Chapter 24

Capacitance and Dielectrics

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 24

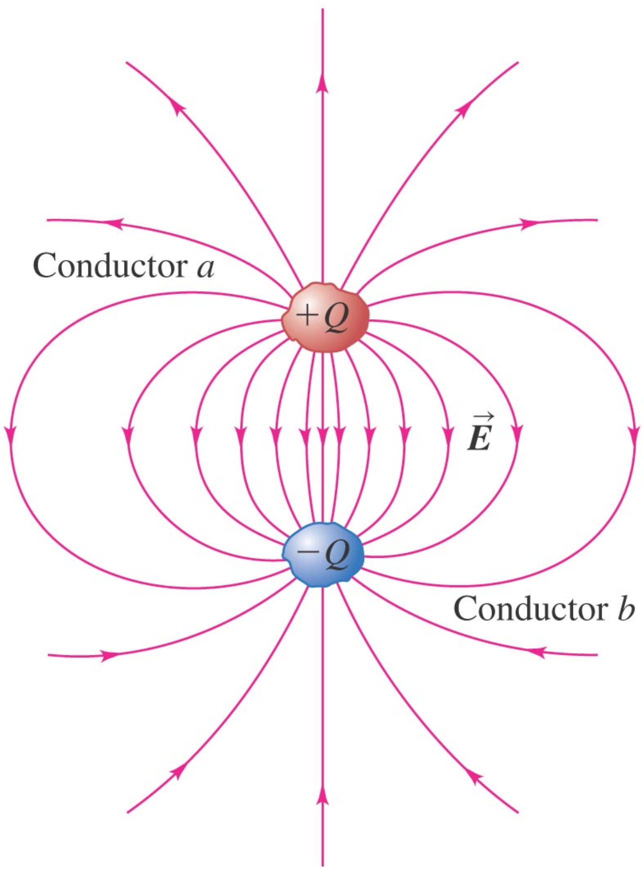
- To understand capacitors and calculate capacitance
- To analyze networks of capacitors
- To calculate the energy stored in a capacitor
- To examine dielectrics and how they affect capacitance

Introduction

- How does a camera's flash unit store energy?
- Capacitors are devices that store electric potential energy.
- The energy of a capacitor is actually stored in the electric field.
- Concept of capacitor is closely related to the idea of potential and potential energy

Capacitors and capacitance

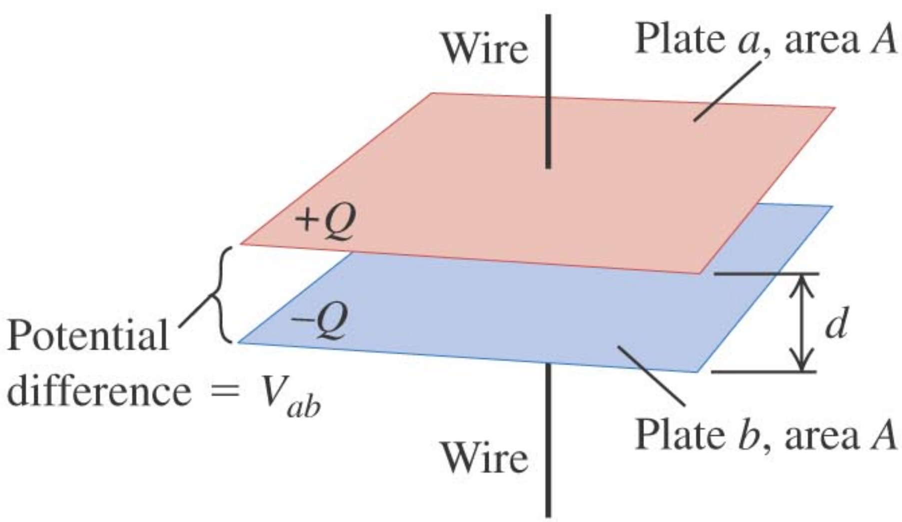
- Any two conductors separated by an insulator form a *capacitor*, as illustrated in Figure 24.1 below.
- The definition of capacitance is $C = Q/V_{ab}$.



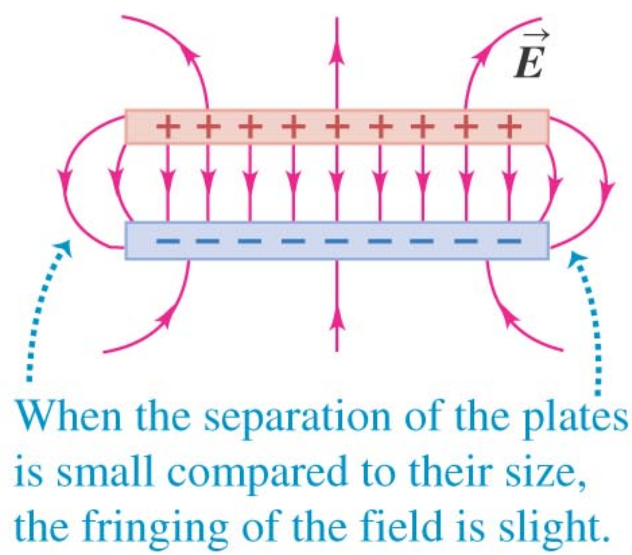
Parallel-plate capacitor

- A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions. (See Figure 24.2 below.)
- The capacitance of a parallel-plate capacitor is: $C = \varepsilon_0 \dfrac{A}{d}$

(a) Arrangement of the capacitor plates

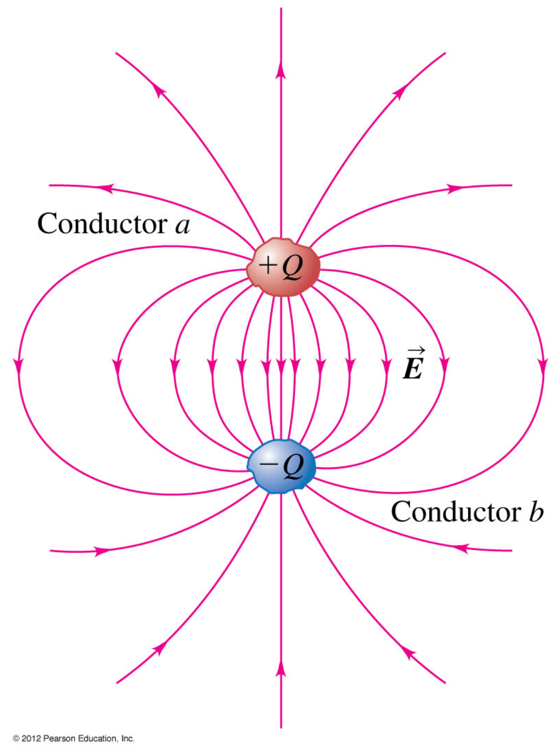


(b) Side view of the electric field $\vec{E}$



Q24.1

The two conductors *a* and *b* are insulated from each other, forming a capacitor. You increase the charge on *a* to $+2Q$ and increase the charge on *b* to $-2Q$, while keeping the conductors in the same positions.



