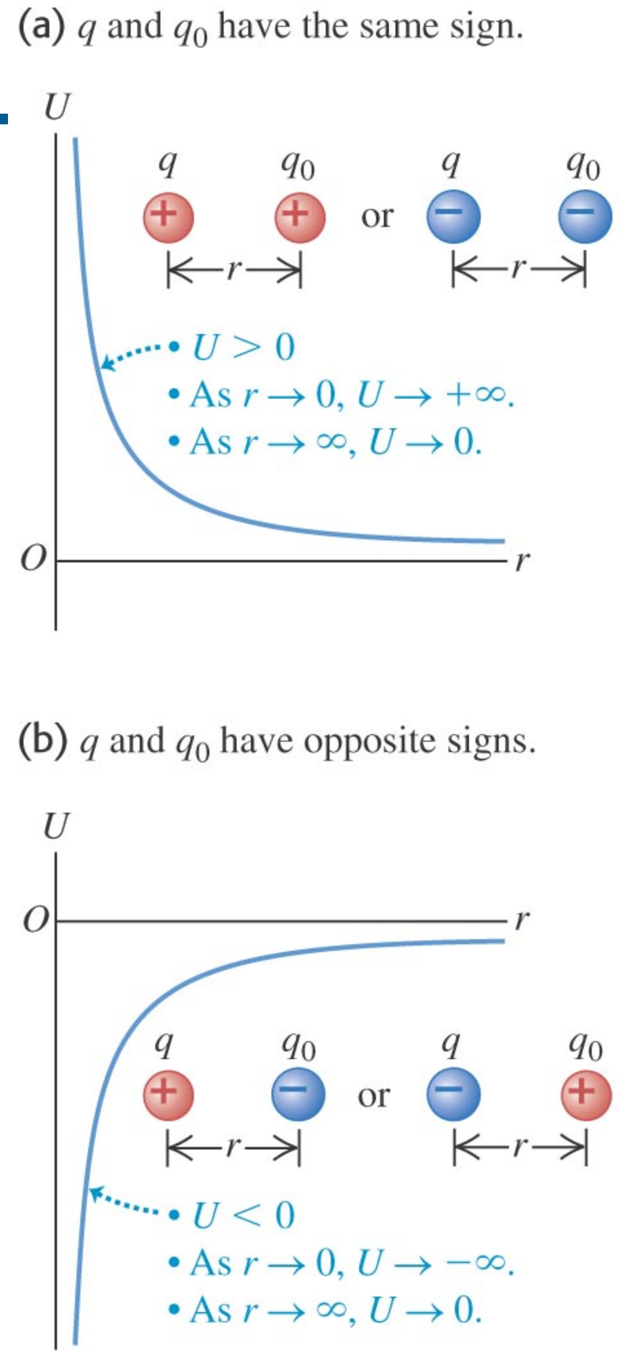
Then the potential energy when the test charge is at any distance r from the charge q is: $U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$

Graphs of the potential energy

- The sign of the potential energy depends on the signs of the two charges.

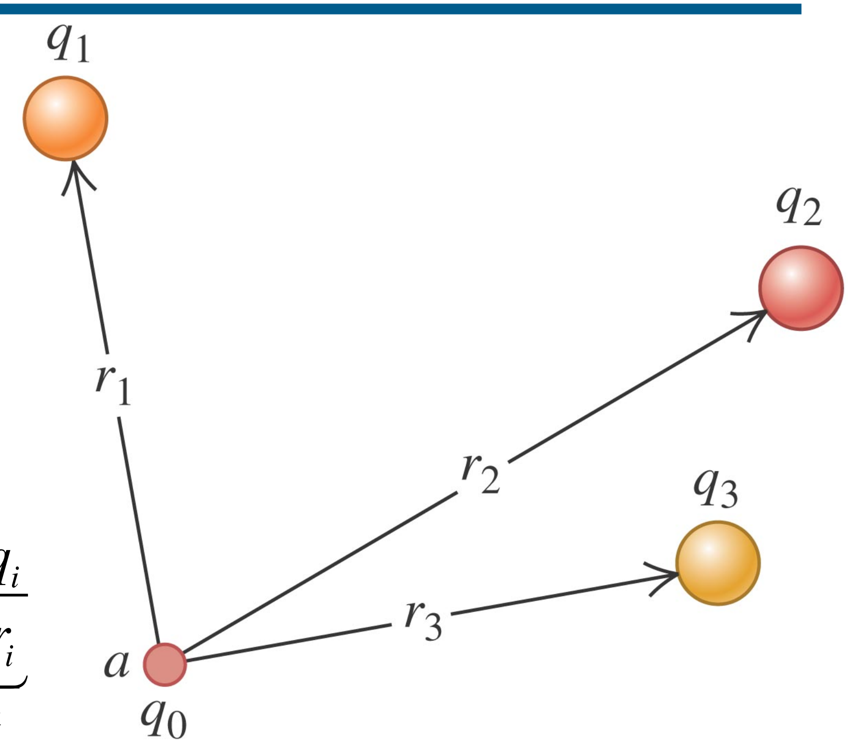
The potential energy when a test charge q_0 is at any distance r from the charge q:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$



Electrical potential with several point charges

- The potential energy associated with q_0 depends on the other charges and their distances from q_0, as shown in figure at the right.



The potential energy of a collection

of charges around the test charge q_0

$$\underbrace{U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right) = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$$

IF we want to bring all of those charges around the test charge without considering the interaction of the charges with each other

Potential energy requred to assemble a group

of charges or to bring them from infinity to their current location:

$$\underbrace{U = \frac{q_0}{4\pi\varepsilon_0}\sum_{i<j} \frac{q_i q_j}{r_{ij}}}$$

This what we invest. All the mutual interactions are taken into account

We impose the condition i<j to avoid counting each charge twice.

Example: A system of point charges

Two point charges are located on the x-axis, $q_1 = \text{-}e$ at $x = 0$ and $q_2 = +e$ at $x = a$.

a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$

b) Find the total potential energy of the system of three charges.

Electric potential

- ***Electric potential*** is ***potential energy per unit charge***.

$$V = \frac{U}{q_0} \rightarrow \textit{f}\text{or a point charge:} V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\text{for a charge distribution: } V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

- We can think of the potential difference between points *a* and *b* in either of two ways. The potential of *a* with respect to *b* ($V_{ab} = V_a - V_b$) equals:
  - ✓ the work done by the electric force when a *unit* charge moves from *a* to *b*. $W_{ab} = -\Delta U = -(U_b - U_a)/q_0$
  - ✓ the work that must be done to move a *unit* charge slowly from *b* to *a* against the electric force.

