Static equilibrium

ME 297-1
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SJSU
Nayer Eradat
Based on Jim Burge's notes and other online resources

Static equilibrium

- Tolerances for optics are very tight so we need good mechanical structure to support them in accurate locations
- Our job is to determine
  - Motion
  - Distortion under forces
- To do this we need to evaluate the system under applied forces and system's reaction forces
- This section is on
  - Definition of applied forces & moments
  - Developing the free body diagrams (FBD)
  - Solving static equilibrium condition
  - Finding reaction forces and moments

Forces

- Forces are vectors
- They have a magnitude and direction
- Forces accelerate an object according to the Newton's second law

$$\mathbf{F} = m\mathbf{a} \qquad \text{or more generally} \qquad \mathbf{F} = \frac{d\mathbf{P}}{dt}$$

- Forces rotate objects
- Forces can stretch a spring scale
- Units for force based on Pound or Newton (SI)
  - 1pound(lb_F)=4.45N
  - 1N=0.22lb

Gravitational forces

*G*ravitational force between objects of mass m_1 and m_2 at a distance r

from each other: $$\mathbf{F} = G\frac{m_1 m_2}{r^2}\mathbf{r}$$

where $G = 6.67384 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$ is the universal gravitational constant.

Acceleration of gravity for nearby objects is: $g = G\frac{m_e}{r_e^2} \simeq 9.8\frac{m}{s^2} = 386\frac{in}{s^2}$

Then the force applied by earth on an object of mas m becomes:

$$W = mg$$

where W is the weight. **Weight is not** the same as **mass.**

Weight is a vector quantity and represents a force.

Mass is a scalar quantity and represents amount of material.

1 kg mass weighs 9.8 N or 2.2 lbs

1 $\mathrm{lb_M}$ is the mass that weighs 1 pound. 1 slug mass weighs 32.2 lbs

Pound and newton are forces, slug and kg are masses

The moment

The moment (torque) is defined as

$$\vec{\tau} = \mathbf{M_A} = \mathbf{r_{AB}} \times \mathbf{F_B}$$

$$\tau = |\mathbf{M_A}| = r_{AB} F_B \sin\theta = r_{AB} \bullet F_\perp = r_\perp \bullet F_B$$