As a result of this change, the capacitance *C* of the two conductors

A. becomes 4 times great.

B. B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

Q24.2

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $–Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

A. V_{ab} becomes 4 times as great.

B. V_{ab} becomes twice as great.

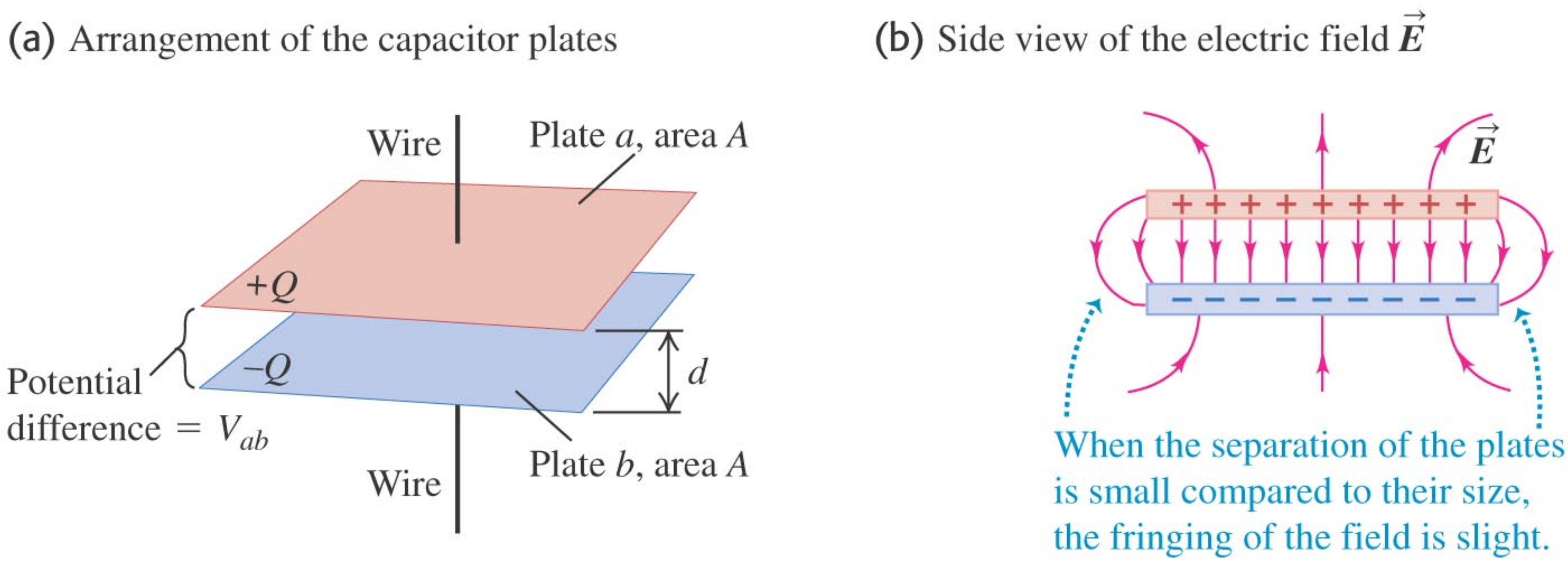
C. V_{ab} remains the same.

D. V_{ab} becomes 1/2 as great.

E. V_{ab} becomes 1/4 as great.

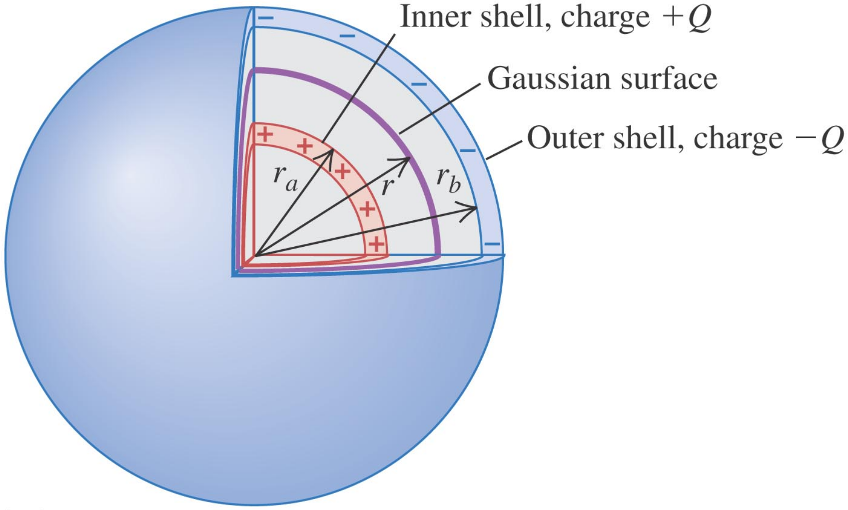
Parallel-plate capacitor

- 24.1 A parallel plate capacitor has a capacitance of 1.0 F if the plates are 1.0mm apart, what is the area of the plates?
- 24.2 The plates of a parallel-plate capacitor in vacuum are 5.00 mm apart and 2.00m^2 in area. A potential difference of 10KV is applied across the capacitor. Compute: a) the capacitance; b) the charge on each plate; c) the magnitude of the electric field in the space between them.



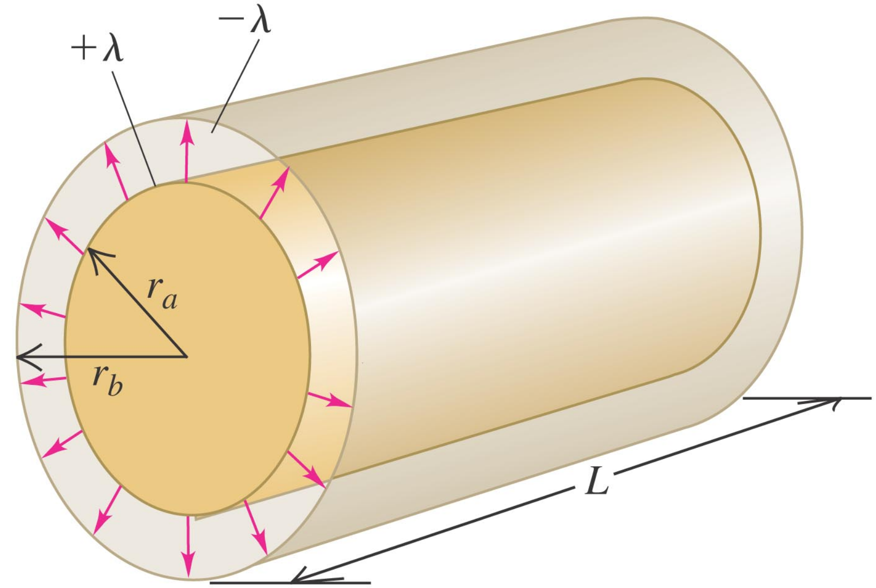
A spherical capacitor

- 24.3: Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge of +Q and outer radius of r_a and the outer shell has total charge of –Q and inner radius of r_b. The shells are attached by thin insulating rods with no effect on capacitance. **Find the capacitance of this spherical capacitor.**



