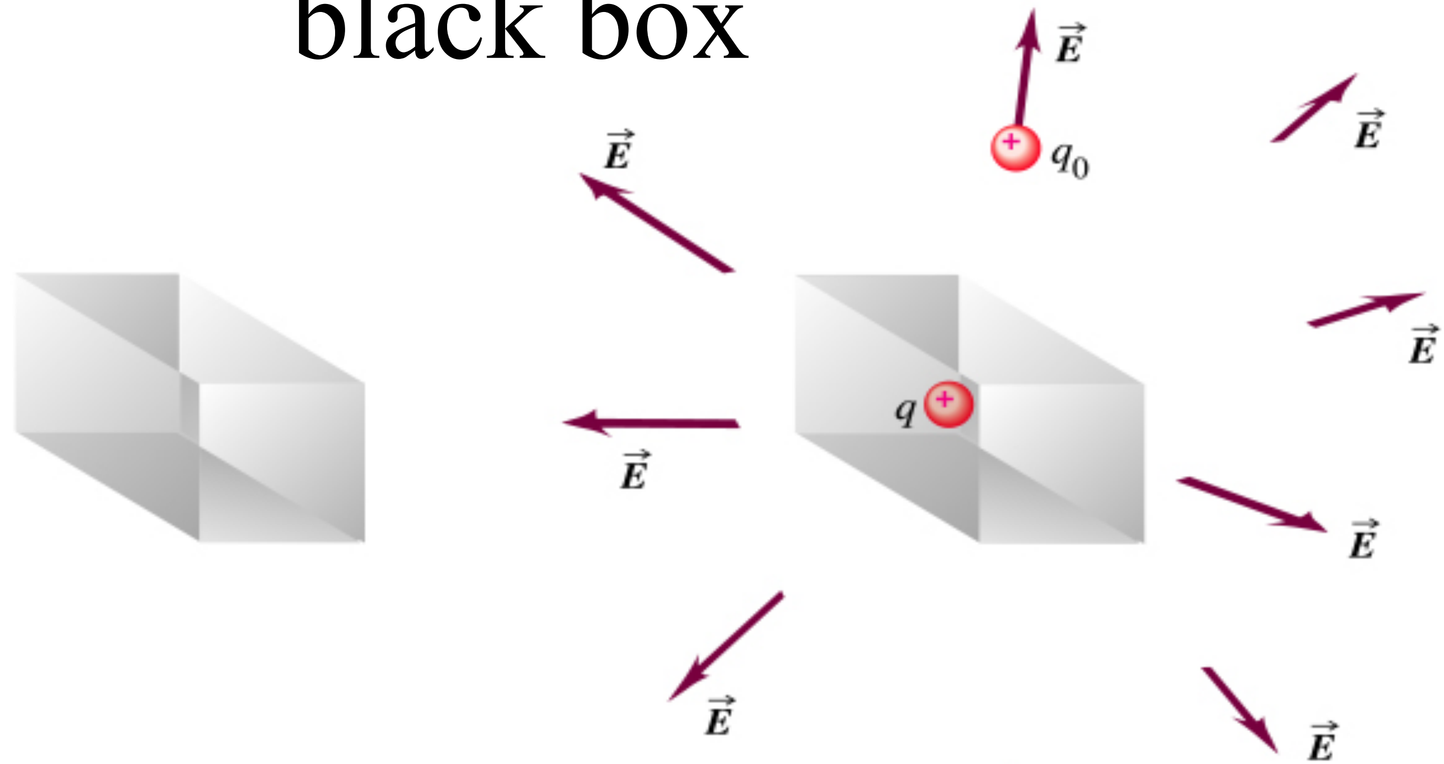
Chapter 22 Gauss's law

- Electric charge and flux (sec. 22.2 & .3)
- Gauss's Law (sec. 22.4 & .5)
- Charges on conductors (sec. 22.6)

Learning Goals for CH 22

- Determine the amount of charge within a closed surface by examining the electric field on the surface!
- Electric flux and how to calculate it.
- How to use Gauss's Law to calculate the electric field due to a symmetric distribution of charges.

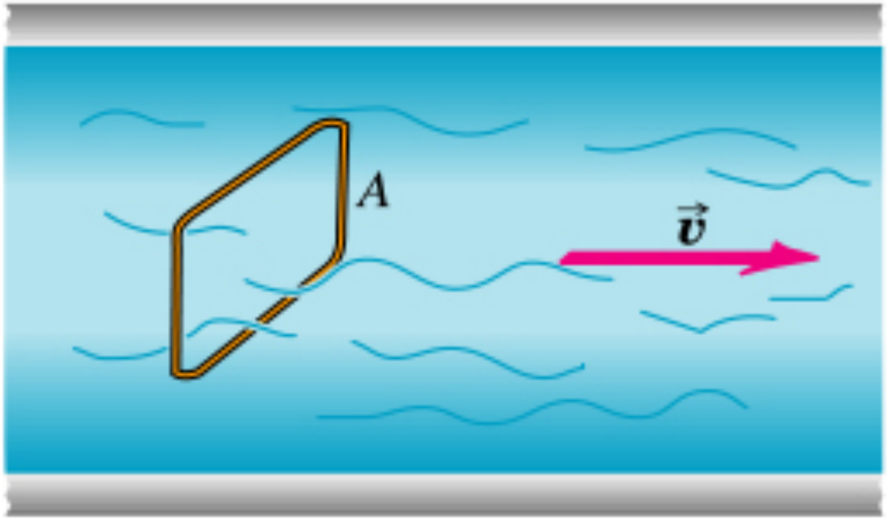
Determining a charge outside of a black box



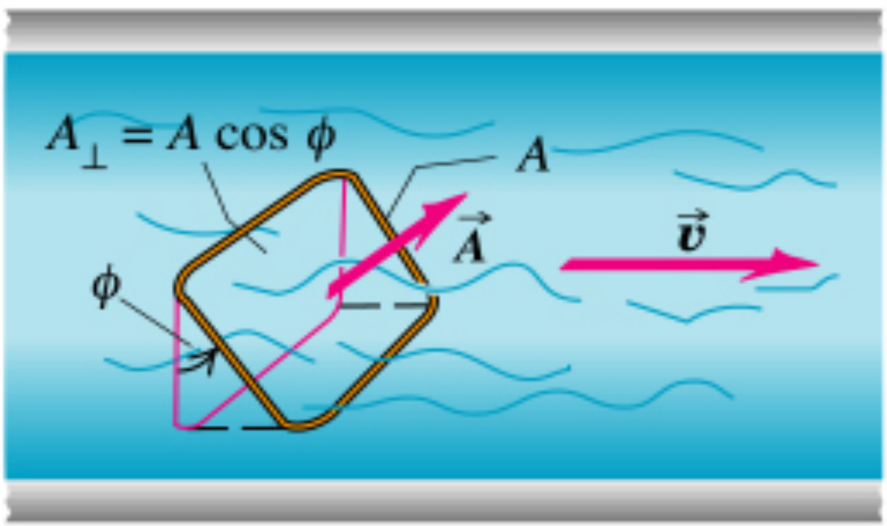
A charge inside a box can be probed with a test charge q_o to measure **E** field outside the box.