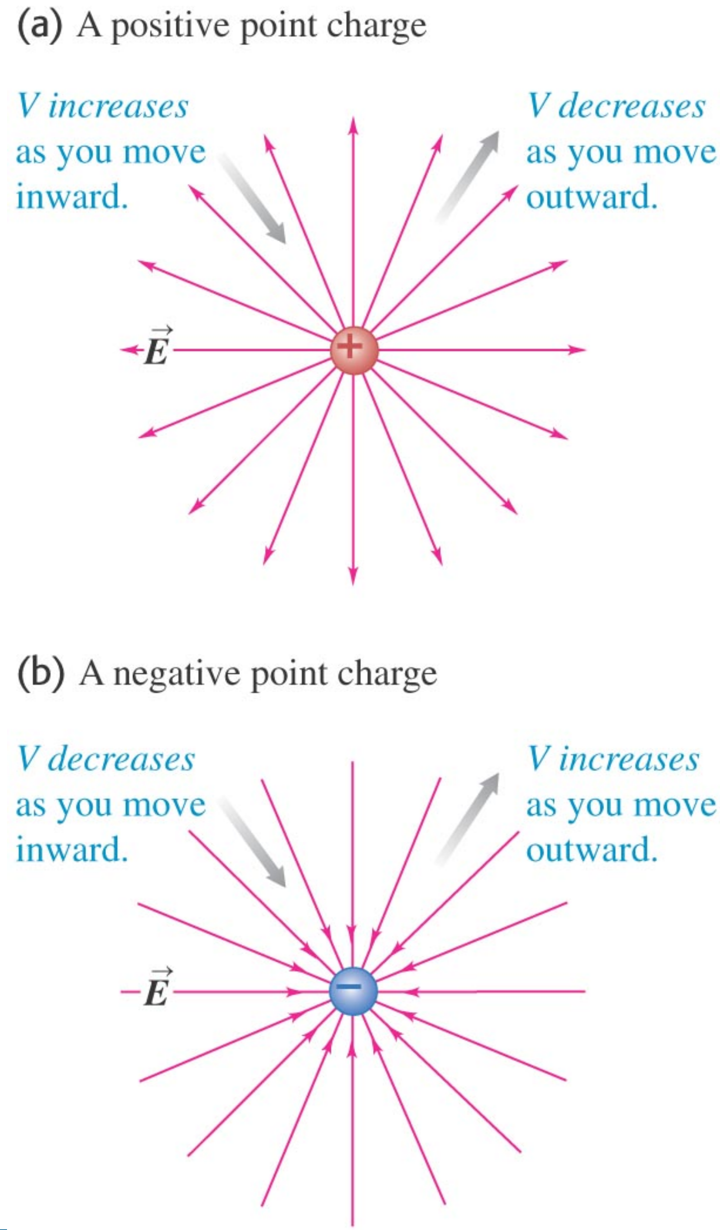
Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*

$$W_{a\rightarrow b} = \int_a^b \mathbf{F}\bullet d\mathbf{l} = \int_a^b q_0 \mathbf{E}\bullet d\mathbf{l}$$

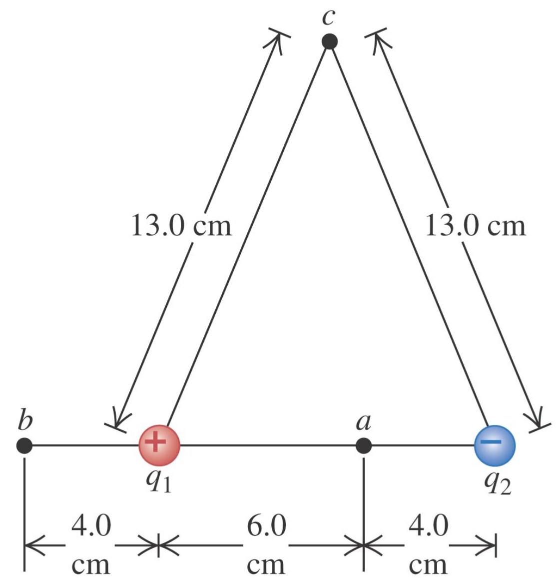
Potential difference between points a and b