A cylindrical capacitor

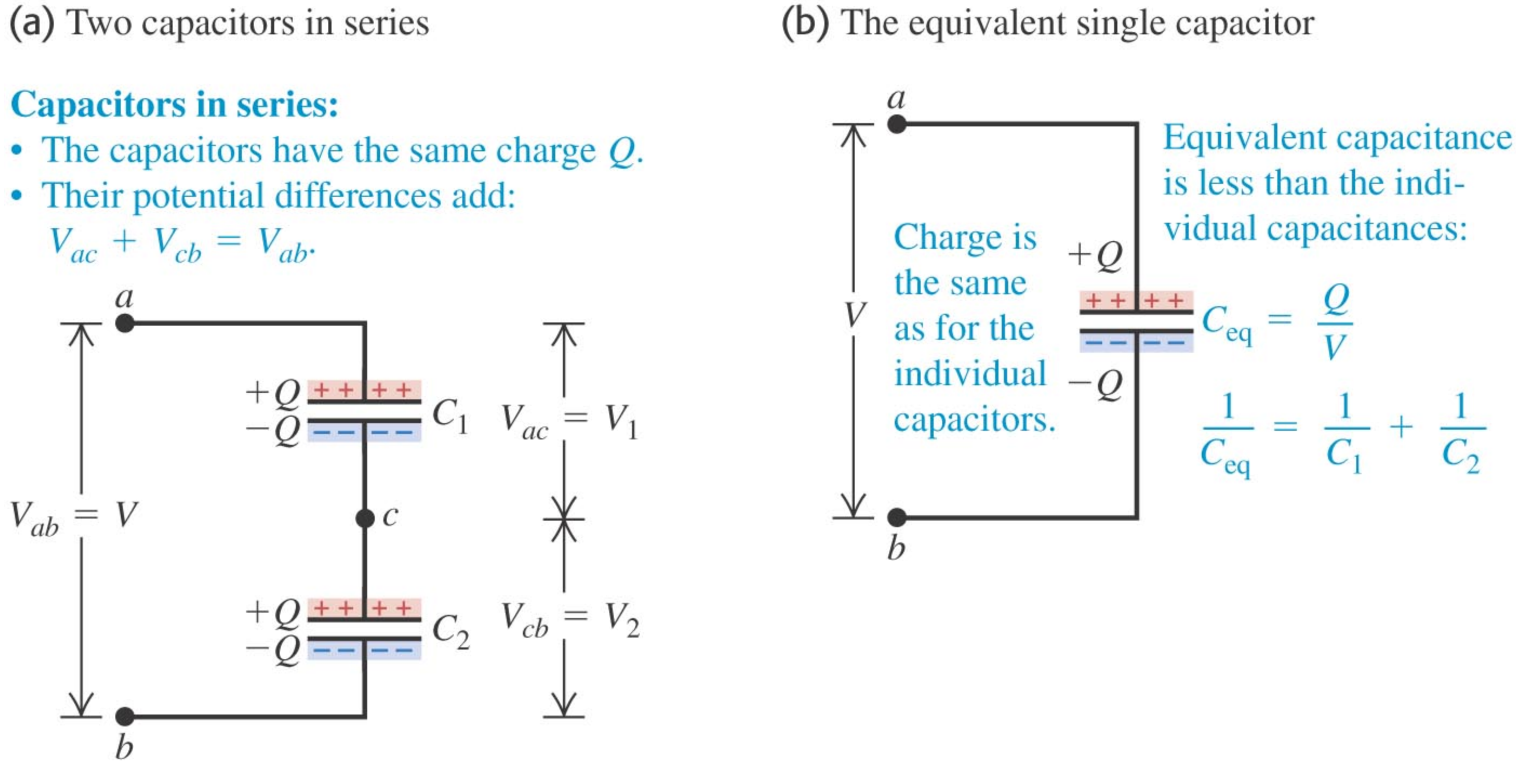
- 24.4) A long cylindrical conductor has a inner radius r_a and linear charge density lambda. It is surrounded by a coaxial cylindrical conducting shell with inner radius r_b and linear charge density –lambda. **Calculate the capacitance per unit length for this capacitor assuming that there is vacuum between the cylinders.**



Capacitors in series

- Capacitors are in *series* if they are connected one after the other, as illustrated in Figure 24.8 below.
- The *equivalent capacitance* of a series combination is given by:

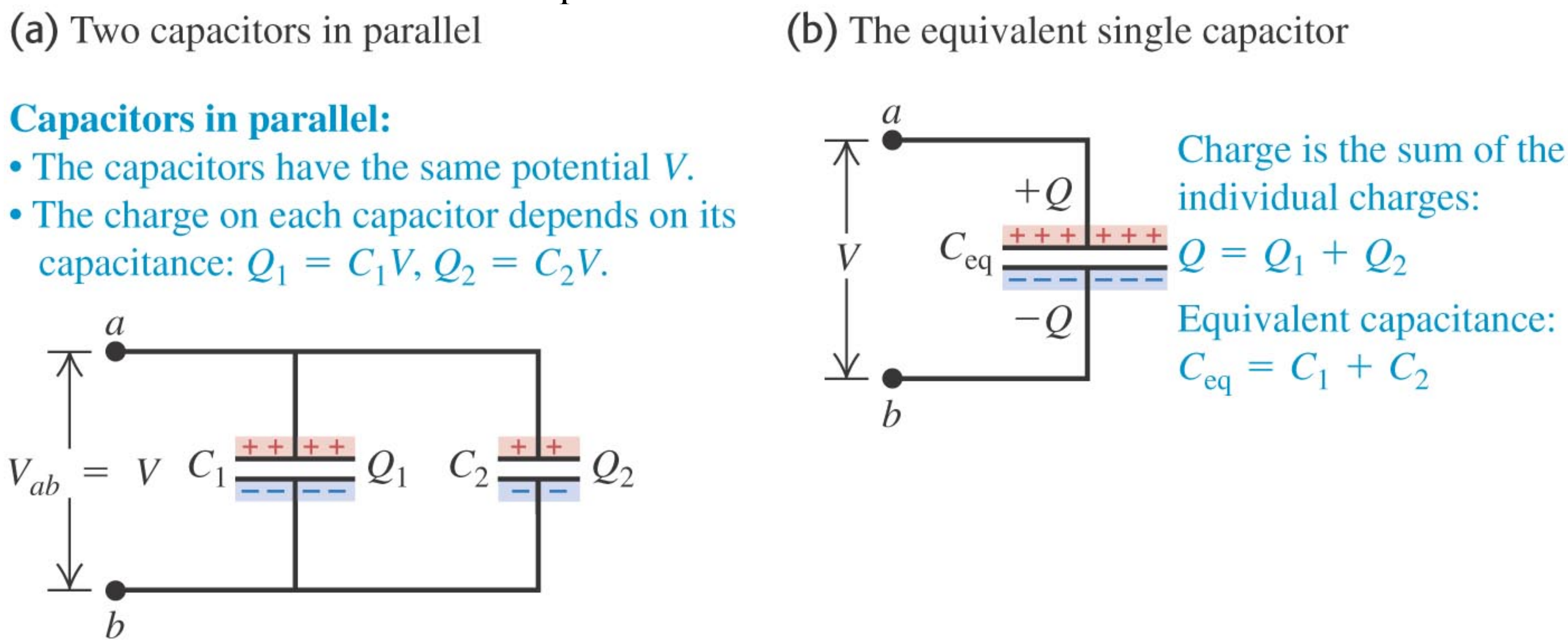
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$



Capacitors in parallel

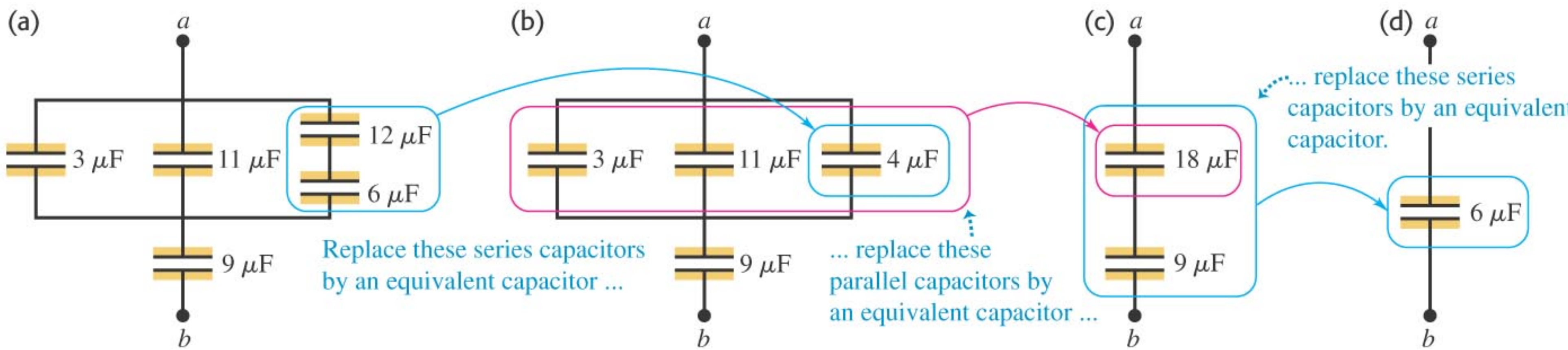
- Capacitors are connected in *parallel* between *a* and *b* if the potential difference V_{ab} is the same for all the capacitors. (See Figure 24.9 below.)
- The *equivalent capacitance* of a parallel combination is the *sum* of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \dots .$$