Electric flux outside a closed surface that encloses a net charge

- We can experience
  - The direction of the electric flux lines outside depends on the sign of the charge enclosed
  - Charges outside the surface do not contribute to the net flux through the surface
  - The net electric flux is directly proportional to the net amount of the charge enclosed by the surface
- We have a quantitative feel for the Gauss's law



(a)

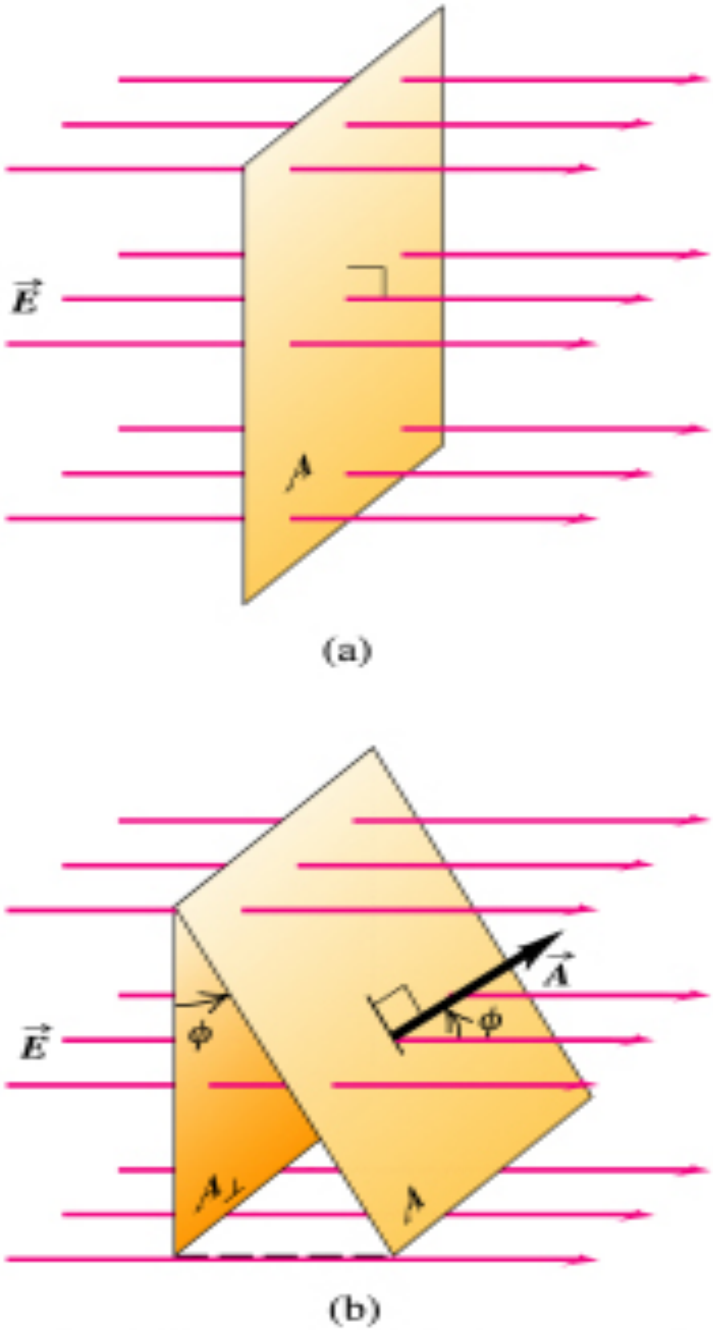


(b)

The volume (V) flow rate (dV/dt) of fluid through the wire rectangle (a) is vA when the area of the rectangle is perpendicular to the velocity vector **v** and (b) is vA cos f when the rectangle is tilted at an angle f.

We will next replace the fluid velocity flow vector **v** with the electric field vector **E** to get to the concept of electric flux F_E.

Volume flow rate through the wire rectangle.



(a) The electric flux through the surface = EA.

(b) When the area vector makes an angle f with the vector **E**, the area projected onto a plane oriented perpendicular to the flow is *A* **perp.** = *A* cos f. The flux is zero when f = 90^o because the rectangle lies in a plane parallel to the flow and no fluid flows through the rectangle

A flat surface in a uniform electric field.

Calculating Electric flux

Electric flux hrough a flat surface due to **uniform E - field**

$\phi_E = EA\cos\phi = EA_{\perp}$

$\phi_E = \mathbf{E}\bullet\mathbf{A}$ where $\mathbf{A} = A\hat{\mathbf{n}}$

Direction of the $\hat{\mathbf{n}}$ depends on the side of the surface we are calculating the flux for.

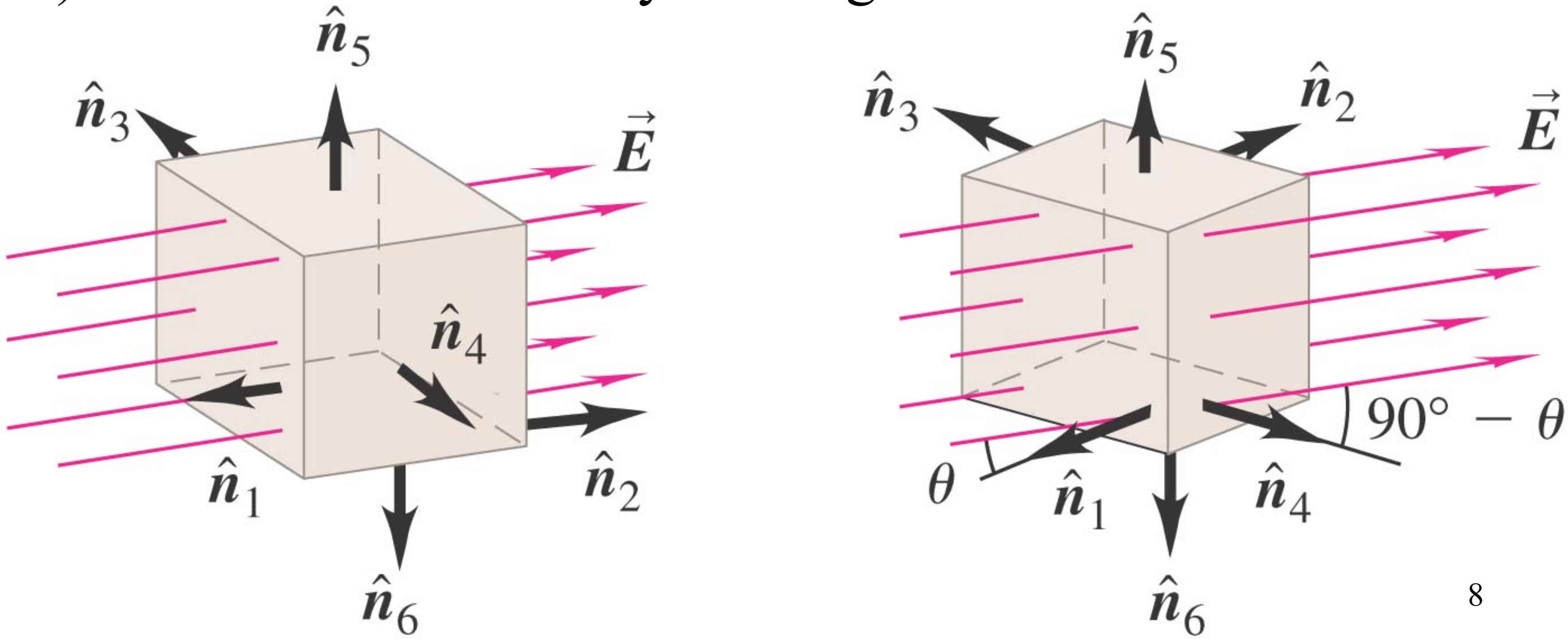
Electric flux through a **non - flat surface** due to a **non - uniform E - field**

$$\phi_E = \int E\cos\phi\, dA = \int E_{\perp}\, dA = \int \mathbf{E}\bullet d\mathbf{A}$$

This is a surface integral and we need to calculate it.

Electric flux through a cube

- A cube of side L is placed in a region of uniform electric field E.
- Find the electric flux through each face of the cube and the total flux when
  a) E is perpendicular to the 2 of the faces of the cube.
  b) The cube is rotated by and angle theta about n5-n6 axis.