$$V_a - V_b = \int_a^b \mathbf{E}\bullet d\mathbf{l} = \int_a^b E\cos\phi\, dl$$



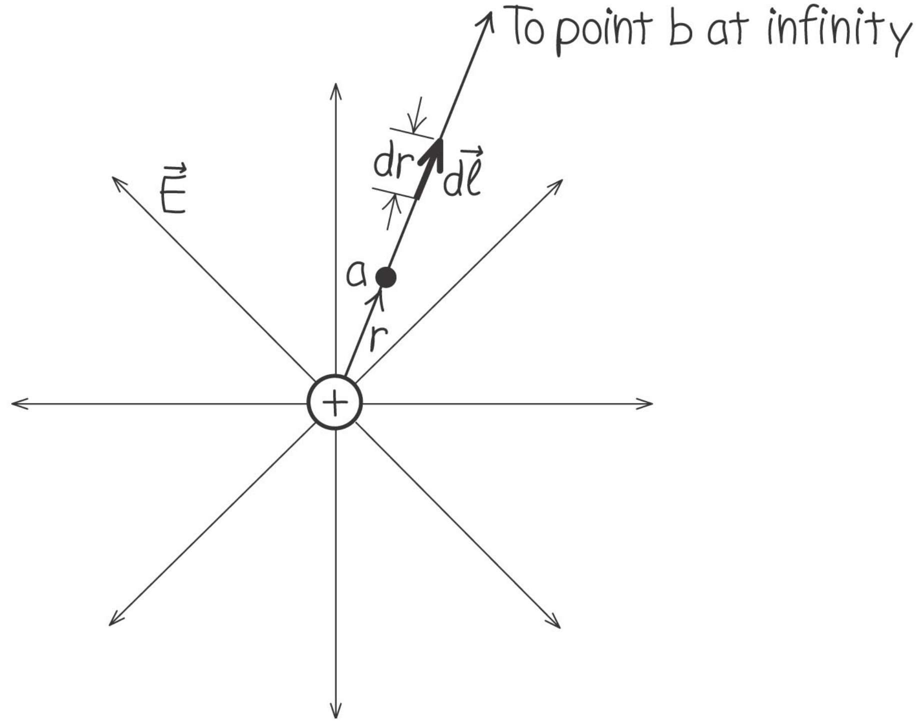
Potential due to two point charges

- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



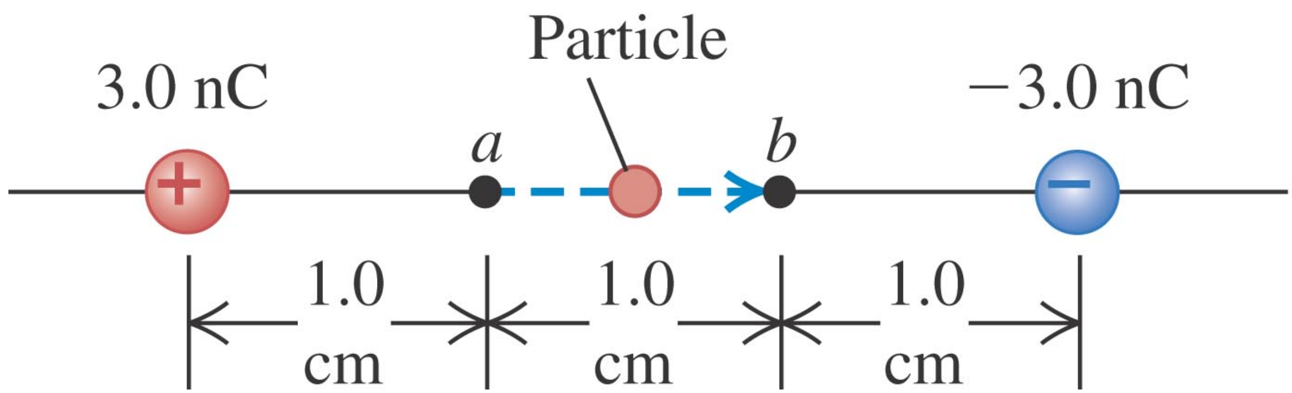
Finding potential by integration

- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



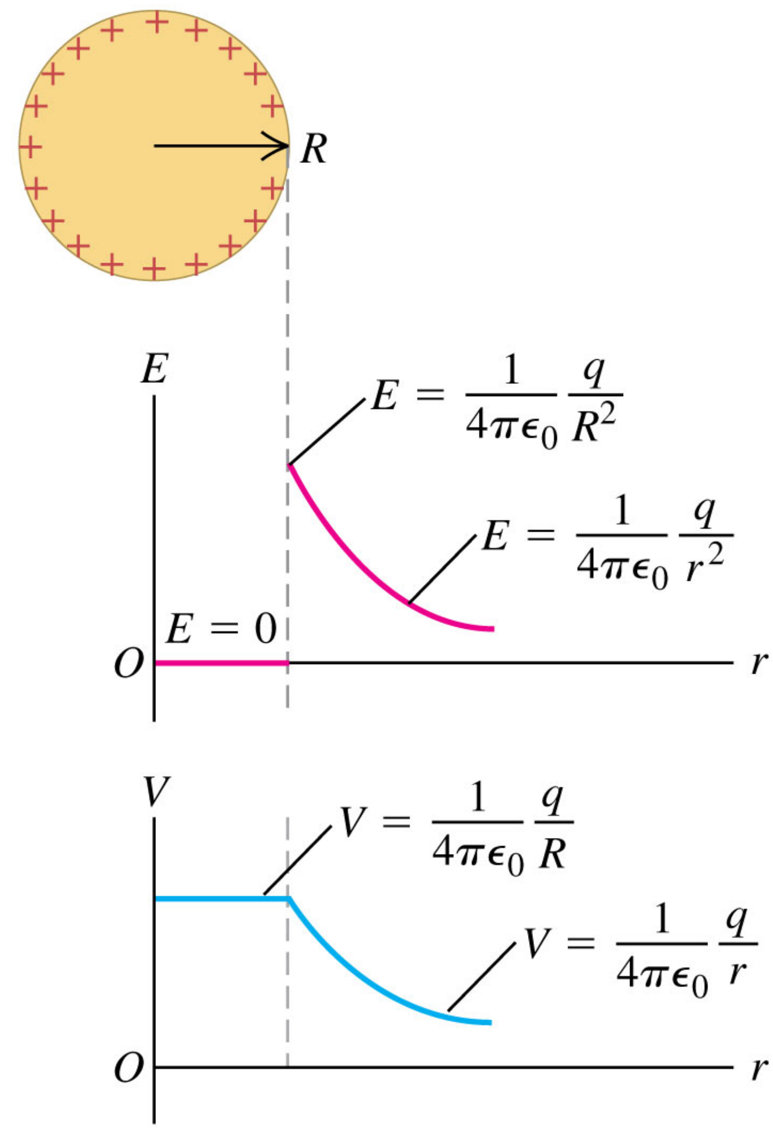
Moving through a potential difference

- Example 23.7 combines electric potential with energy conservation. Follow this example using Figure 23.15 below.