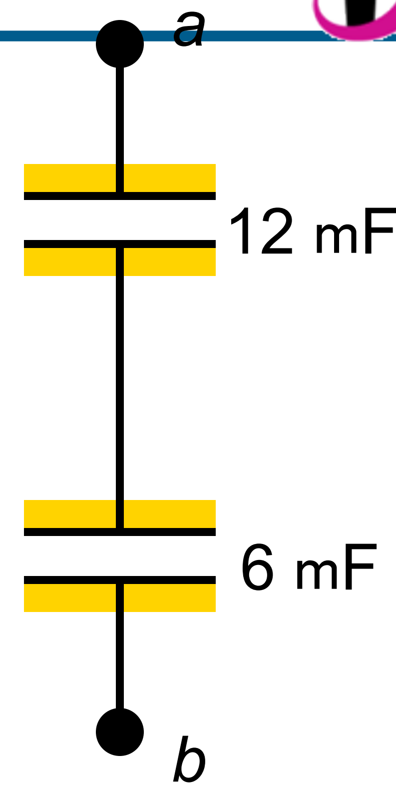
Calculations of capacitance: Problem solving strategy

- Set up: Make a drawing, identify parallel and series arrangements, the plate at higher potential has the positive Q and the plate at lower potential has the negative.
- Series capacitors have the same charge
- Parallel capacitors have the same voltage across.
- Simplify the more complicated combinations by identifying the parts that are parallel or series.
- Evaluate



Q24.3

A 12-mF capacitor and a 6-mF capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?



A. C_{eq} = 18 mF

B. C_{eq} = 9 mF

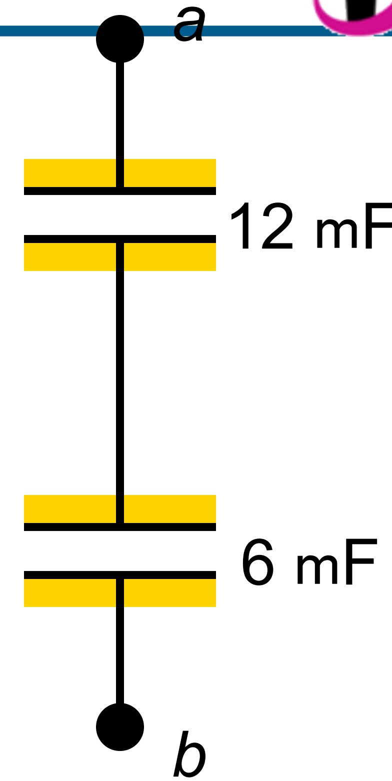
C. C_{eq} = 6 mF

D. C_{eq} = 4 mF

E. C_{eq} = 2 mF

Q24.4

A 12-mF capacitor and a 6-mF capacitor are connected together as shown. If the charge on the 12-mF capacitor is 24 microcoulombs (24 mC), what is the charge on the 6-mF capacitor?



A. 48 mC

B. 36 mC

C. 24 mC

D. 12 mC

E. 6 mC

Q24.5

A 12-mF capacitor and a 6-mF capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?

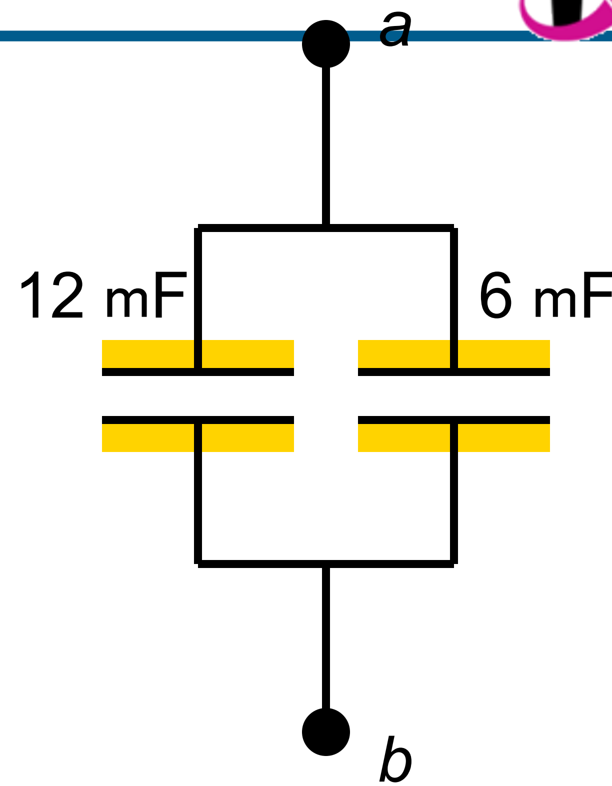
A. C_{eq} = 18 mF

B. C_{eq} = 9 mF

C. C_{eq} = 6 mF

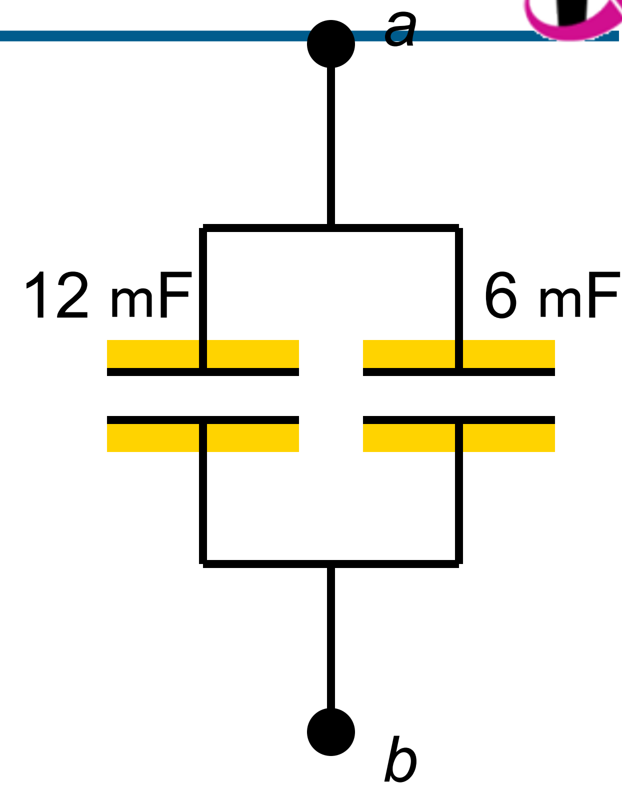
D. C_{eq} = 4 mF

E. C_{eq} = 2 mF



Q24.6

A 12-mF capacitor and a 6-mF capacitor are connected together as shown. If the charge on the 12-mF capacitor is 24 microcoulombs (24 mC), what is the charge on the 6-mF capacitor?