Gauss's law
(Carl Friedrich Gauss 1777-1855)

- Gauss's law is an alternative to the Coulomb's law
- It is a different way of expressing the relationship between the electric charge and electric field

$$\phi_E = \oint \mathbf{E} \bullet d\mathbf{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$\phi_E = \oint E \cos\phi \, dA = \oint E_{\perp} \, dA = \frac{Q_{encl}}{\varepsilon_0}$$

Electric flux through a sphere of radius R centered on a point charge q

Start from:

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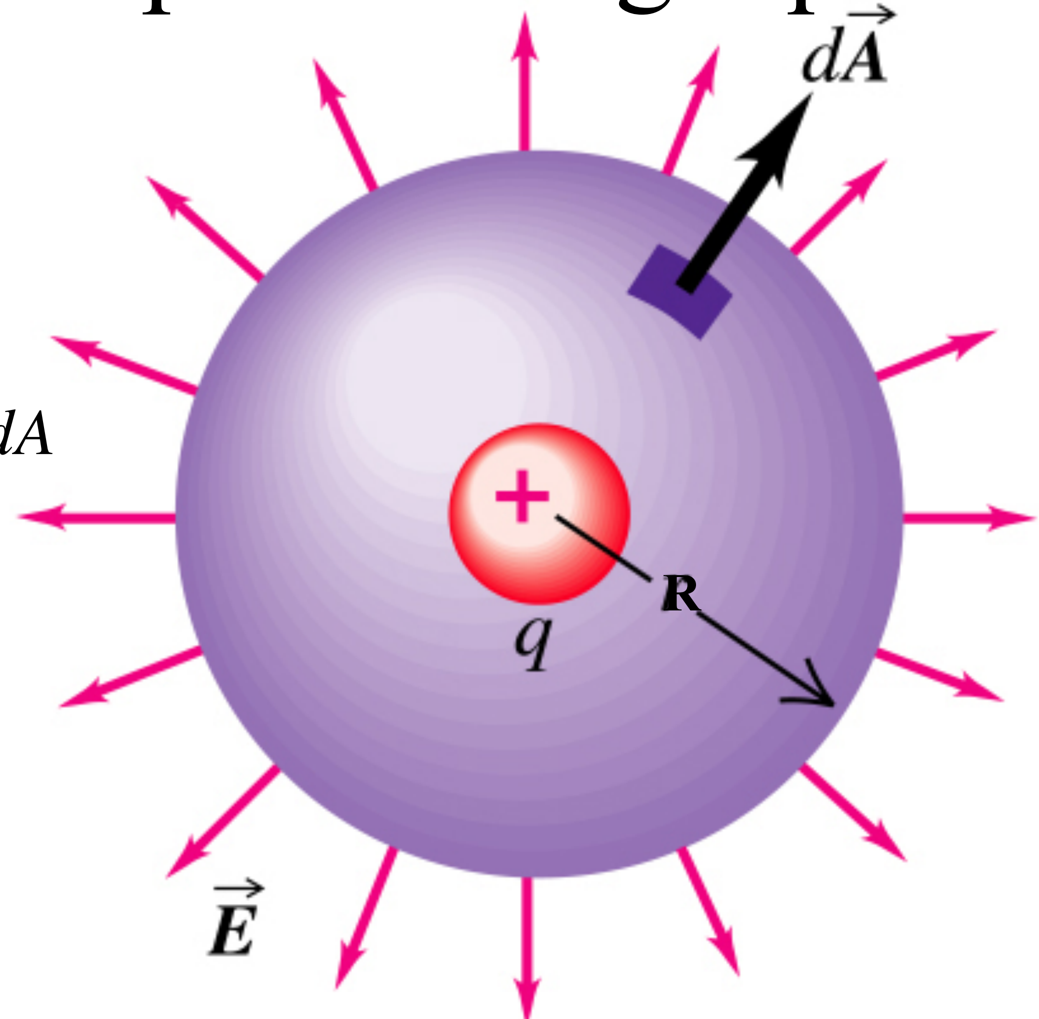
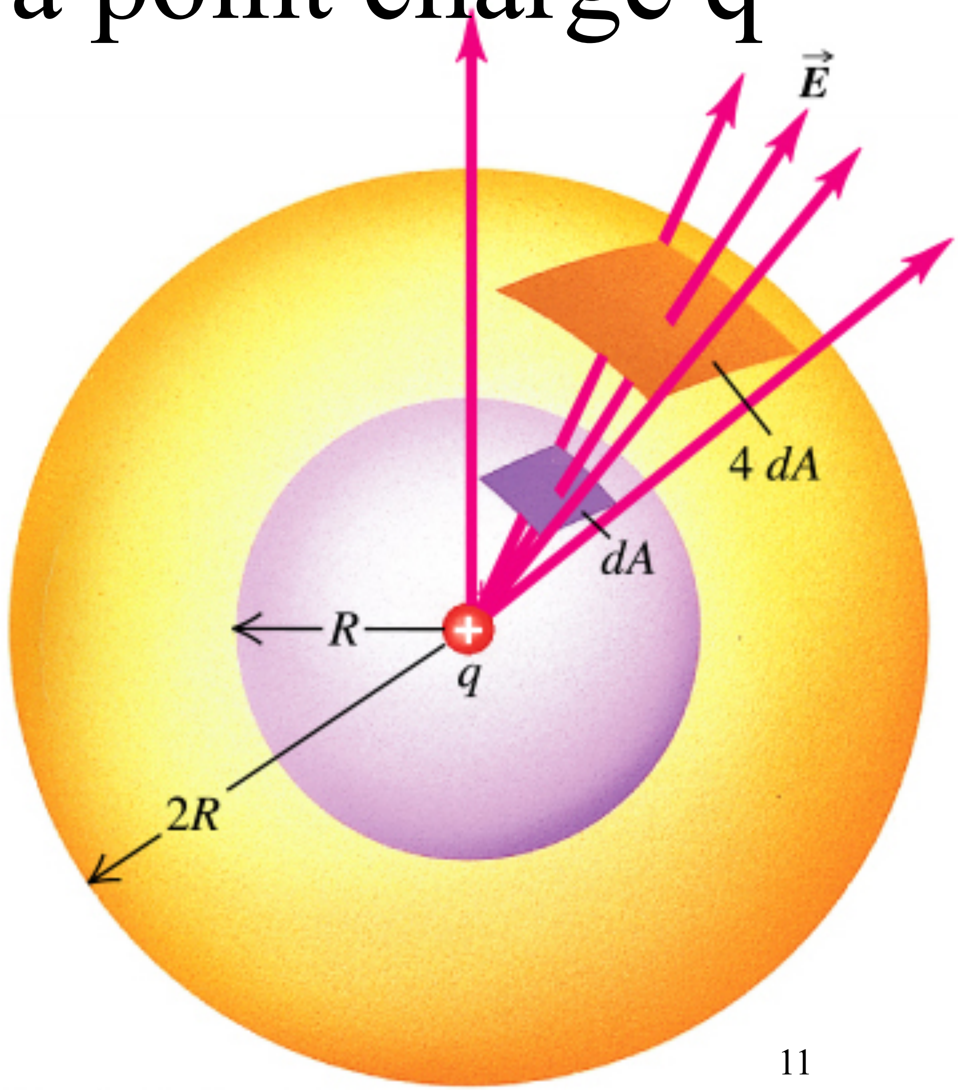


Electric flux through a sphere of radius 2R centered on a point charge q

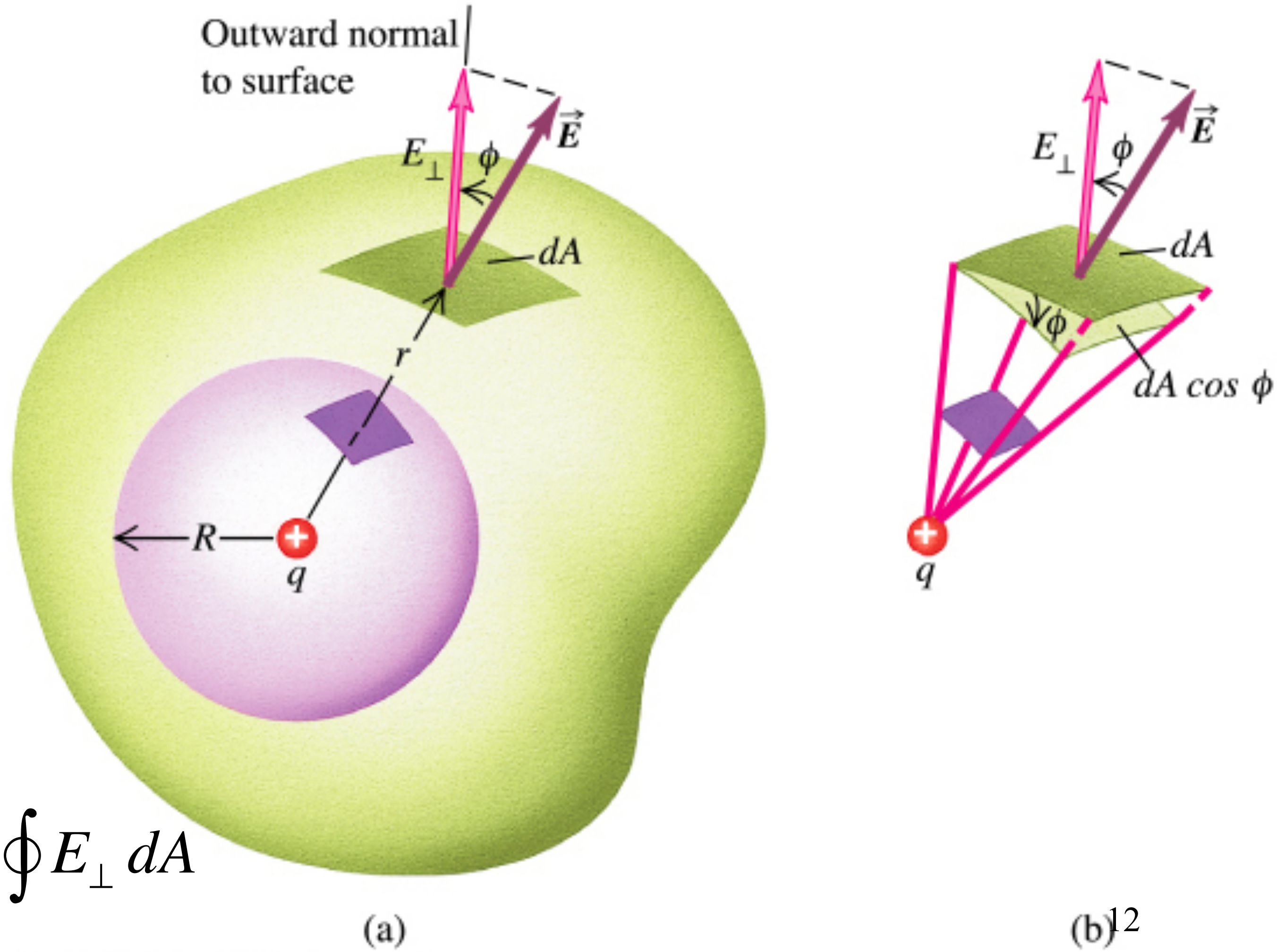
Start from:

$$\phi_E = \oint \mathbf{E} \bullet d\mathbf{A}$$

$$\phi_E = \oint E \cos\phi \, dA = \oint E_\perp \, dA$$



Electric flux through a non-spherical surface enclosing a point charge q



Start from:

$$\phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$$

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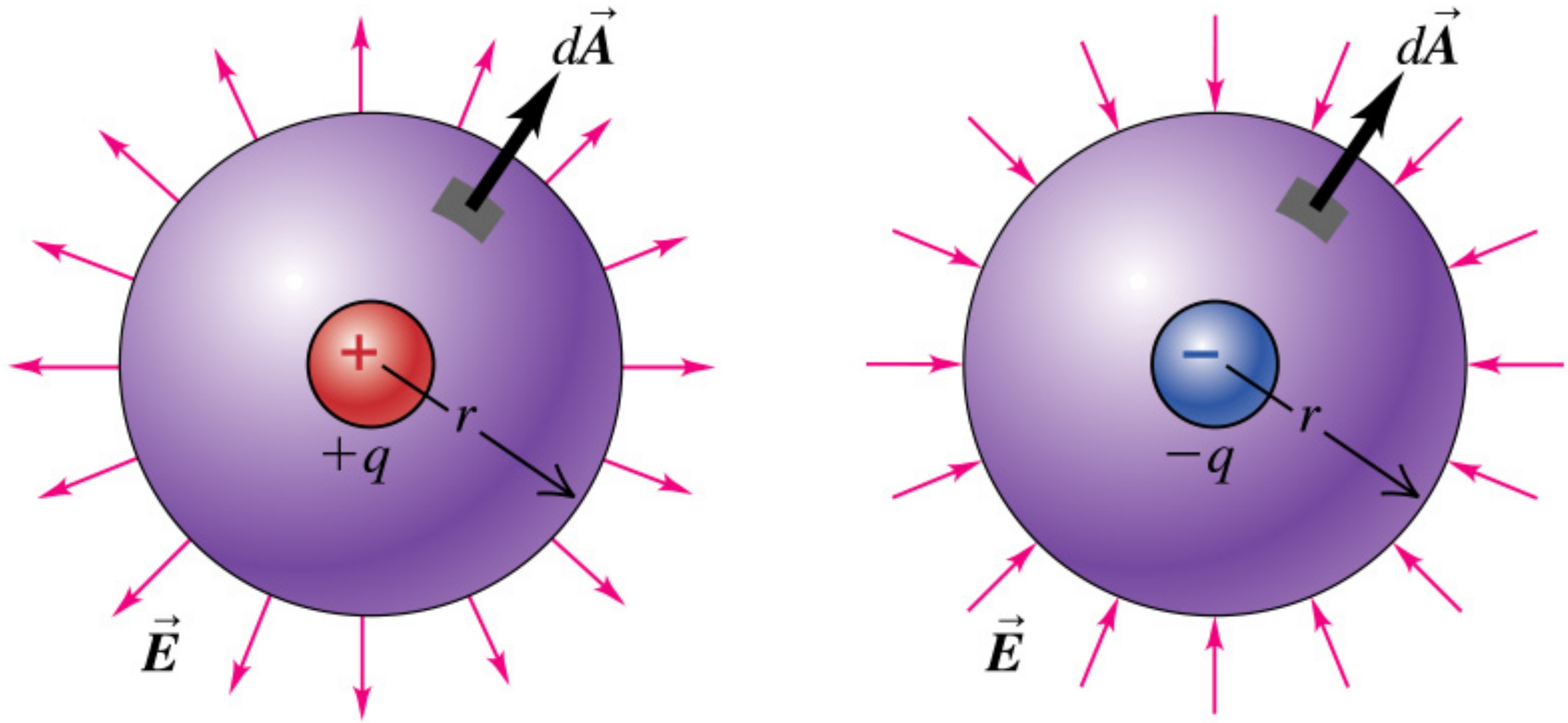
Spherical Gaussian surfaces around a positive and negative point charge.



(a) Gaussian surface around positive charge: positive (outward) flux

(b) Gaussian surface around negative charge: negative (inward) flux

Applications of the Gauss's law

- The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by permittivity of vacuum
- Gauss's Law can be used to calculate the magnitude of the E field vector:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)}$$

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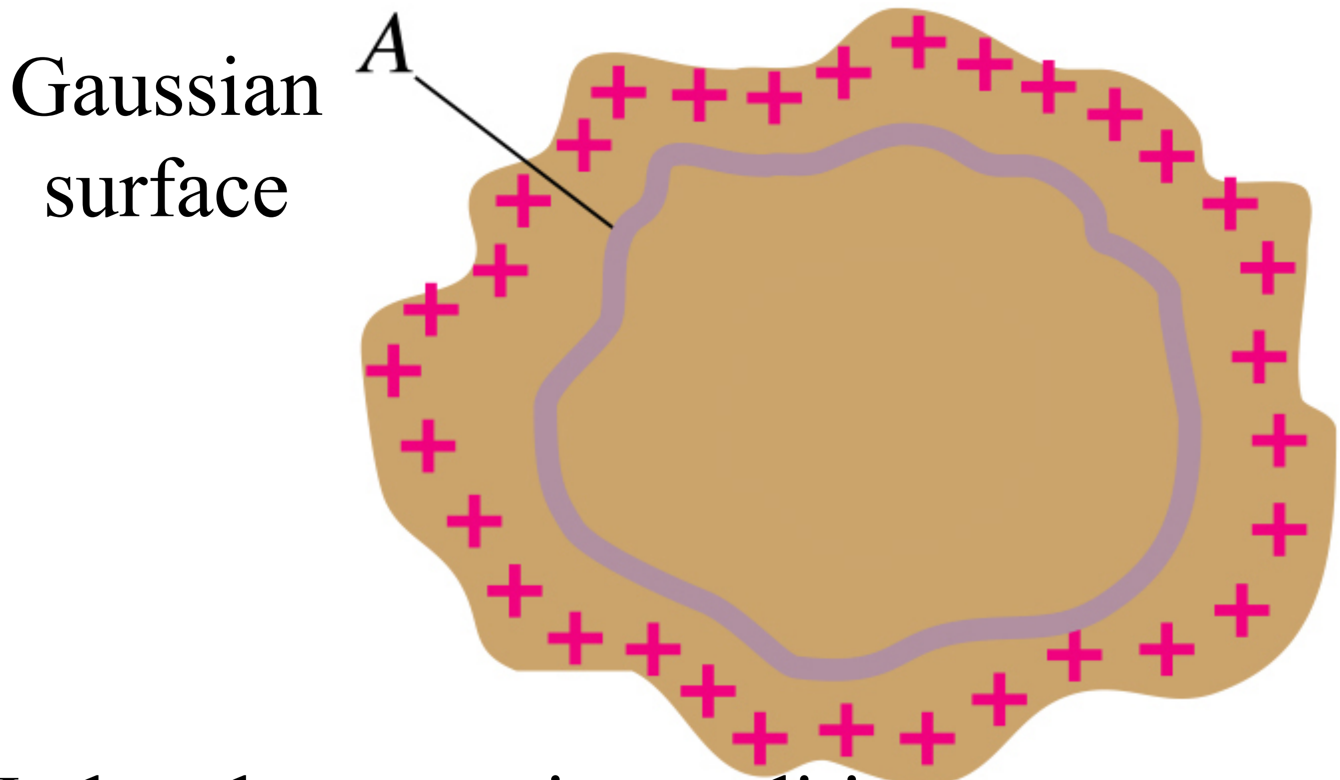
5. Determine the value of Q_{encl} from your figure and insert it into Gauss's equation.
6. Solve the equation for the magnitude of **E**.