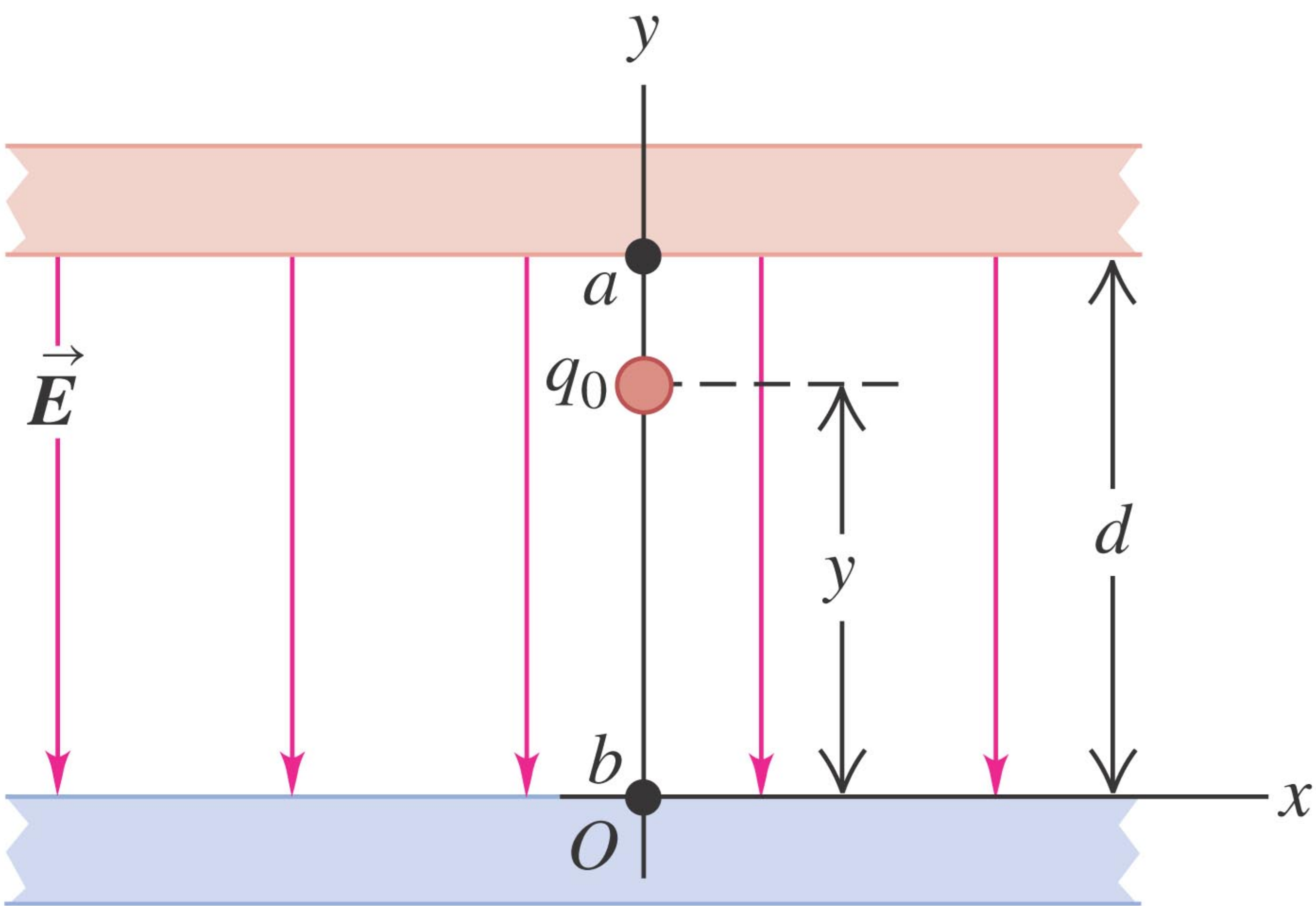
Calculating electric potential

- Read Problem-Solving Strategy 23.1.
- Follow Example 23.8 (a charged conducting sphere) using Figure 23.16 at the right.



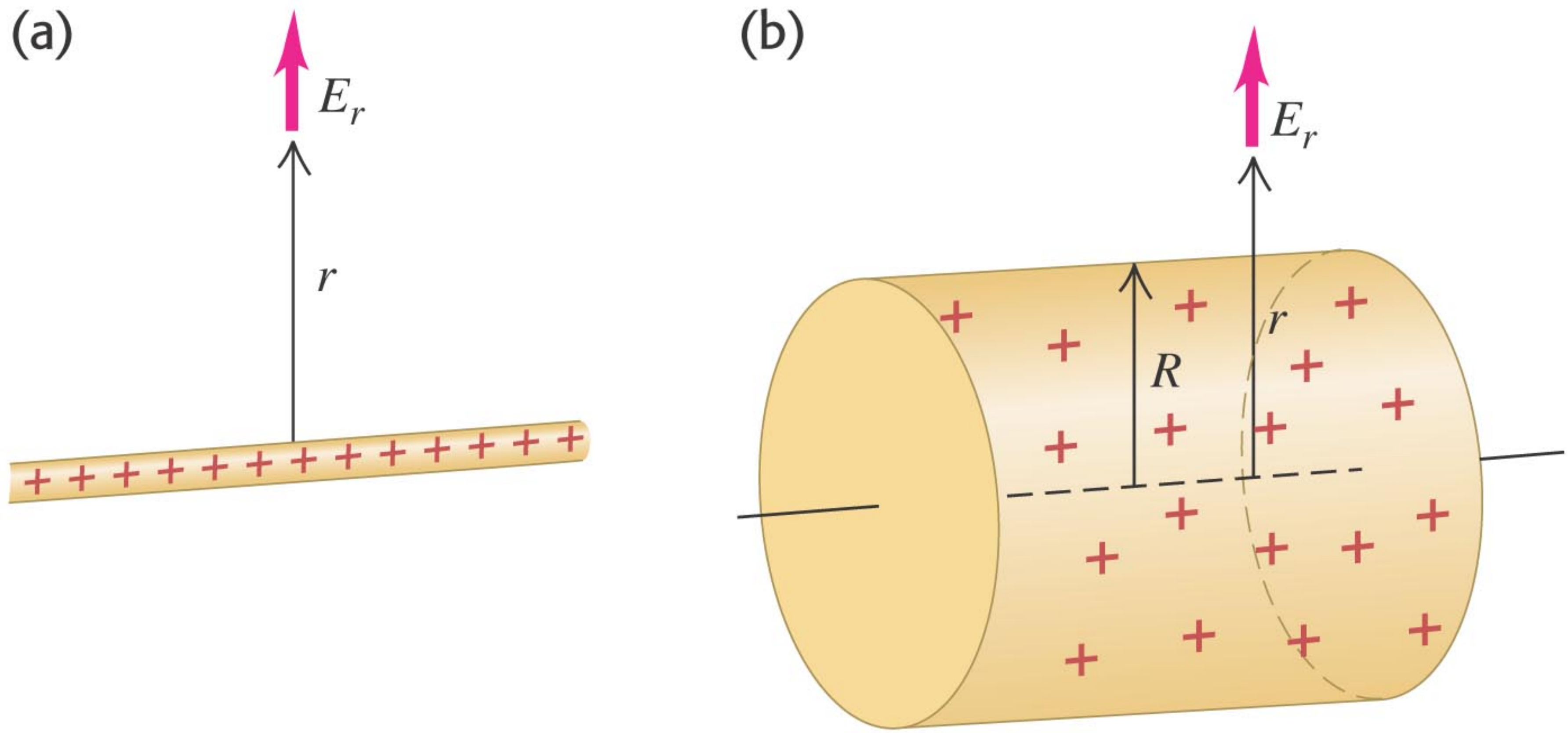
Oppositely charged parallel plates

- Follow Example 23.9 using Figure 23.18 below.



