1) Dispersion and Abbe number

$\frac{dn}{d\lambda} < 0$ Positive (Normal) Dispersion

for a positive dλ, dn is negative

$\frac{dn}{d\lambda} > 0$ Negative (Anomalous) Dispersion

for a positive dλ, dn is positive

$$V_d = \frac{n_d - 1}{n_f - n_c} \qquad \lambda_f := 480 \cdot nm \qquad \lambda_c := 643.8 \cdot nm \qquad \lambda_d := 587.6 \cdot nm$$

n is always greater than 1 so top of equation is always positive

looking at dispersion in terms of the Abbe number $\frac{n_f - n_c}{\lambda_f - \lambda_c} = \frac{dn}{d\lambda}$

A) what kind of Abbe V number offers large dispersion.

for a large Abbe V n_f-n_c would need to be small. This would result in a small dispersion.

·a small Abbe V number would have a large dispersion typical values are for heavier flint glasses have V_d ranging from 30 to 40

B) What is the sign of the Abbe V number for a glass with

$\lambda_c > \lambda_f$ so $\lambda_f - \lambda_c < 0$

a) negative dispersion $n_c > n_f$ for negative dispersion

and the abbe number would be negative

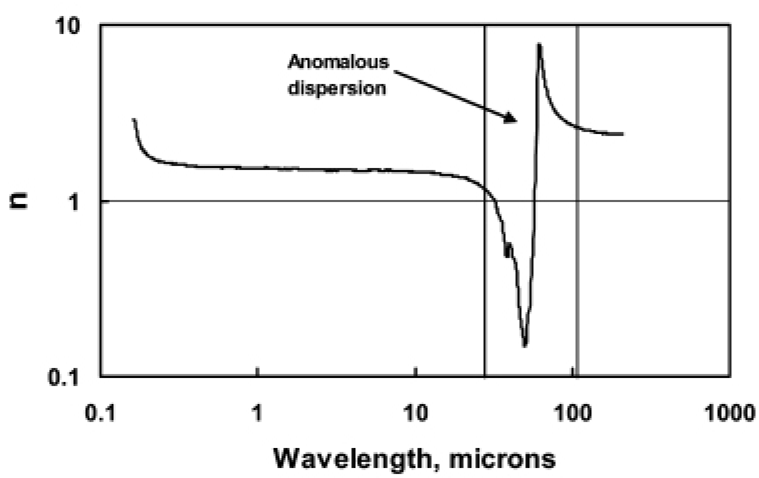
b) positive dispersion $n_c < n_f$ for positive dispersion

Abbe number is also positive

c) no dispersion. The numerator would be zero so the Abbe V number would be infinity

C. Find 3 glasses from a catalog with

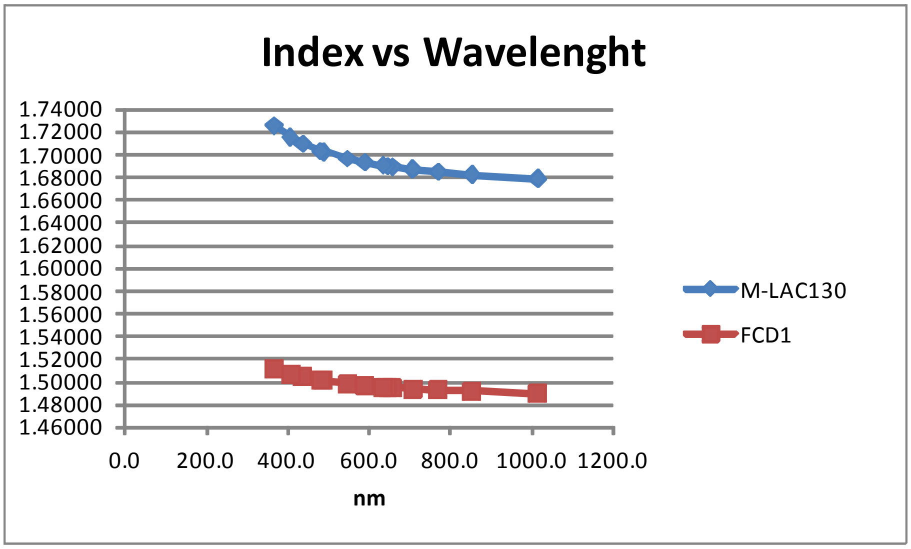
a) Negative dispersion: NaCL but only in the IR



Normal and anomalous dispersion for single-crystal NaCl over the visible and infrared spectral regions

http://spie.org/x33587.xml

b) Positive dispersion M-Lac130 from Hoya Abbe 53
c) Minimal Dispersion. FCD1 from Hoya Abbe 81