A conductor under electrostatic situation

- This means there is no moving charges.
- All the fields on charges are balanced such that the net force on the charges is zero.
- Question:
  - **How the charges of a piece of a conductor are arranged under the electrostatic condition?**

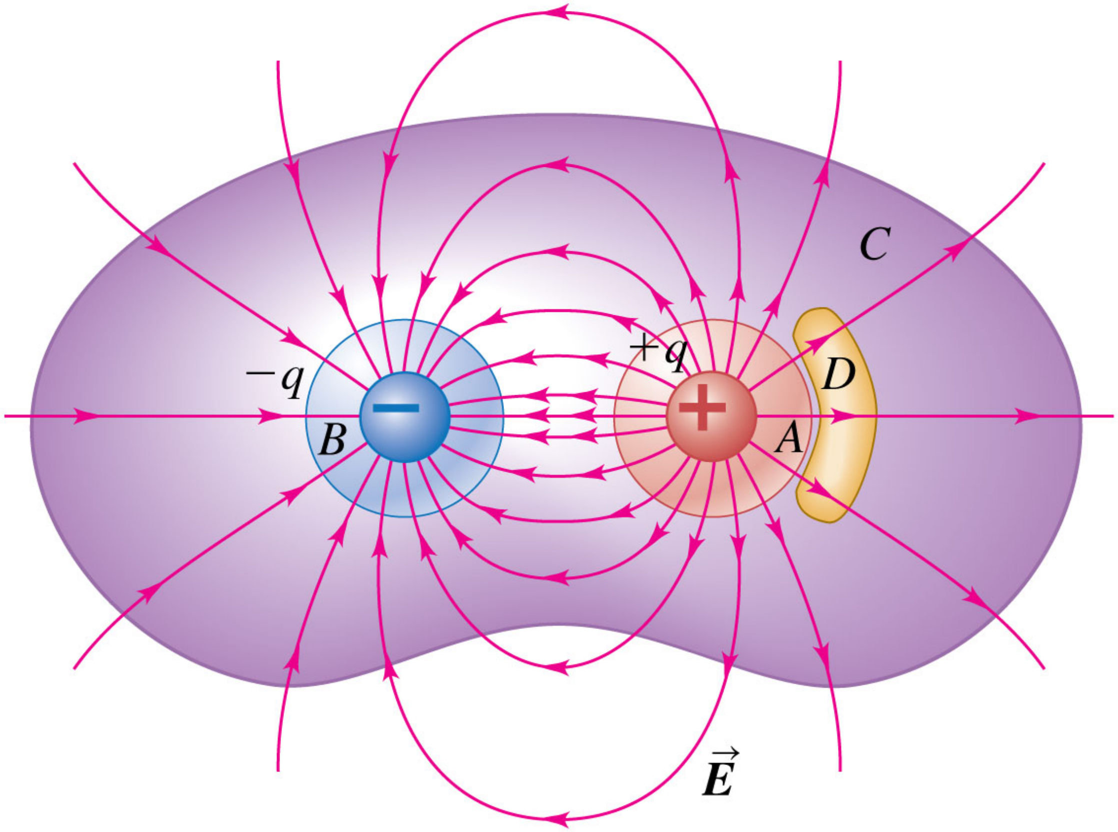
Find the electric field inside a conductor using the Gauss's law



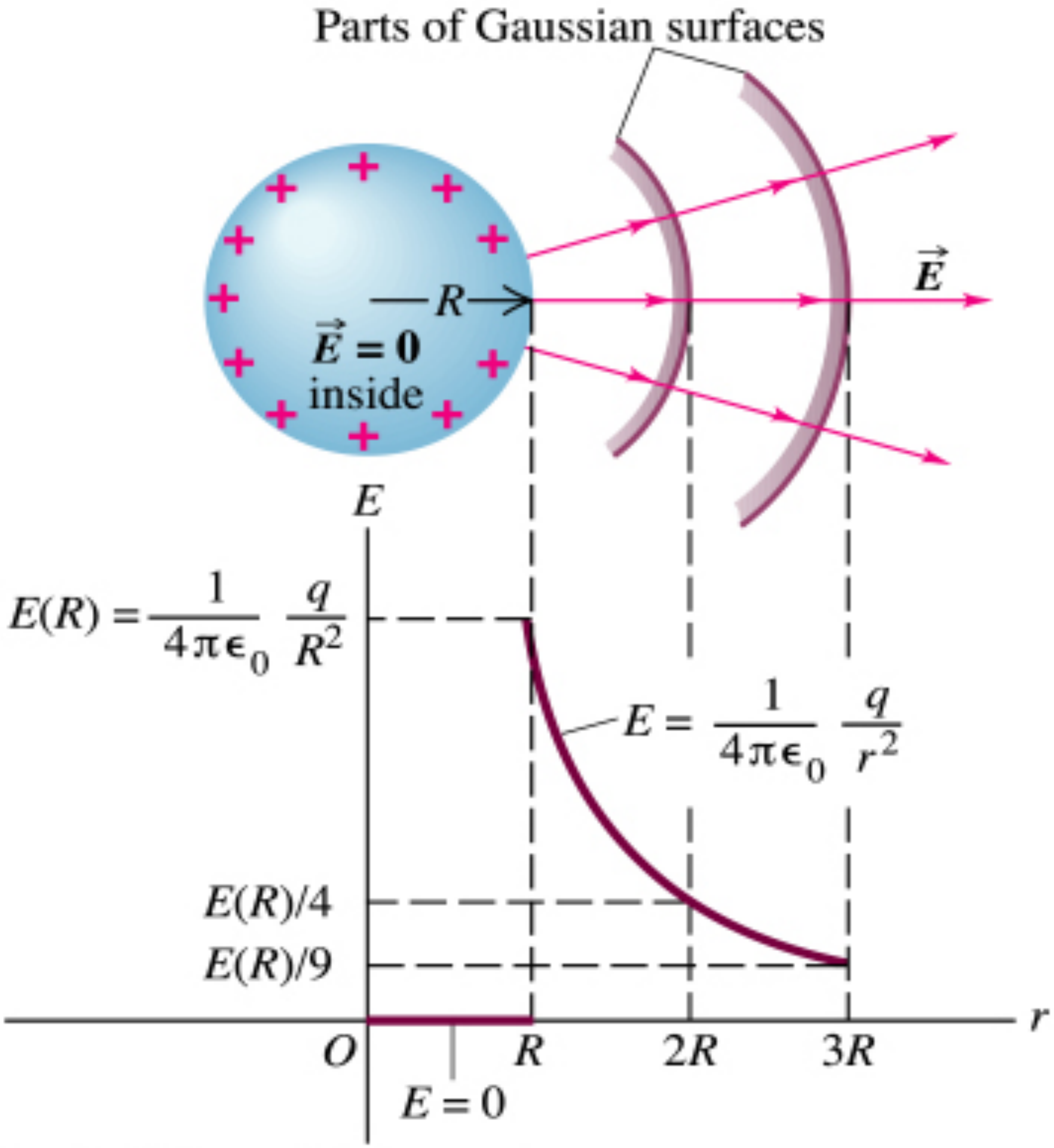
Under electrostatic conditions, any excess charge resides entirely on the surface of a solid conductor.