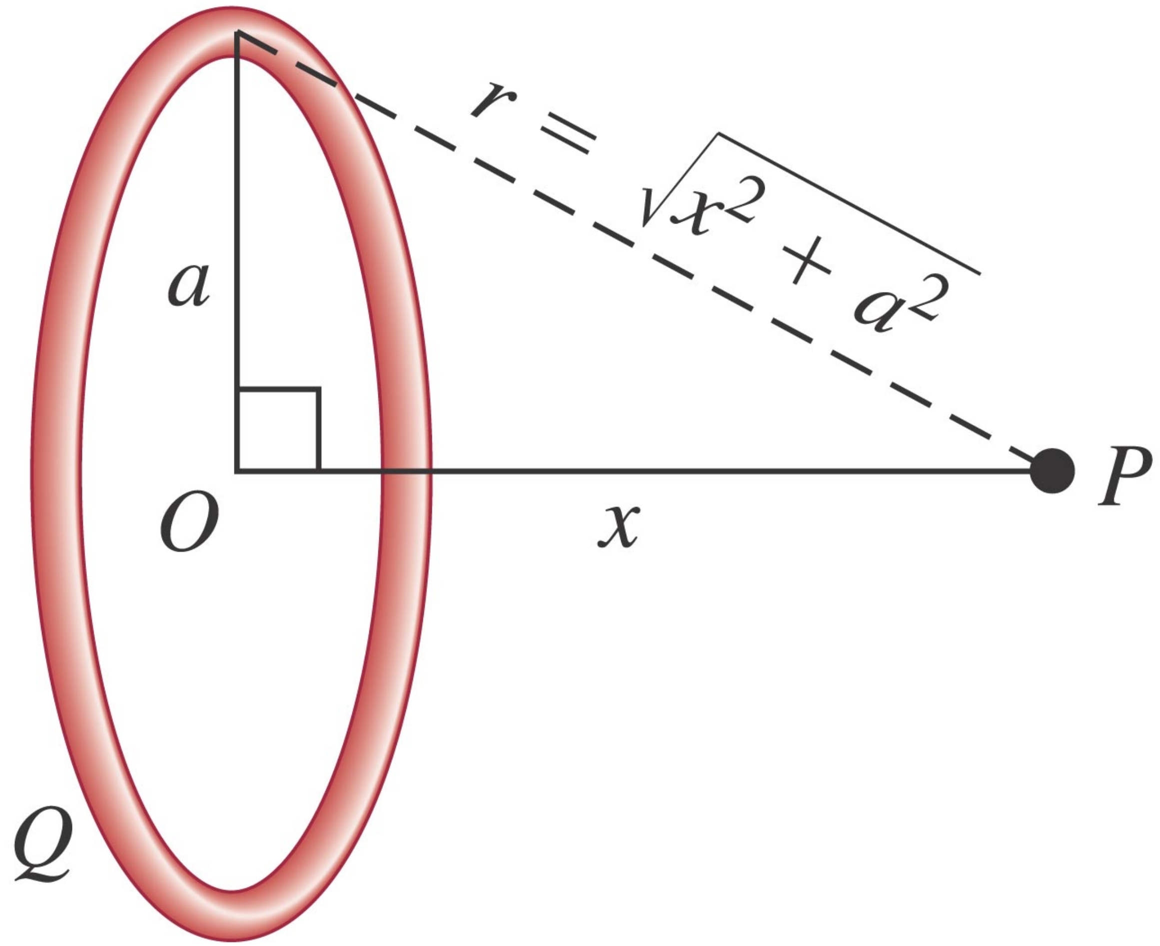
An infinite line charge or conducting cylinder

- Follow Example 23.10 using Figure 23.19 below.



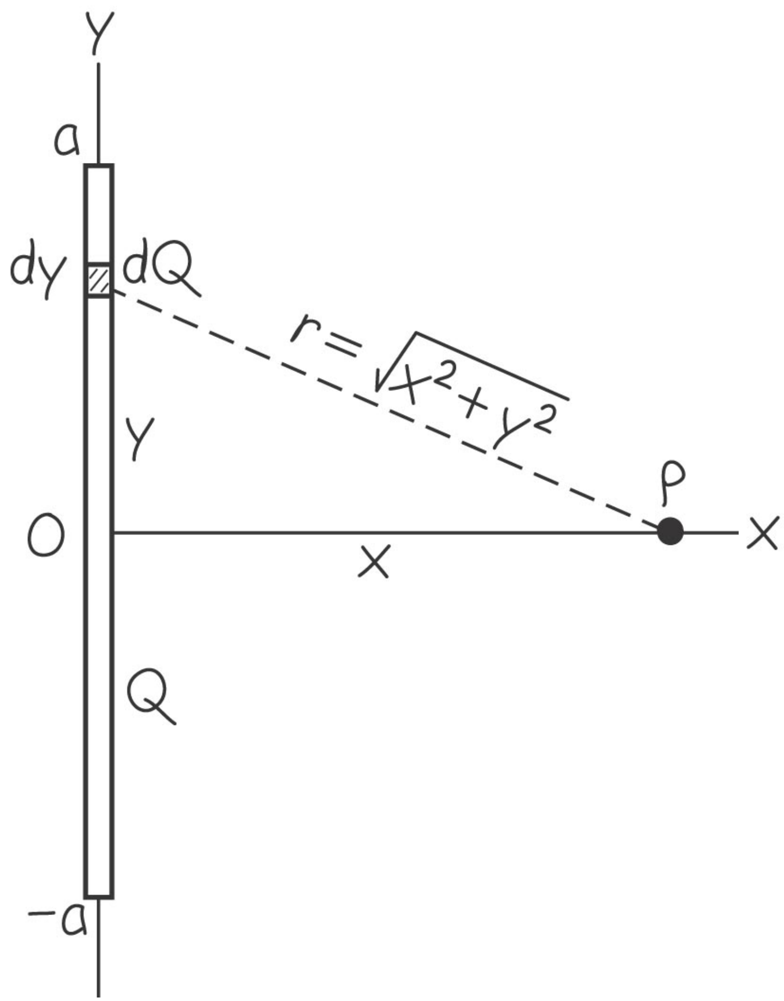
A ring of charge

- Follow Example 23.11 using Figure 23.20 below.