Units are in-lb or N.m

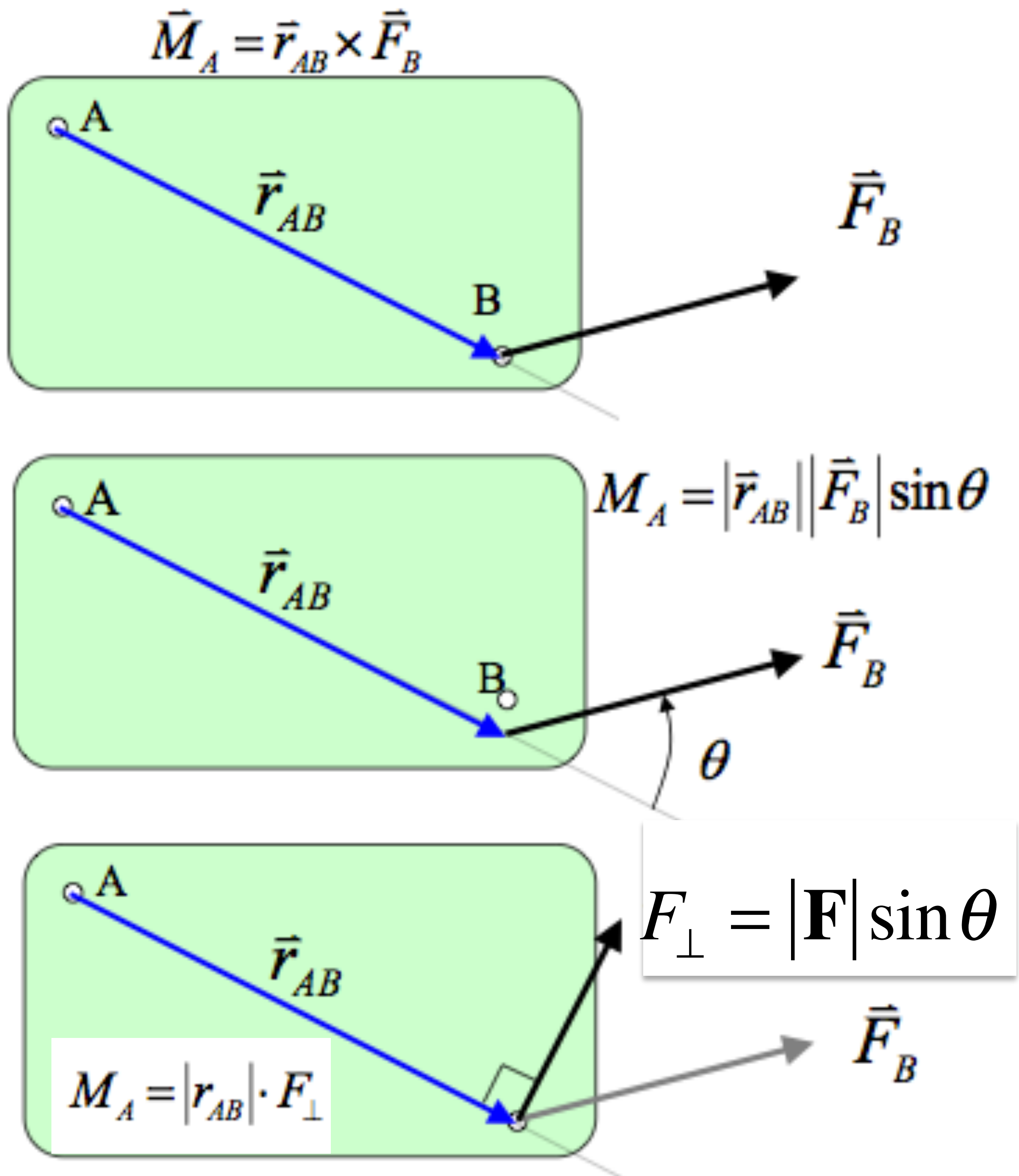
1 N.m = 8.84 in.lb

Forces without moment (F || r) can cause **tanslational** motion

Forces with moment (F not prallel to r) can cause **translational and rotational** motion

Moments are "twisting forces". They make things rotate

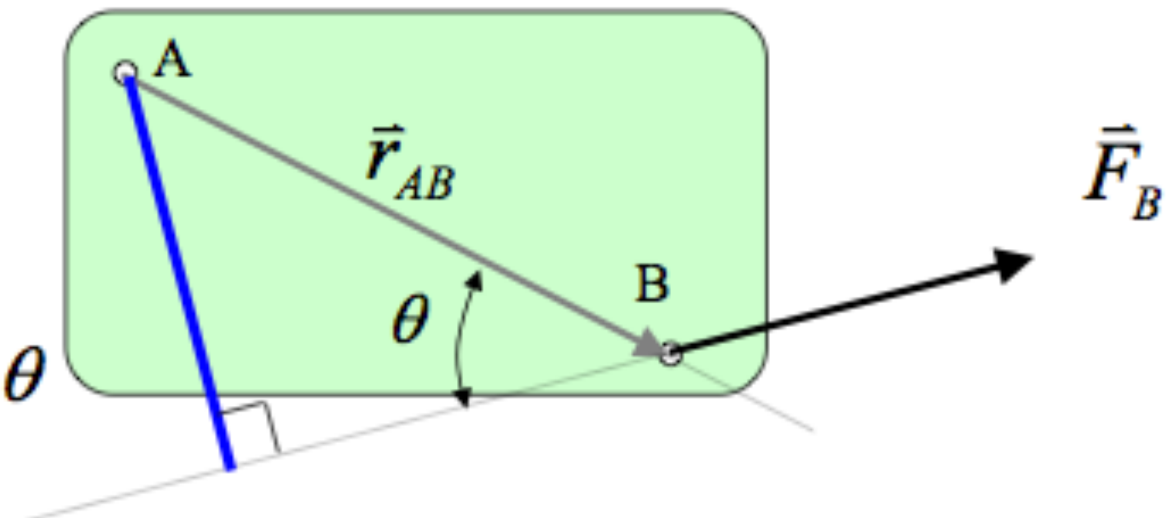
Defining moment (torque) from applied force