Electric flux around a dipole

- Calculate or guess the amount of electric flux through each of the surfaces A, B, C, D



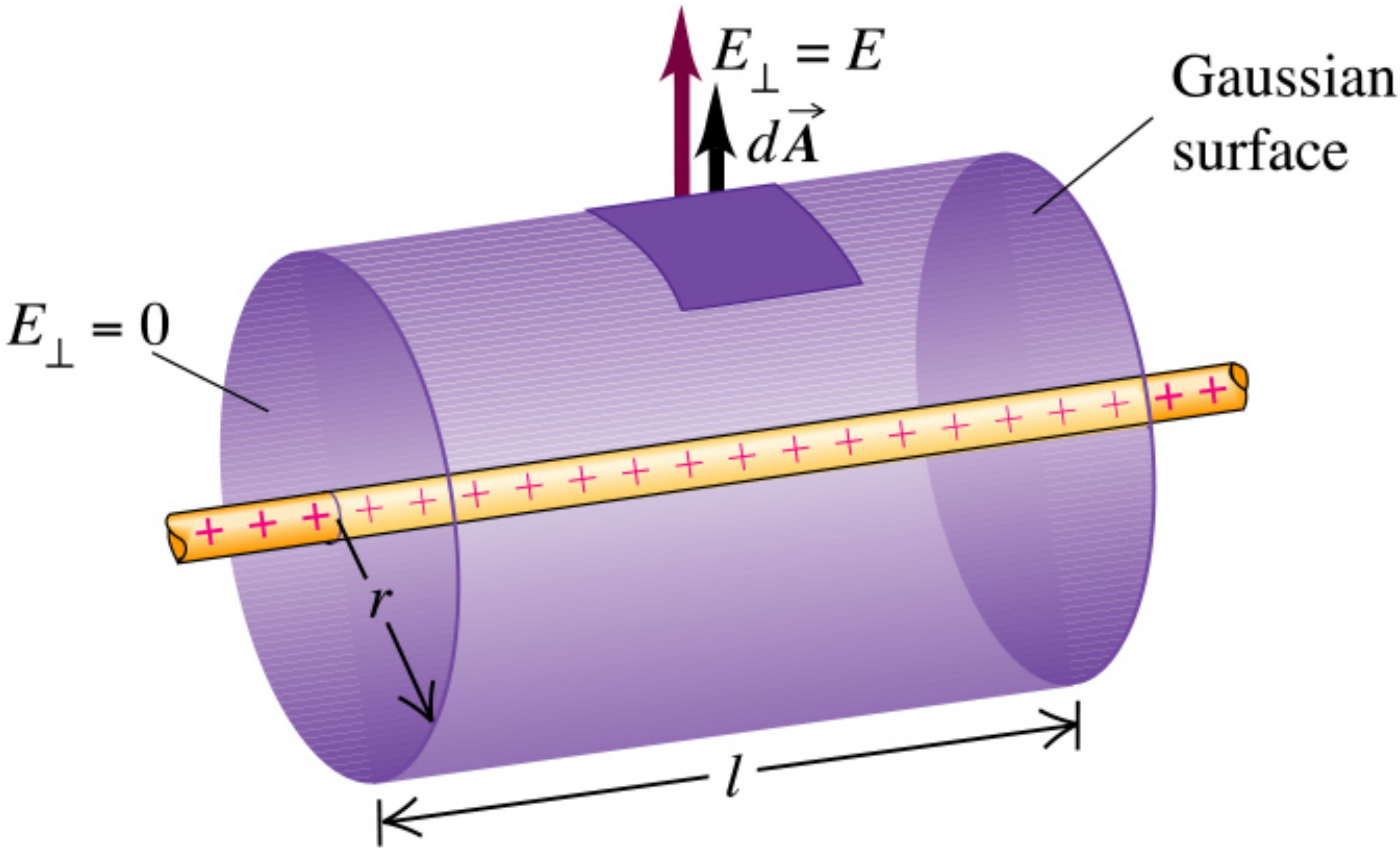
Electric field of a hollow spherical conductor (Ex22.5)



Under electrostatic conditions the electric field inside a solid conducting sphere is zero. Outside the sphere the electric field drops off as $1 / r^2$, as though all the excess charge on the sphere were concentrated at its center.

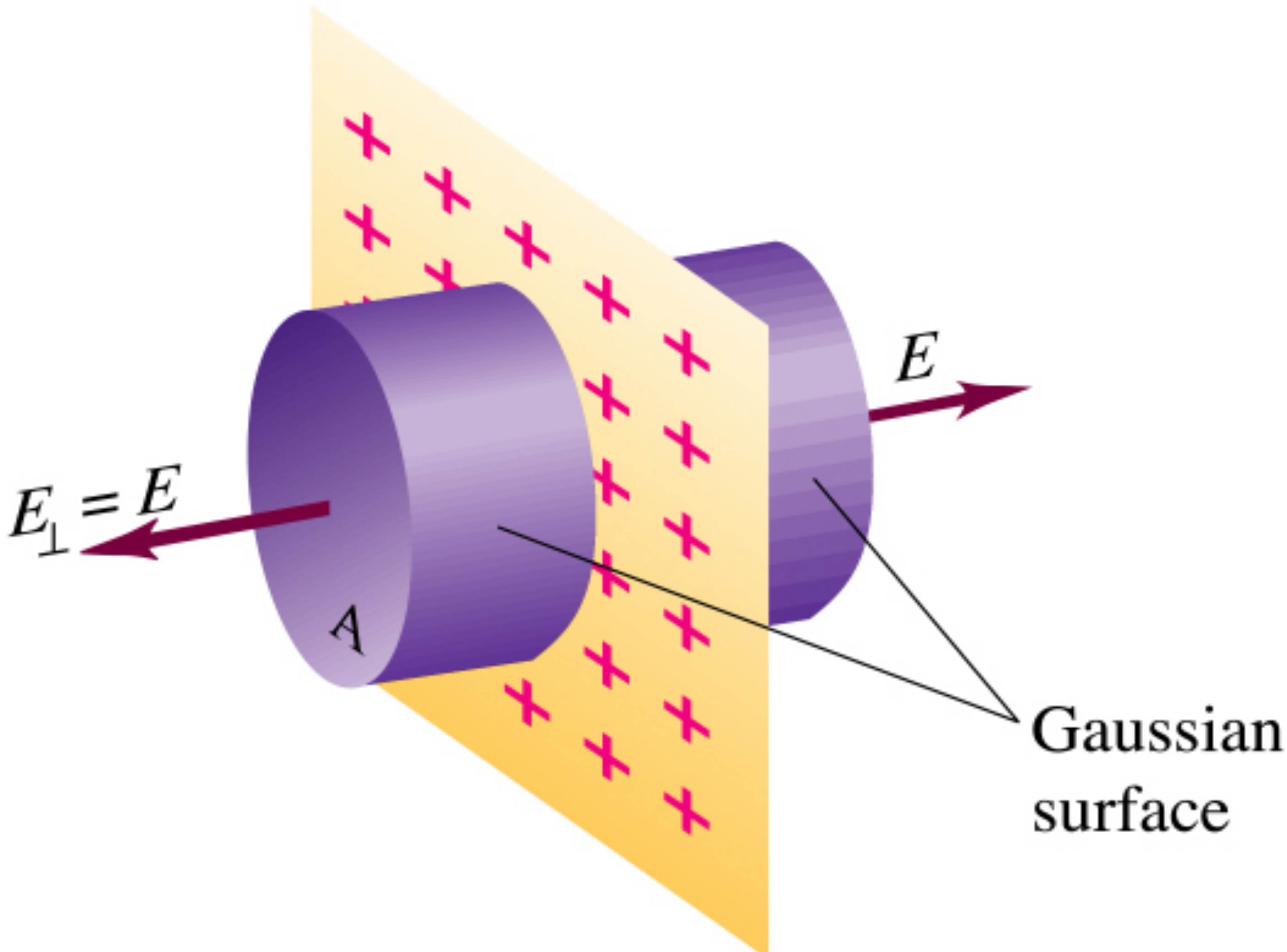
Electric field = **zero** (electrostatic) inside a solid conducting sphere

Electric field around a infinitely long line charge (22.6)



A coaxial cylindrical **Gaussian surface** is used to find the electric field outside an infinitely long charged wire.