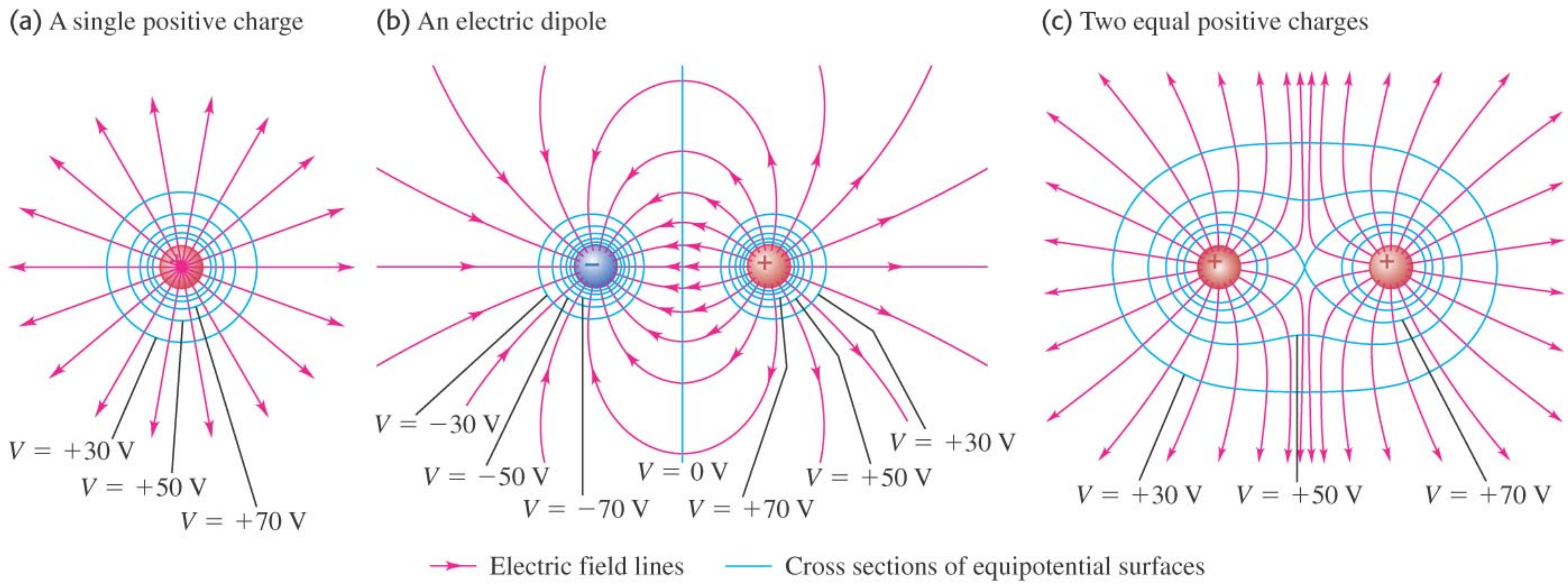
A finite line of charge

- Follow Example 23.12 using Figure 23.21 below.



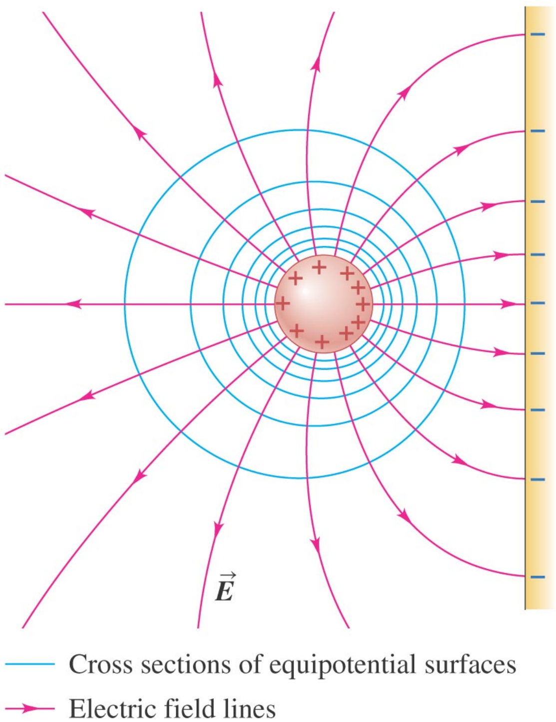
Equipotential surfaces and field lines

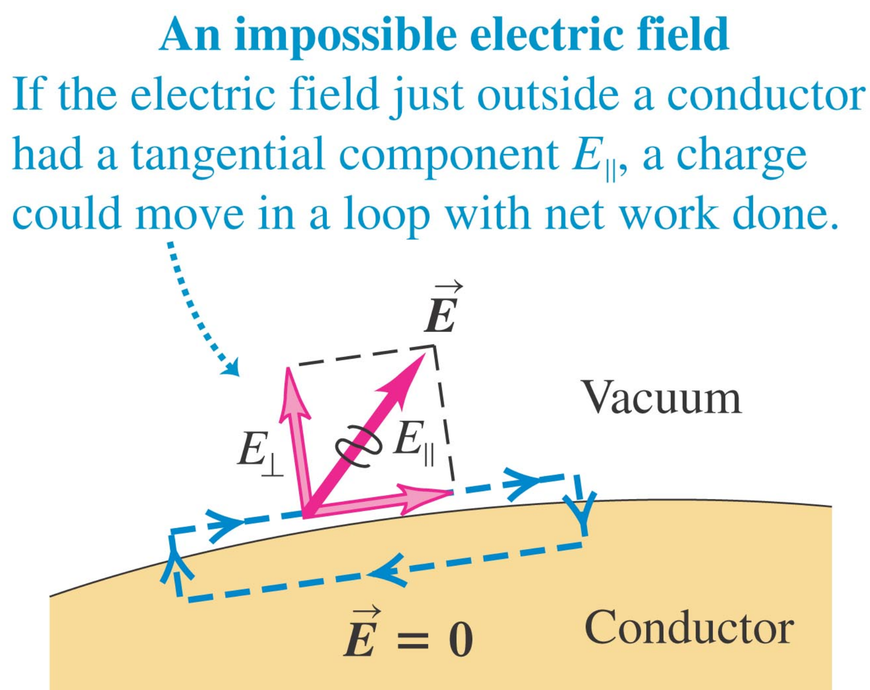
- An *equipotential surface* is a surface on which the electric potential is the same at every point.
- Figure 23.23 below shows the equipotential surfaces and electric field lines for assemblies of point charges.
- Field lines and equipotential surfaces are always mutually perpendicular.



Equipotentials and conductors

- When all charges are at rest:
  - ✓ the surface of a conductor is always an equipotential surface.
  - ✓ the electric field just outside a conductor is always perpendicular to the surface (see figures below).
  - ✓ the entire solid volume of a conductor is at the same potential.





Potential gradient

- Read in the text the discussion of *potential gradient*.
- Follow Example 23.13 which looks at a point charge.
- Follow Example 23.14 which deals with a ring of charge.