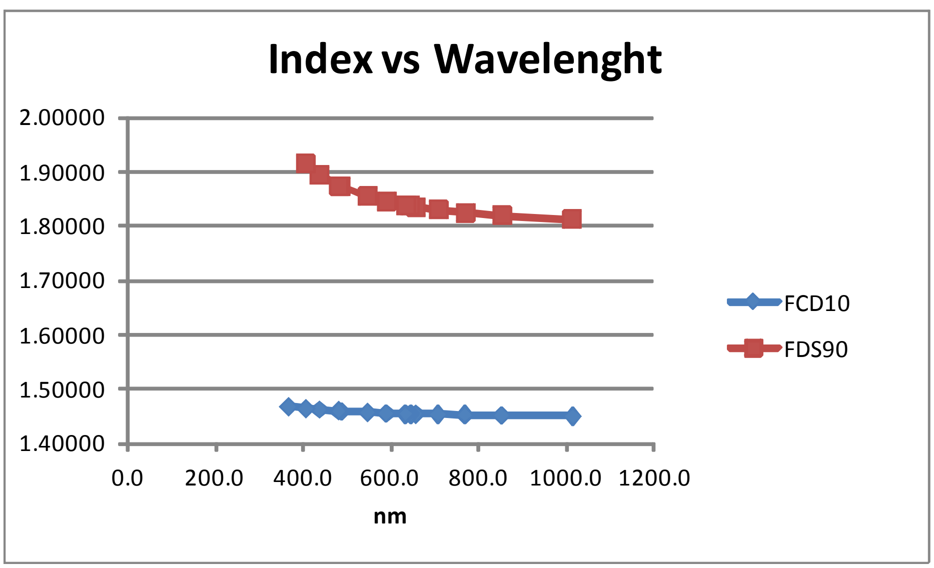


D) what can we say about dispersion of glass in the IR range based on the Abbe V number

nothing abbe V is defined only in the visible range. Alternate wavelengths such as the λ_K λ_{Hg} and λ_{Cs} could be used to determine an alternate measurement

E) Find the catalog information for a low and high dispersion glass and explain the meaning of the information listed for them

Low Dispersion. FCD10 from Hoya Abbe 90.27
High Dispersion FDS90 from Hoya Abbe 23.78



3) A camera focused between 3 meters & infinity

• A point and shoot camera company claims their digital camera can take sharp images of the objects located between 1m and infinity.
The pixel size is 3X3 (micron2), Lens aperture is 5mm in diameter, lens focal length is 10mm.
a) Evaluate validity of the claim numerically for green light (550nm). Draw diagrams if necessary.
b) What happens when you consider red (750 nm) and blue (450nm) light?
For which wavelength the focus is better?
c) What if the the diameter of the lens is 2.5 mm and f is 5 mm
d) What if the diameter is 10 mm and the focal length is 10mm.
e) Organize your findings as a function of f# which is f/D and wavelength in the form of an easy rule to remember.

Should look at diffraction and defocus $f := 10 \cdot mm$ $D := 5 \cdot mm$ $pixel := 3 \cdot \mu m$

First lets look at diffraction limit based on wavelength

Airy disk calcs $\lambda_r := 750 \cdot nm$ $\lambda_g := 550 \cdot nm$ $\lambda_b := 450 \cdot nm$

$\theta = 1.22 \cdot \frac{\lambda}{D}$ $\theta = \frac{x}{f}$ assumes small angle $x = 1.22 \cdot \frac{\lambda \cdot f}{D} = 1.22 \cdot \lambda \cdot fnum$

$x_r := 2 \cdot 1.22 \cdot \frac{\lambda_r \cdot f}{D}$ $x_g := 2 \cdot 1.22 \cdot \frac{\lambda_g \cdot f}{D}$ $x_b := 2 \cdot 1.22 \cdot \frac{\lambda_b \cdot f}{D}$

$x_r = 3.66 \cdot \mu m$ $x_g = 2.684 \cdot \mu m$ $x_b = 2.196 \cdot \mu m$

$pixel = 3 \cdot \mu m$

Pixel size OK for blue and green if we consider only one pixel as limit, red woould bleed into adjacent pixel. Would be OK for 2 pixel as min COC.

all visible wavelengths would produce an airy disk smaller than a single pixel

Next lets look at the focal point for an object at infinity, 1m and 10 m.