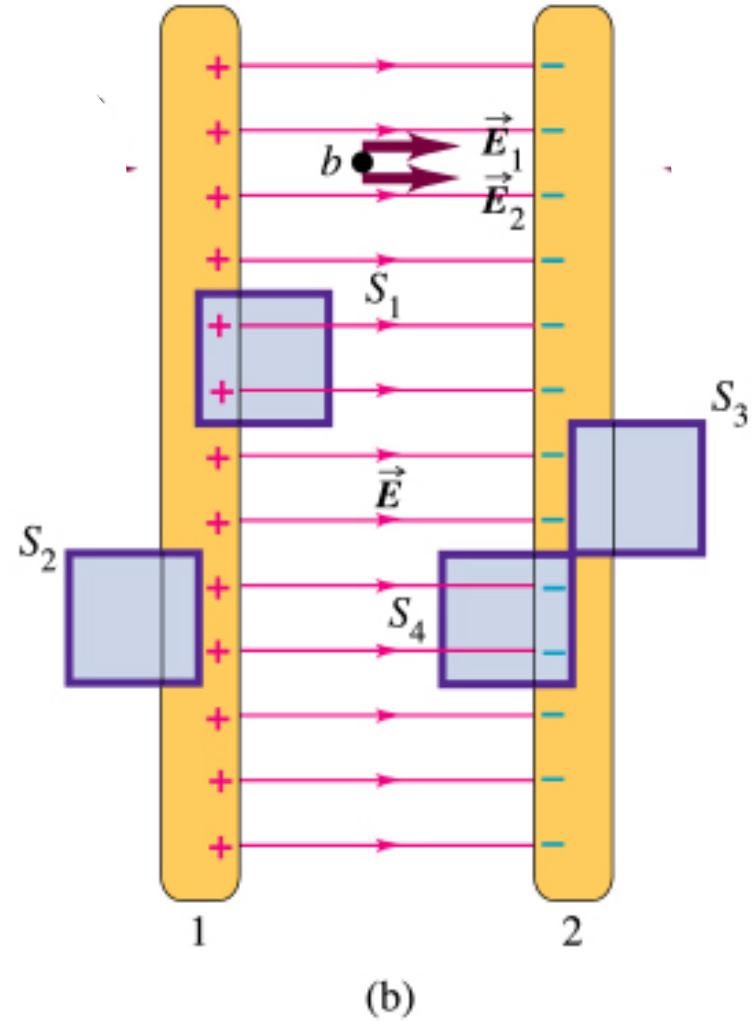
Field of an infinite plane sheet of charge (22.7)



A **cylindrical Gaussian surface** is used to find the electric field of an infinite plane sheet of charge.

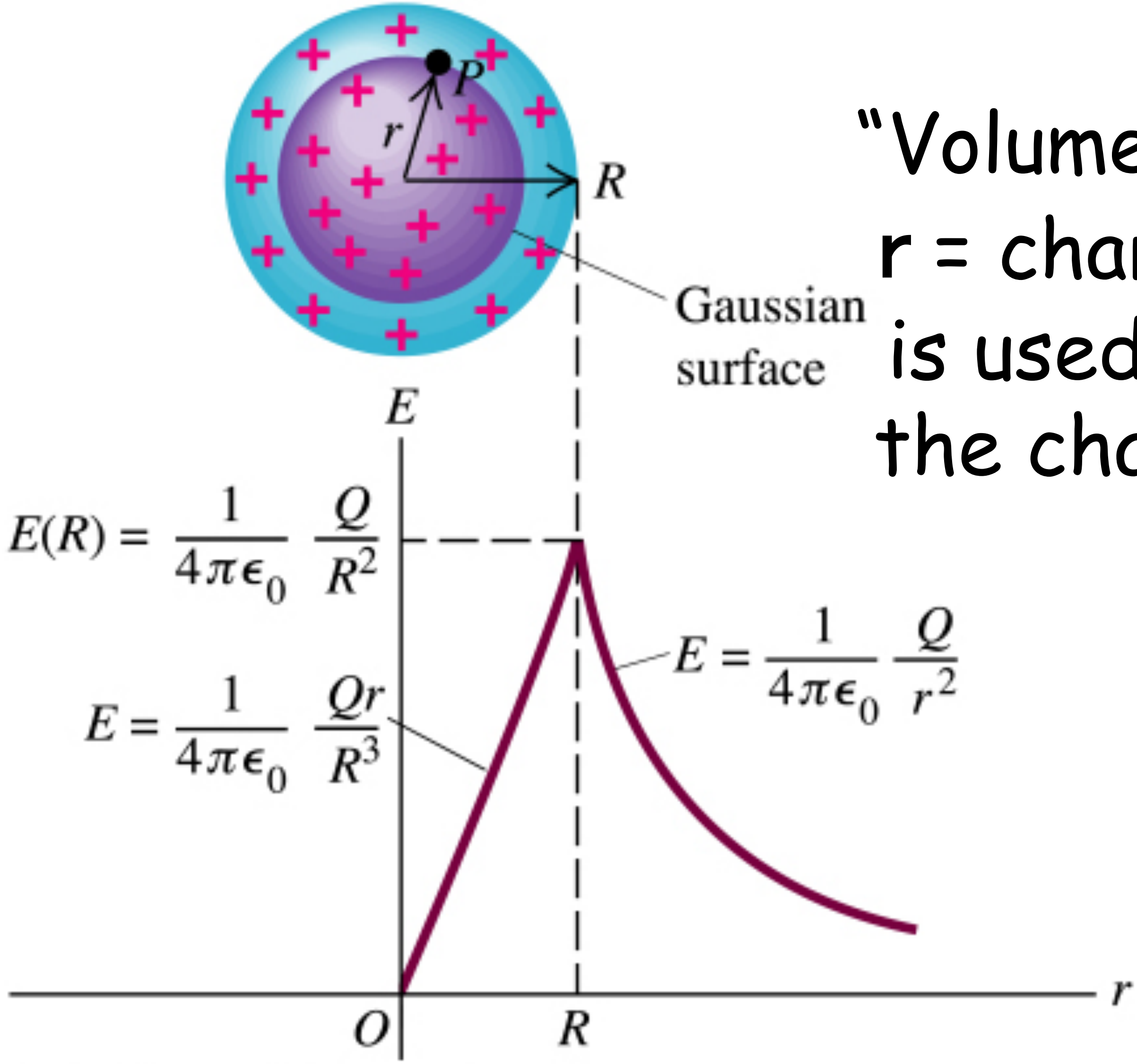
Field between oppositely charged parallel conducting plates (22.8)

Ignoring edge effects



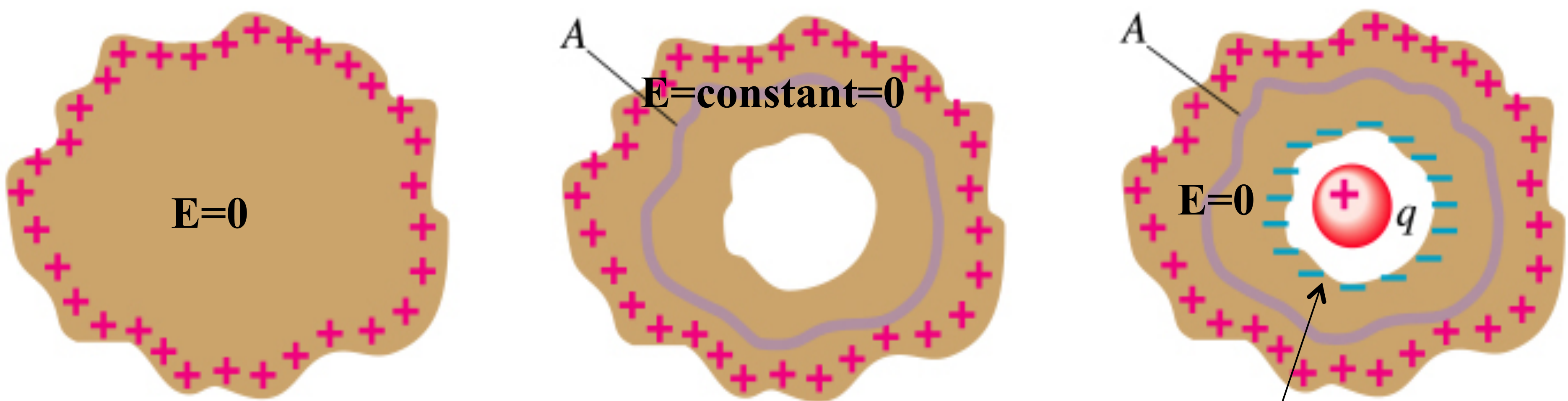
Electric field between two (large) oppositely charged parallel plates.