Q23.1

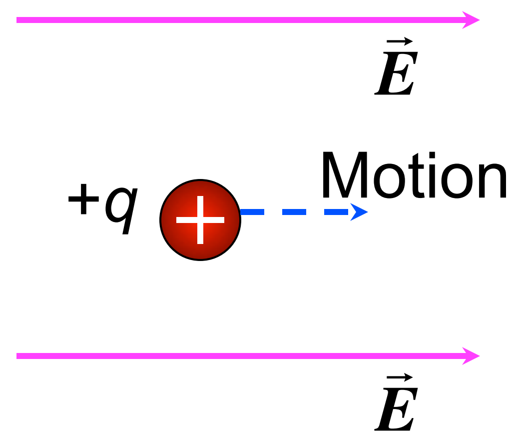
When a positive charge moves in the direction of the electric field,

A. the field does positive work on it and the potential energy increases.

B. the field does positive work on it and the potential energy decreases.

C. the field does negative work on it and the potential energy increases.

D. the field does negative work on it and the potential energy decreases.



Q23.2

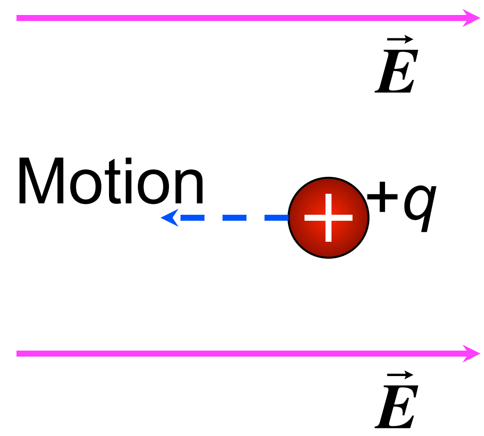
When a positive charge moves opposite to the direction of the electric field,

A. the field does positive work on it and the potential energy increases.

B. the field does positive work on it and the potential energy decreases.

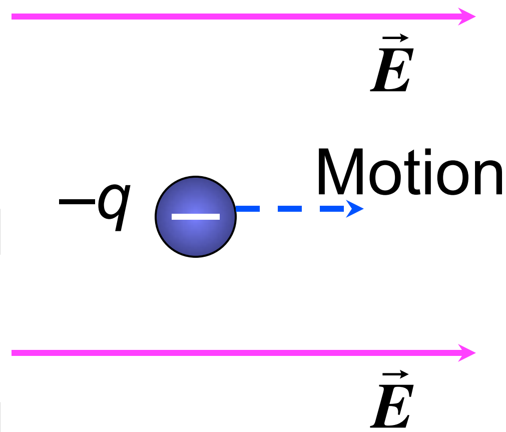
C. the field does negative work on it and the potential energy increases.

D. the field does negative work on it and the potential energy decreases.



Q23.3

When a negative charge moves in the direction of the electric field,



A. the field does positive work on it and the potential energy increases.

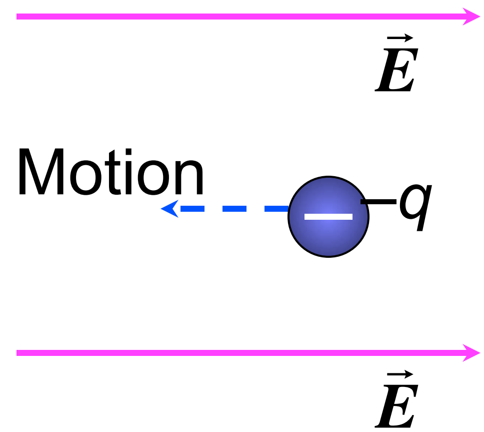
B. the field does positive work on it and the potential energy decreases.

C. the field does negative work on it and the potential energy increases.

D. the field does negative work on it and the potential energy decreases.

Q23.4

When a negative charge moves opposite to the direction of the electric field,



A. the field does positive work on it and the potential energy increases.

B. the field does positive work on it and the potential energy decreases.

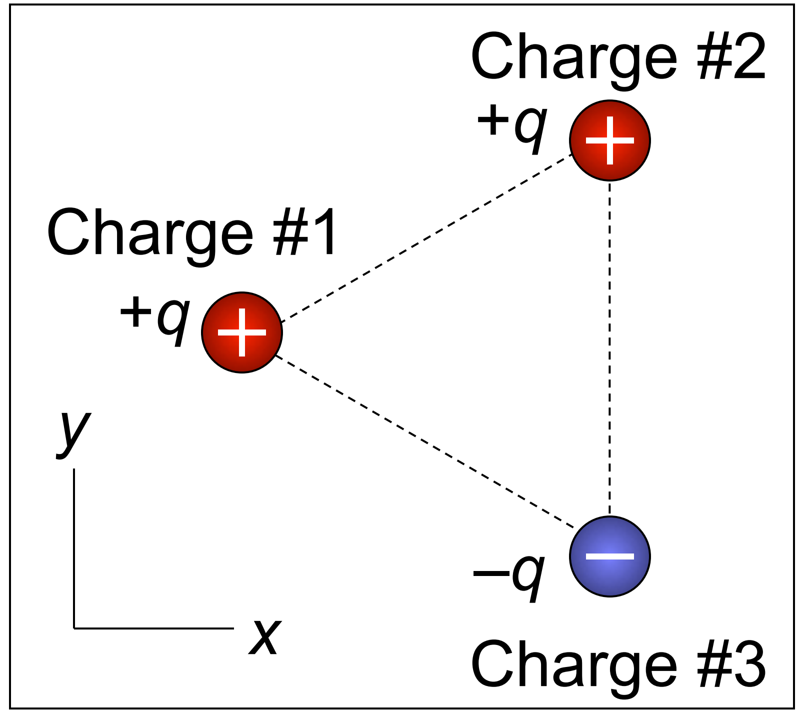
C. the field does negative work on it and the potential energy increases.