A. 48 mC

B. 36 mC

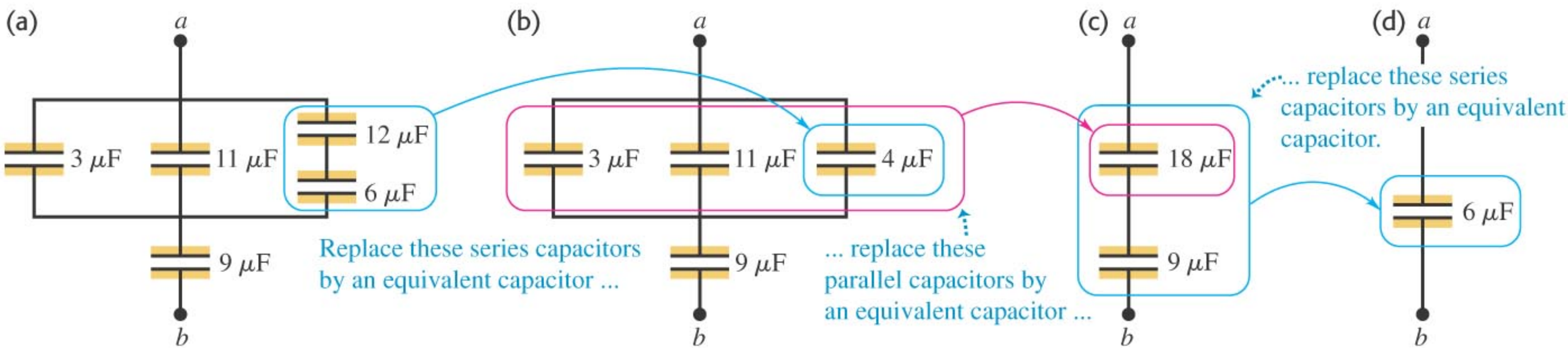
C. 24 mC

D. 12 mC

E. 6 mC

Calculations of capacitance: a capacitor network

- 24.6: Find the equivalent capacitance of the combination shown in Fig.



Energy stored in a capacitor

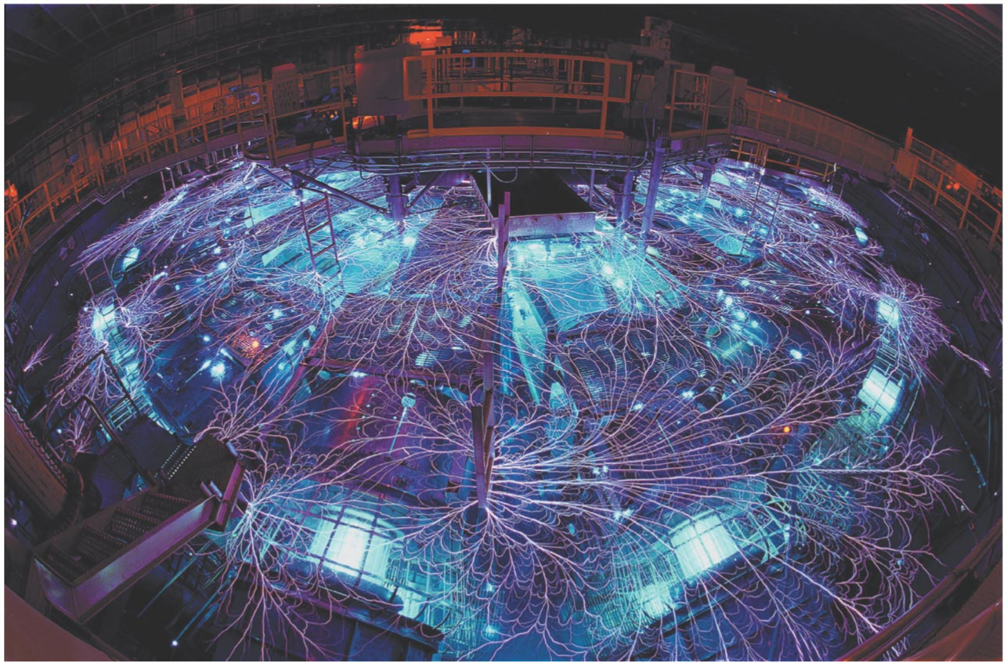
The **potential energy stored in a capacitor** is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

The capacitor energy is stored in the electric field between the plates.

The **energy density** is $u = \frac{1}{2}\varepsilon_0 E^2$

The Z machine shown below can produce up to 2.9 x10^{14} W using capacitors in parallel!



Energy stored in a capacitor

The **potential energy stored in a capacitor** = work required to charge it.

Potential across the plates of the capacitor due to small charge q: $v = q/C$

$$dW = v\,dq = \frac{q}{C}dq \rightarrow W = \int_0^W dW = \frac{1}{C}\int_0^W q\,dq = \frac{1}{C}\frac{Q^2}{2}$$

Potential energy of the uncharged capacitor = zero.

Then $U = U_b - U_a = U_b = W = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV$

Analogy:

Streatched spring	Charged capacitor
$U = \frac{1}{2}kx^2$	$U = \frac{1}{2}\frac{1}{C}Q^2$

Capacitance: ability to store **charge** $(Q = CV)$ and **energy** $\left(U = \dfrac{1}{2}CV^2\right)$

Electric energy density

Energy per unit volume stored in the electric field of a capacitor

is called energy diensity. $u = \dfrac{U}{Volume}$

Exampe for a parallel plate capacitor:

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\varepsilon_0 \frac{A}{d}V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

Electric energy density in vacuum

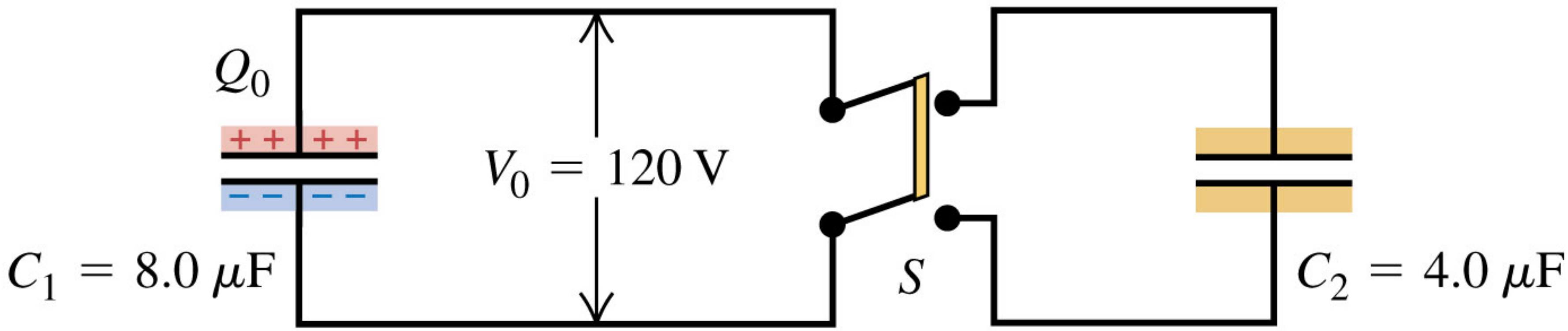
$$u = \frac{1}{2}\varepsilon_0 E^2$$

this is independent of geometry so can be applied to any capacitor.

Electric field energy and **electric potential energy** are the same things.