Field of a uniformly charged non-conducting sphere (22.9)



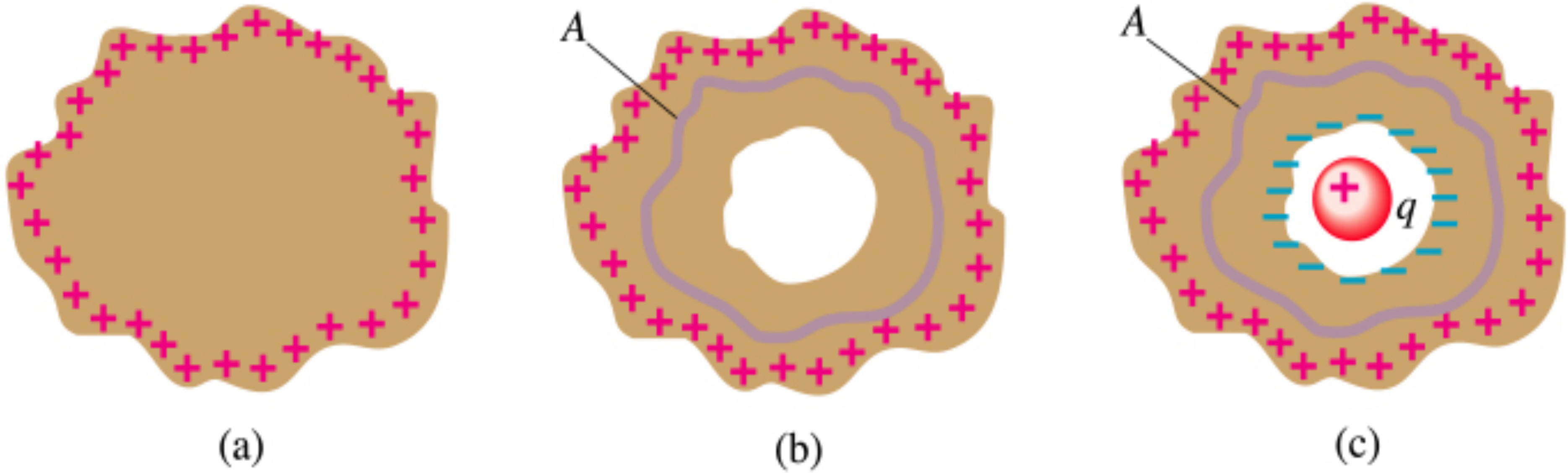
"Volume charge density":
r = charge / unit volume
is used to characterize
the charge distribution.

E-field within a charged conductor



(a) (b) (c)

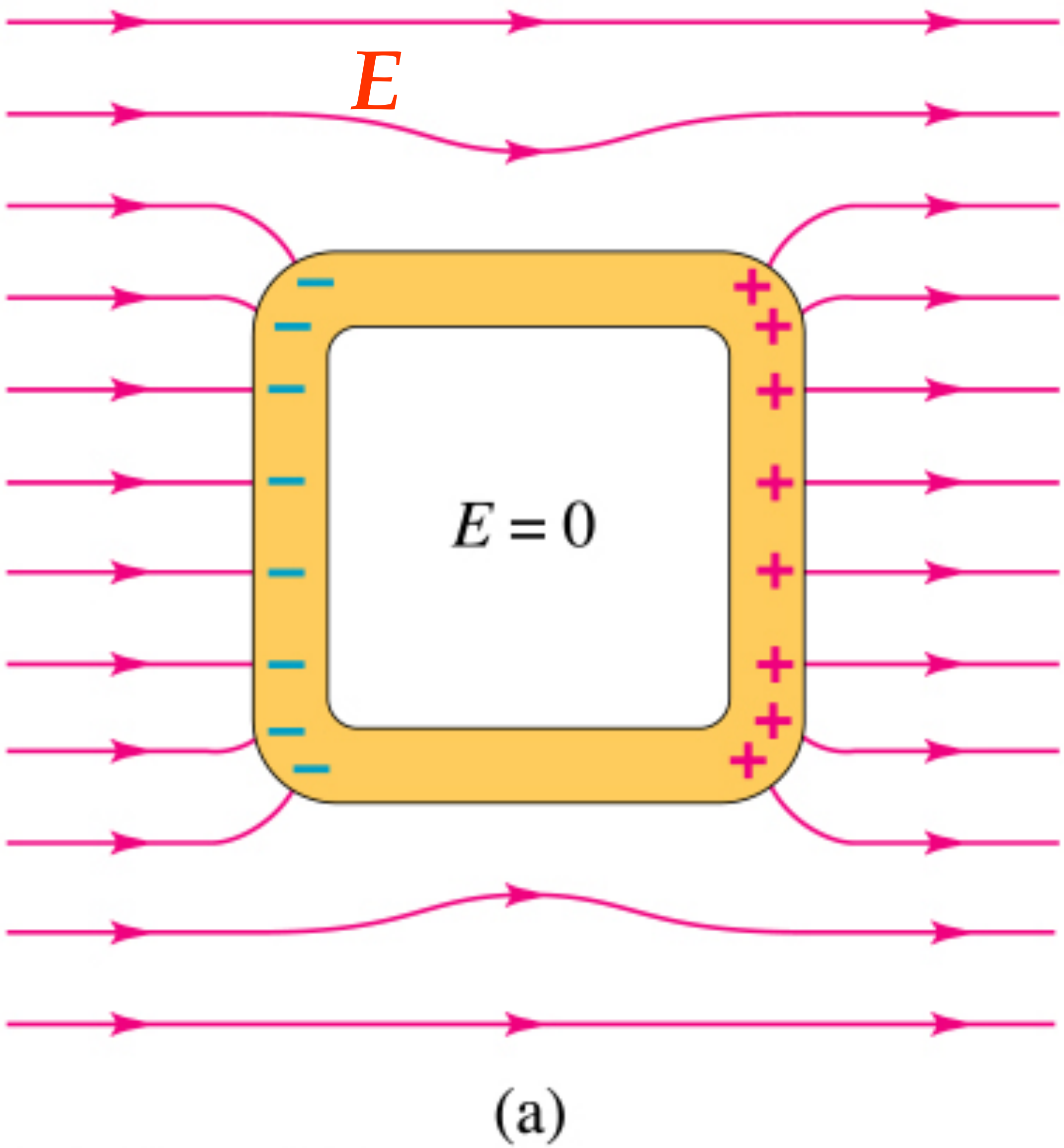
Induced charge Inside on the conductor surface to make E=0 inside the conductor



The solution of this problem lies in the fact that the electric field inside a conductor is zero and if we place our **Gaussian surface** inside the conductor (where the field is zero), the charge **enclosed** must be zero (+ q – q) = 0.

Find electric charge q on surface of hole in the charged conductor.

A Faraday Cage



A **Gaussian surface** drawn inside the conducting material of which the box is made must have zero electric field on it (field inside a conductor is zero). If the **Gaussian surface** has zero field on it, the charge enclosed must be zero per **Gauss's Law**.

The ***E*** field inside a conducting box (a "Faraday cage") in an electric field.

Chapter 23 Electric Potential

• Electric potential energy	(sec. 23.1)
• Electric potential	(sec. 23.2)
• Calculating elec. potential	(sec. 23.3)
• Equipotential surfaces	(sec. 23.4)
• Potential gradient	(sec. 23.5)

Learning Goals - we will learn: ch 23

- How to calculate the electric potential energy (U) of a collection of charges.
- The definition and significance of electric potential (V).
- How to use the electric potential to calculate the electric field (E).