D. the field does negative work on it and the potential energy decreases.

Q23.5

The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of the system of three charges is



A. positive.

B. B. negative.

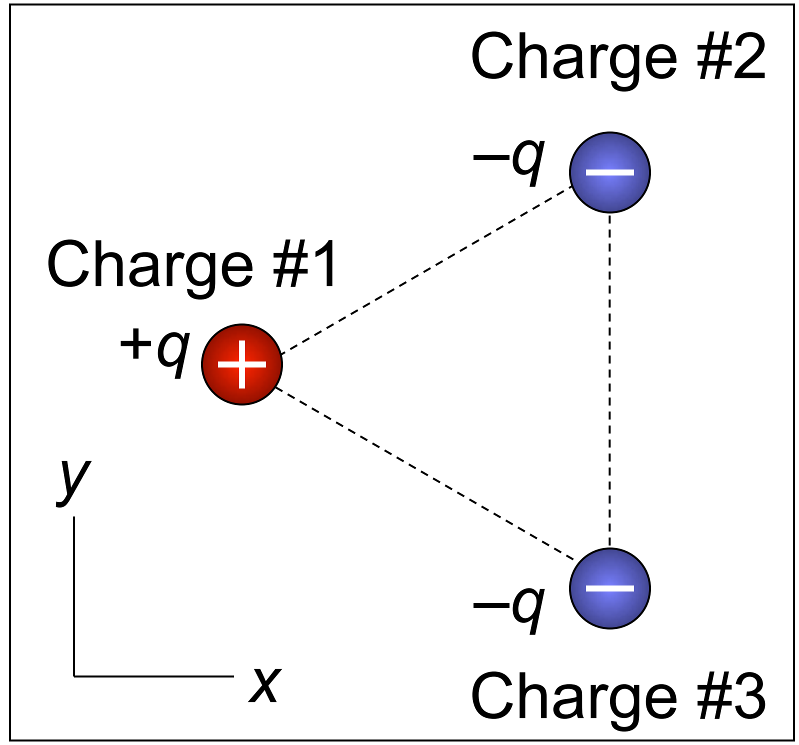
C. zero.

D. not enough information given to decide

Q23.6

The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of the system of three charges is



A. positive.

B. B. negative.

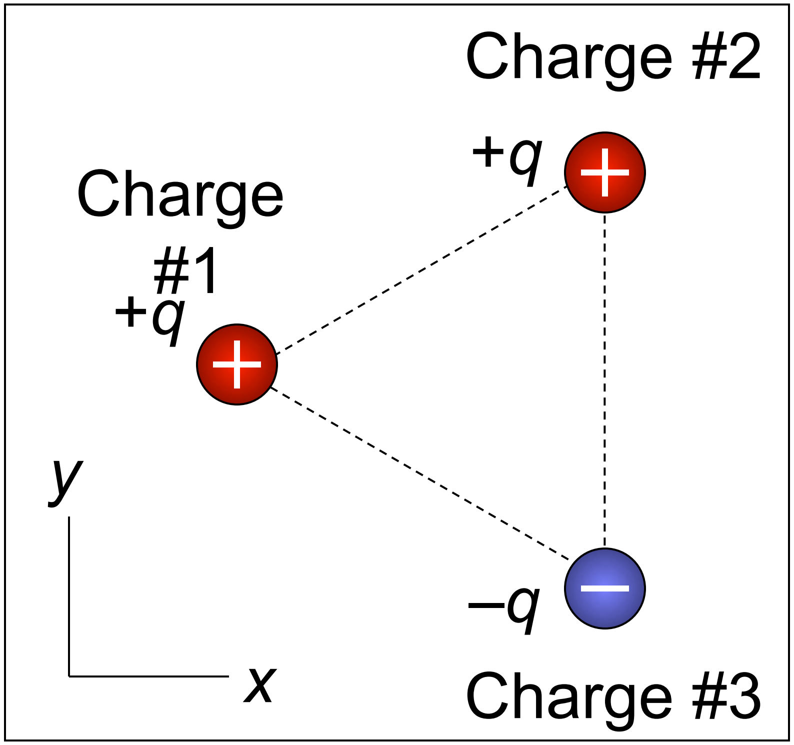
C. zero.

D. not enough information given to decide

Q23.7

The electric potential due to a point charge approaches zero as you move farther away from the charge.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is



A. positive.

B. B. negative.

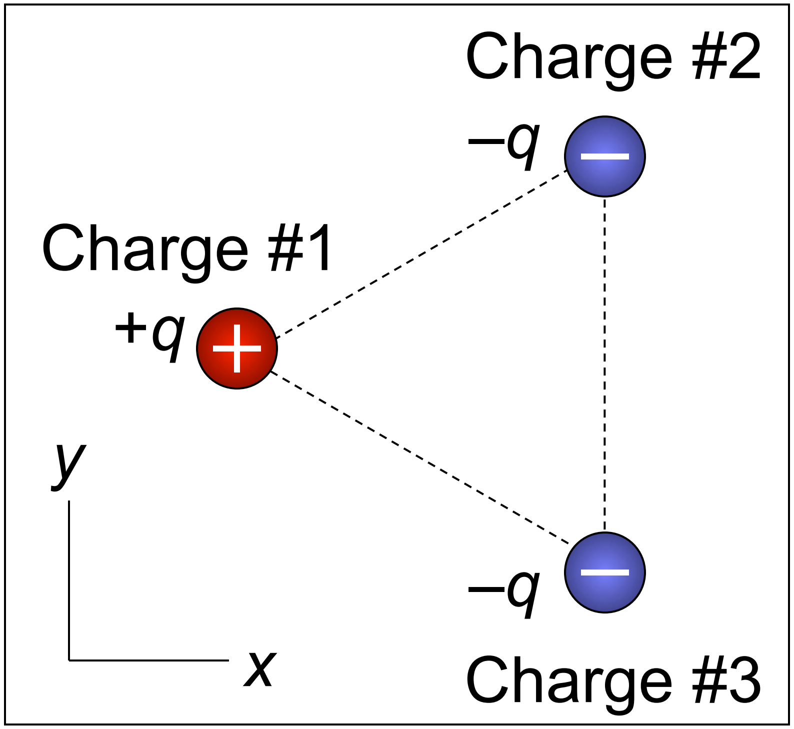
C. zero.

D. not enough information given to decide

Q23.8

The electric potential due to a point charge approaches zero as you move farther away from the charge.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is



A. positive.

B. B. negative.

C. zero.

D. not enough information given to decide

Q23.9

Consider a point *P* in space where the electric potential is zero. Which statement is correct?

A. A point charge placed at *P* would feel no electric force.

B. The electric field at points around *P* is directed toward *P*.

C. The electric field at points around *P* is directed away from *P*.

D. none of the above

E. not enough information given to decide

Q23.10

Where an electric field line crosses an equipotential surface, the angle between the field line and the equipotential is

A. zero.

B. between zero and 90°.

C. 90°.

D. not enough information given to decide

Q23.11

The direction of the electric potential gradient at a certain point

A. is the same as the direction of the electric field at that point.

B. is opposite to the direction of the electric field at that point.

C. is perpendicular to the direction of the electric field at that point.

D. not enough information given to decide