Some examples of capacitor energy

- 24.7 We charge C_1 by connecting it to potential difference of V_0.The switch S is initially open. Once the C_1 is charged the source of potential is disconnected.
  A) Charge on C^1 if the switch is left open.
  B) What is the energy stored in C_1?
  C) The capacitor C_2 is initially uncharged. We close the S what is the potential difference across each capacitor and charge on them?
  D) What is the total energy of the system after we close the switch S?



Electric field energy example

24.8 Want to store 1.00J of electric potential energy in a volume of 1.00m^3 in vacuum.

A) What is the magnitude of electric filed required?

B) If the field magnitude is 10 times larger, how much energy is stored per cubic meter?

24.9 The spherical capacitor of problem 24.3 has chares +Q and –Q on its inner and outer conductors. Find the electric potential energy stored in the capacitor

A) Using the capacitance found earlier

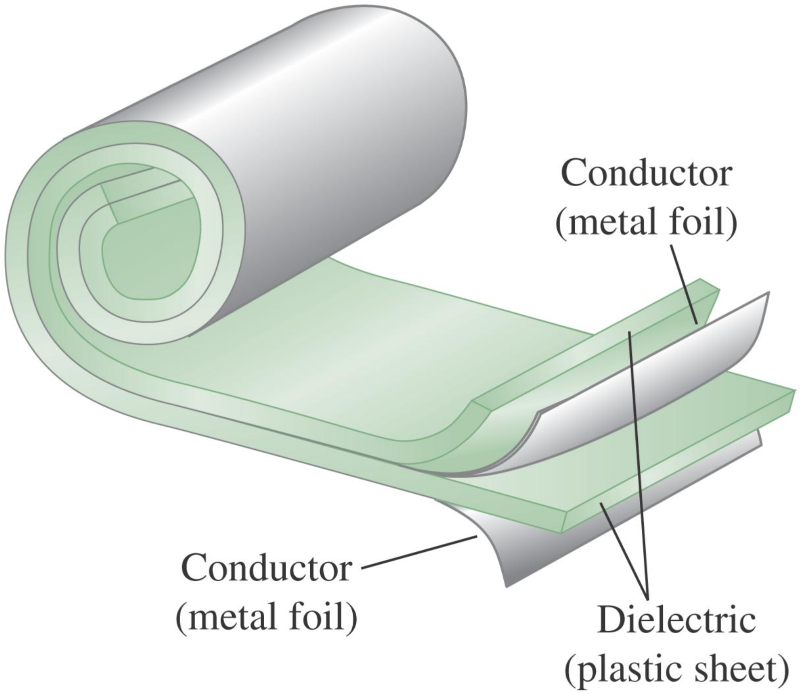
B) By integrating the electric energy density

Dielectrics

- A *dielectric* is a non-conducting material. Most capacitors have dielectric between their plates. (See Figure 24.13 at upper right.)

- The *dielectric constant* of the material is:

$$K = \frac{\varepsilon}{\varepsilon_0} > 1$$

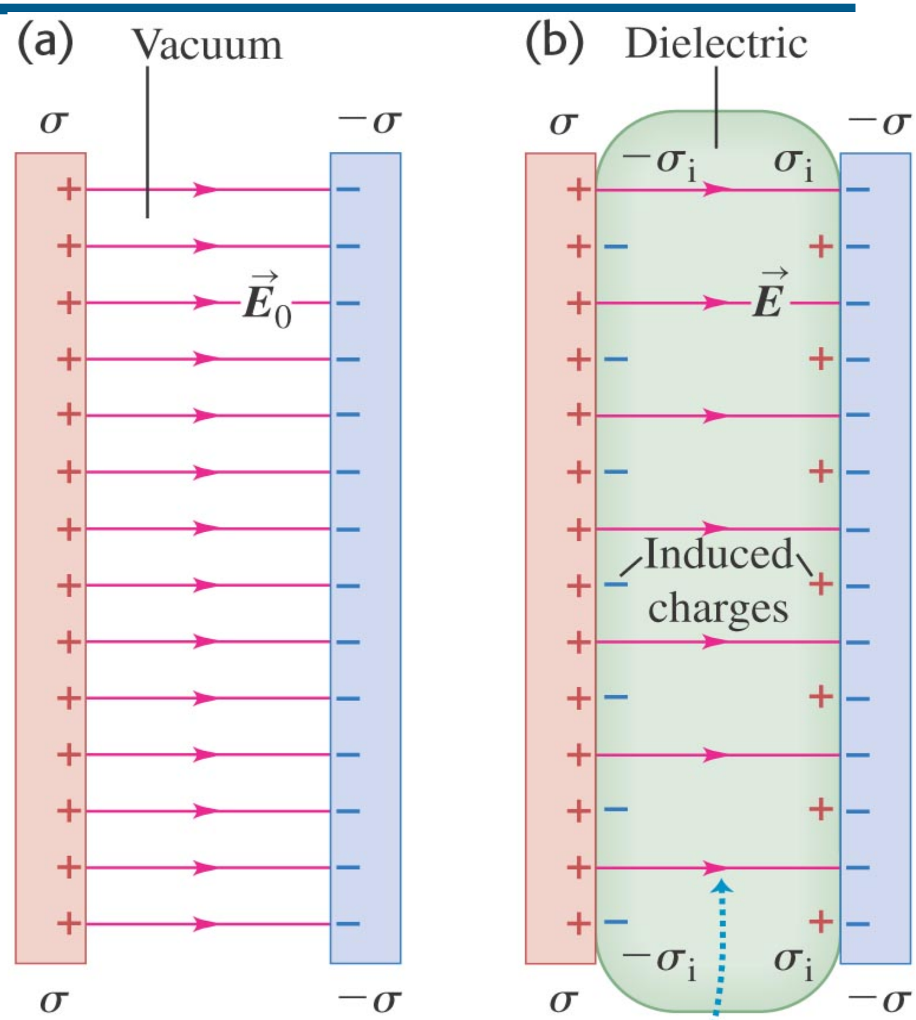


Dielectrics

- Dielectric *increases* the capacitance and the energy density by a factor K.
- Figure shows how the dielectric affects the electric field between the plates.

$$K = \frac{\varepsilon}{\varepsilon_0} > 1$$

$$\underbrace{C_0 = \varepsilon_0 \frac{A}{d}}_{\text{In vacuum}} \qquad \underbrace{C = \varepsilon \frac{A}{d}}_{\text{In dielectric}}$$



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

Dielectrics

For a capacitor when dielectric material inserted in the electric field while the charge is kept constant, the E-field reduces by a factor of K

$E = E_0 / K$

The new surface charge density must be reduced bu the Q is constant. That means the charges innside the dielectric are redistrubuted (polarized) to reduce net surface charge density.

$$E_0 = \frac{\sigma}{\varepsilon_0};\ E = \frac{\sigma - \sigma_i}{\varepsilon_0} \rightarrow \frac{E}{E_0} = \frac{1}{K} = \frac{\sigma - \sigma_i}{\sigma} = 1 - \frac{\sigma_i}{\sigma} \rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

Permitivity: $K\varepsilon_0 = \varepsilon$

E-field of capacitor in dielectrics: $E = \dfrac{\sigma}{\varepsilon}$ & the capacitanc: $C = \varepsilon \dfrac{A}{d}$