Defining moment (torque) from applied force

$$M_A = r_\perp \cdot |F|$$

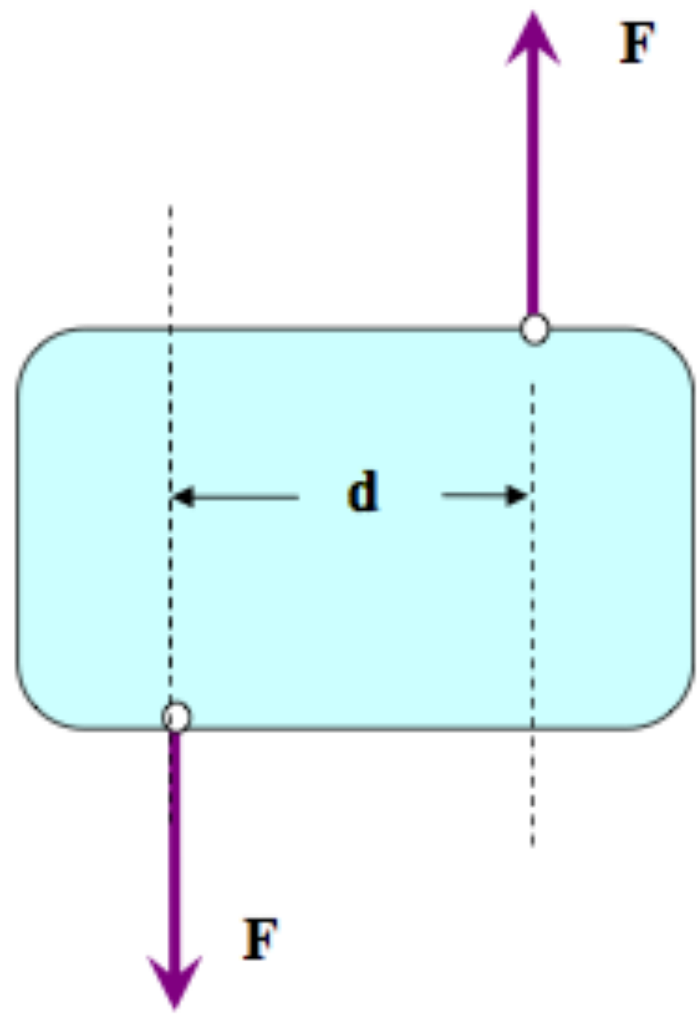
$$r_\perp = |\vec{r}_{AB}| \sin\theta$$



Force couples

- Two forces, equal and opposite in direction, which do not act in the same line are force couples and cause a pure moment.
- r is measured from the axis of rotation

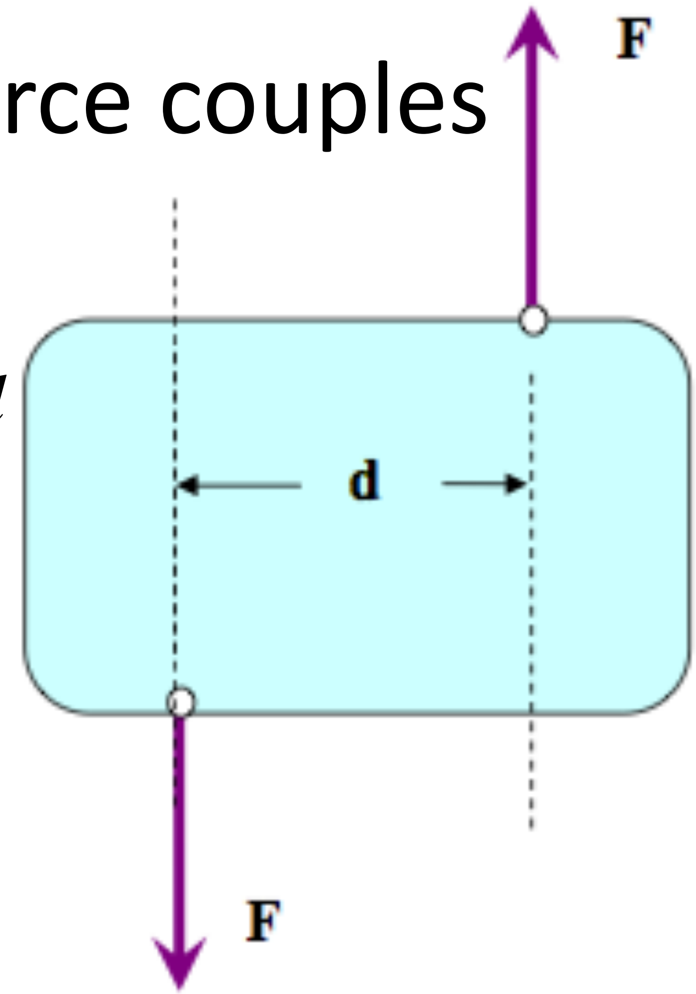
$$\vec{\tau} = \mathbf{M}_{\mathbf{A}} = \mathbf{r}_{\mathbf{AB}} \times \mathbf{F}_{\mathbf{B}}$$