Case 1) Object at infinity

$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_{in}}$ $s_{in} := \infty \cdot m$ $\frac{1}{s_{in}} = 0 \frac{1}{m}$

$s_{2in} := \frac{1}{\frac{1}{f} - \frac{1}{s_{in}}}$ $s_{2in} = 10 \cdot mm$

Case 1) Object at 1 m

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_{21}} \qquad s_1 := 1 \cdot m \qquad s_{21} := \frac{1}{\frac{1}{f} - \frac{1}{s_1}} \qquad s_{21} = 10.101 \cdot mm$$

$$\Delta I_{in1} := s_{21} - s_{2in} = 0.101 \cdot mm$$

So the circle of confusion can be calculated as the height extreme ray coming from edge of lens to the focal

$$\frac{D}{s_{21}} = \frac{COC_{21}}{\Delta I_{in1}} \qquad COC_{21} := \frac{D \cdot \Delta I_{in1}}{s_{21}} = 50 \cdot \mu m$$

$pixel = 3 \cdot \mu m$ much larger than 2 pixels

Case 2) Object at 3 m

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_{23}} \qquad s_3 := 1 \cdot m \qquad s_{23} := \frac{1}{\frac{1}{f} - \frac{1}{s_3}} \qquad s_{23} = 10.101 \cdot mm$$

$$\Delta I_{in3} := s_{23} - s_{2in} = 0.101 \cdot mm \qquad \frac{D}{s_{31}} = \frac{COC_{31}}{\Delta I_{in1}} \qquad COC_{31} := \frac{D \cdot \Delta I_{in3}}{s_{23}} = 50 \cdot \mu m$$

$pixel = 3 \cdot \mu m$ much larger than 2 pixels

If we want to have sharp images of object at infinity and at 1 or 3 meters we can't use this lens diameter and focus combination.

So lets assume we want to calculate at what distance we could still have a COC that was equal to 2 pixels. One method is the calculate the hyper focus. In this case we will calculate the closest distance an object can be in focus and objects at infinity will still be acceptably sharp.

http://en.wikipedia.org/wiki/Hyperfocal_distance

$$COC := 2 \cdot pixel \qquad fnum := \frac{f}{D} = 2$$

$$H_f := \left(\frac{f^2}{fnum \cdot COC} + f \right) \quad \text{Hyper focus} \qquad H_f = 8.343\ m$$

To determine if chromatic aberration is an issue we need more information about the lens type, and material, doublet, triplet, achromatic etc.

a) Evaluate validity of the claim numerically for green light (550nm). Draw diagrams if necessary

Claim is false due to COC from geometric optics see above. Would be OK for Airy disk only.

b) What happens when you consider red (750 nm) and blue (450nm) light? For which wavelength the focus is better?

Claim would still be dominated by COC due to geometric, if we only consider difraction, blue would be better and red worse than green. (See above)

c) What if the the diameter of the lens is 2.5 mm and f is 5 mm

$f_c := 5 \cdot mm$ $\quad D_c := 2.5 \cdot mm$ $\quad COC := 2 \cdot pixel$ $\quad fnum_c := \frac{f_c}{D_c} = 2$

$$H_{fc} := \left(\frac{f_c^{\,2}}{fnum_c \cdot COC} + f \right)$$ Hyper focus $\quad H_{fc} = 2.093\ m$ $\quad$ would work at 3m but not at 1.

$$x = 2 \cdot 1.22 \cdot \frac{\lambda \cdot f}{D} = 1.22 \cdot \lambda \cdot fnum$$ fnum did not change so diffraction still OK

d) What if the diameter is 10 mm and the focal length is 10mm.

$f_d := 10 \cdot mm$ $\quad D_d := 10 \cdot mm$ $\quad COC = 6 \cdot \mu m$ $\quad fnum_d := \frac{f_d}{D_d} = 1$

$$H_{fd} := \left(\frac{f_d^{\,2}}{fnum_d \cdot COC} + f_d \right)$$ $H_{fd} = 16.677\ m$ $\quad$ Worse than original Will not work

$$x = 1.22 \cdot \frac{\lambda \cdot f}{D} = 2 \cdot 1.22 \cdot \lambda \cdot fnum$$ fnum is 1/2 of original so diffraction is even better

e) Organize your findings as a function of f# which is f/D and wavelength in the form of an easy rule to remember.

$$H_f = \frac{f^2}{fnum \cdot COC} + f$$ if f<< than object distance we can simplify

$$H_f = \frac{f^2}{fnum \cdot COC}$$

$$COC = \frac{f^2}{H_f \cdot fnum}$$ $\quad COC = 2 \cdot 1.22 \cdot \lambda \cdot fnum$ $\quad$ Simple rules either could dominate