Table 24.1—Some dielectric constants

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric breakdown

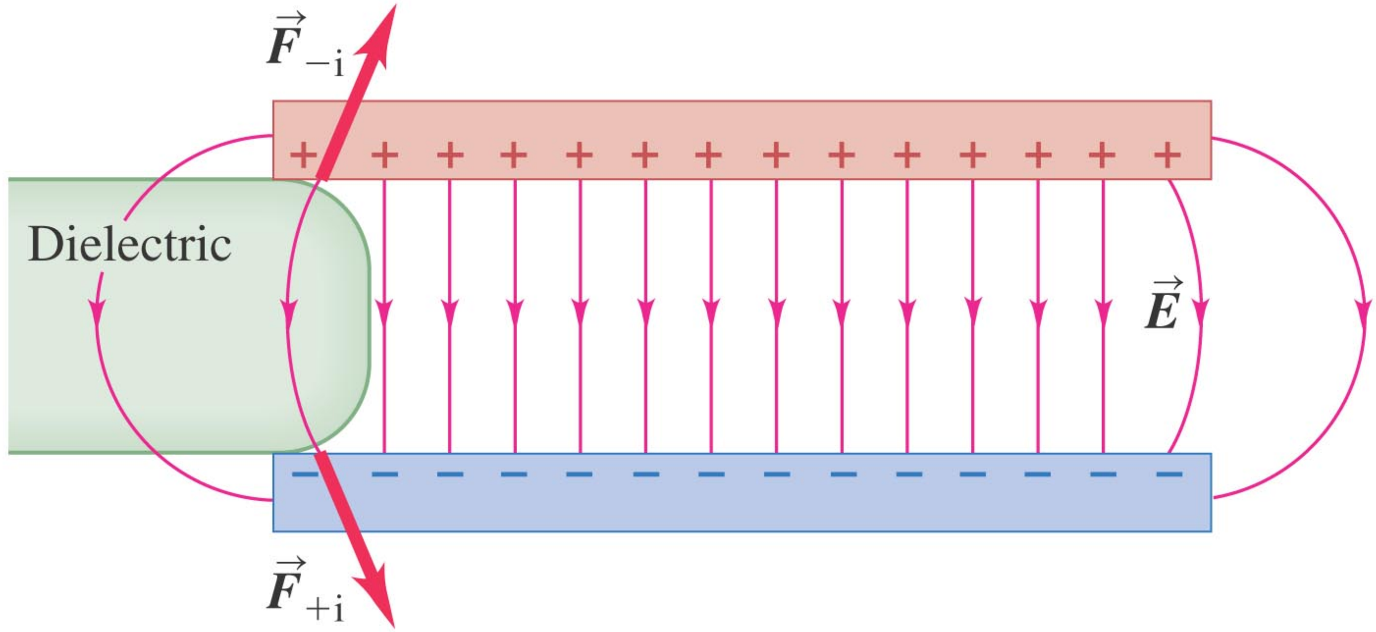
- If the electric field is strong enough, ***dielectric breakdown*** occurs and the dielectric becomes a conductor.
- The ***dielectric strength*** is the maximum electric field the material can withstand before breakdown occurs.
- Table 24.2 shows the **dielectric strength** of some insulators.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

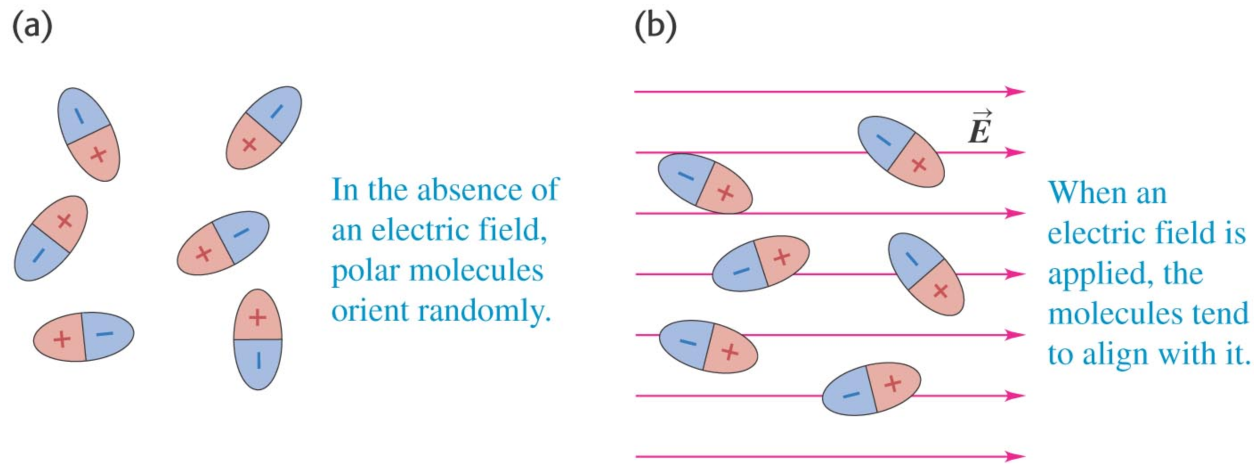
Examples with and without a dielectric

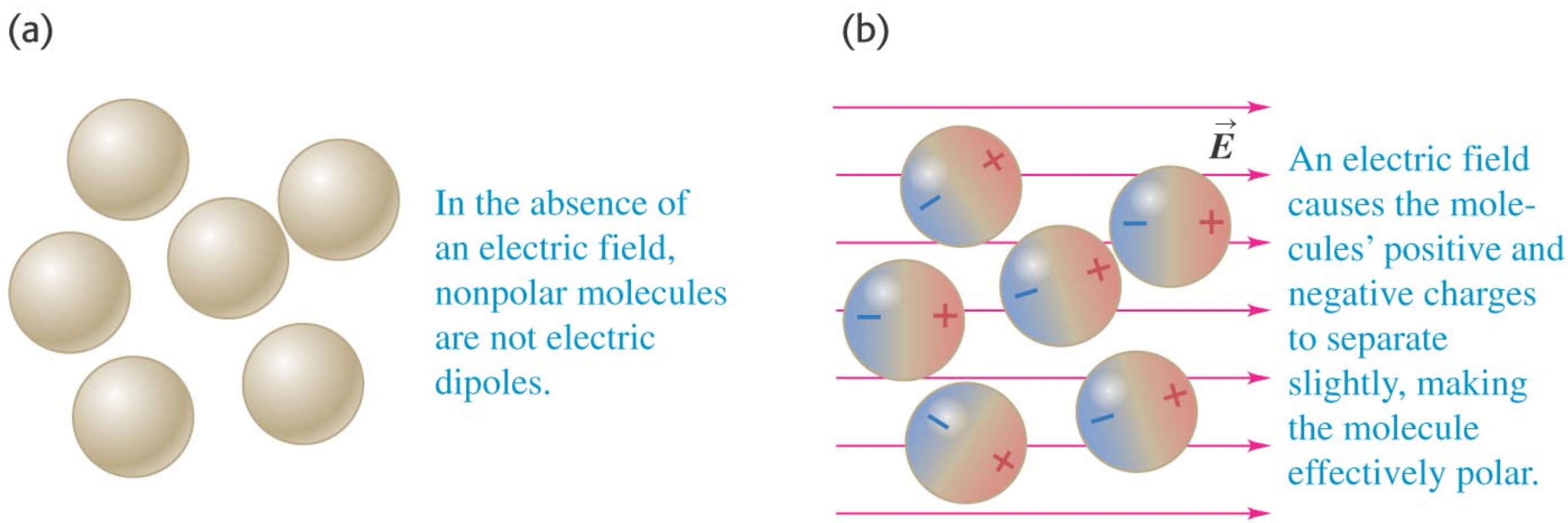
- Refer to Problem-Solving Strategy 24.2.
- 24.10
- 24.11



Molecular model of induced charge - I

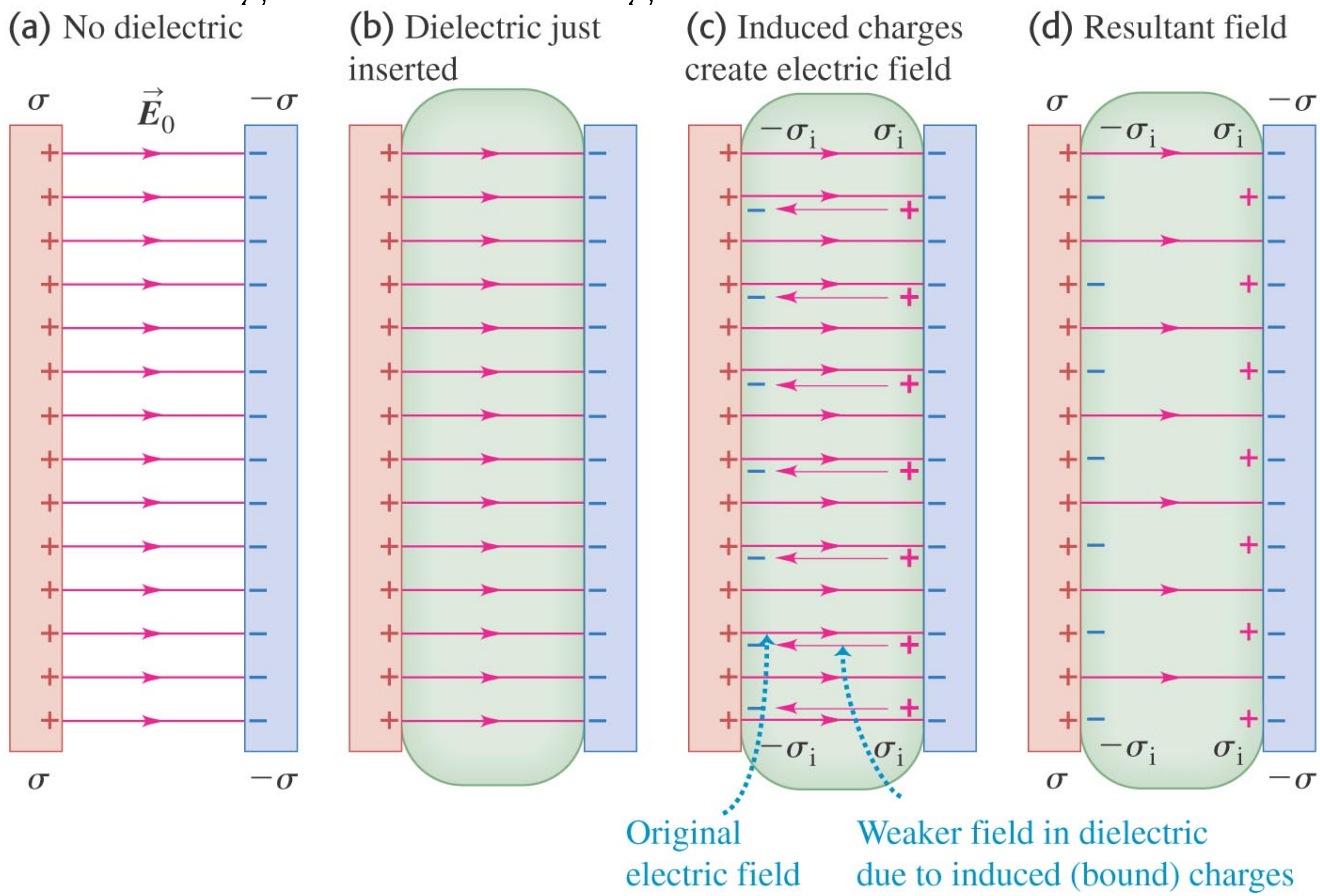
- Figures 24.17 (right) and 24.18 (below) show the effect of an applied electric field on polar and nonpolar molecules.





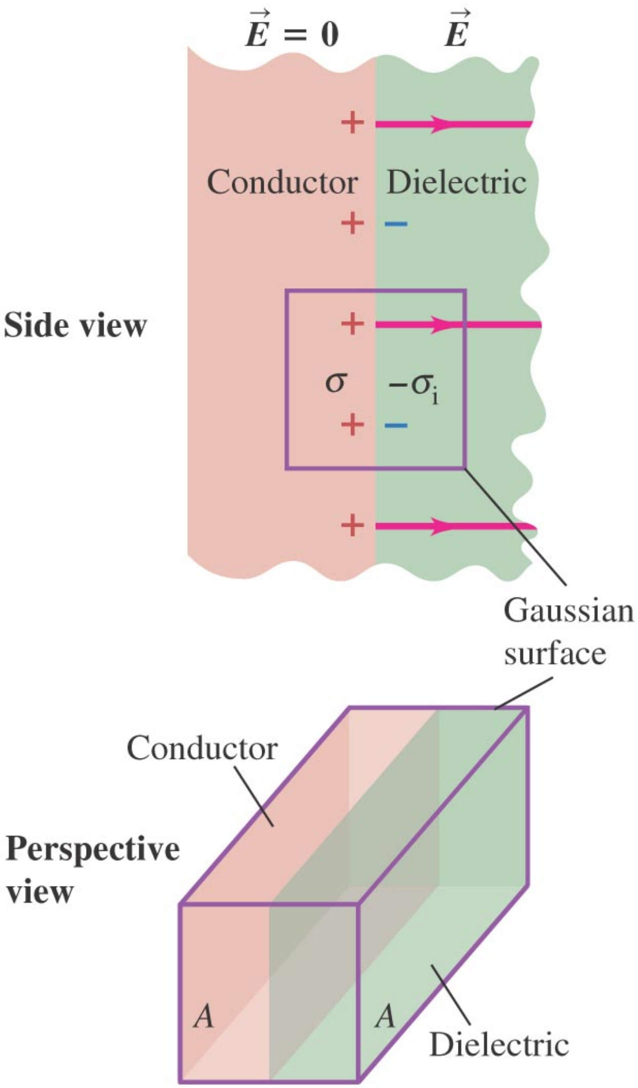
Molecular model of induced charge - II

- Figure 24.20 below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.



Gauss's law in dielectrics

- Follow the text discussion of Gauss's law in dielectrics, using Figure 24.22 at the right.
- Follow Example 24.12 for a spherical capacitor



Q24.7

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $–Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.

B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

Q24.8

You slide a slab of dielectric between the plates of a parallel-plate capacitor. As you do this, the *charges* on the plates remain constant.

What effect does adding the dielectric have on the *potential difference* between the capacitor plates?

A. The potential difference increases.

B. The potential difference remains the same.

C. The potential difference decreases.

D. not enough information given to decide

Q24.9

You slide a slab of dielectric between the plates of a parallel-plate capacitor. As you do this, the *charges* on the plates remain constant.

What effect does adding the dielectric have on the *energy stored* in the capacitor?

A. The stored energy increases.

B. The stored energy remains the same.

C. The stored energy decreases.

D. not enough information given to decide

Q24.10

You slide a slab of dielectric between the plates of a parallel-plate capacitor. As you do this, the *potential difference* between the plates remains constant.

What effect does adding the dielectric have on the *amount of charge* on each of the capacitor plates?

A. The amount of charge increases.

B. The amount of charge remains the same.

C. The amount of charge decreases.

D. not enough information given to decide

Q24.11

You slide a slab of dielectric between the plates of a parallel-plate capacitor. As you do this, the *potential difference* between the plates remains constant.

What effect does adding the dielectric have on the *energy stored* in the capacitor?

A. The stored energy increases.

B. The stored energy remains the same.

C. The stored energy decreases.

D. not enough information given to decide