Pure moment caused by force couples

M = F d

$$\tau = F\frac{d}{2}\sin 90^0 + (-F)\left(-\frac{d}{2}\right)\sin 90^0 = Fd$$



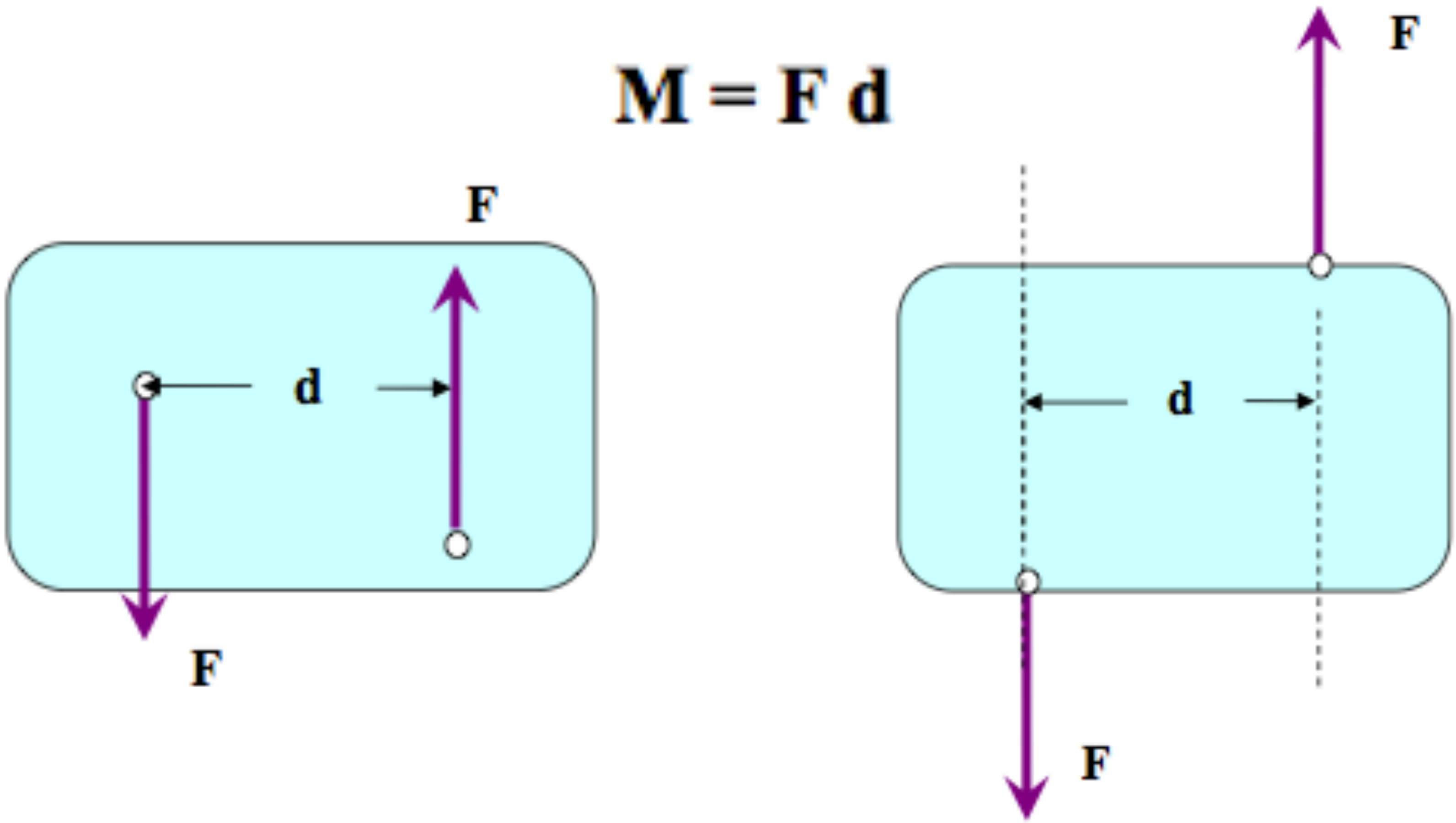
Direction of moment is determined by the right hand rule. If r is the direction of index finger, and F is in the direction of middle finger, then the thumb points at the direction of the torgue.

Direction of the torque is the direction of the axis of rotation.

Direction of the rotation is from the first vetor (F) towards the second vector (r) or the fingers of the right hand.

Effect of the force action point
For vectors direction matters not location

$$M = F\,d$$



Special cases

- **Unstretchable Cable:**
  - **Can only transmit tension along direction of cable**
  - **No compression**
  - **No moment**
  - **No lateral force**
- **Tension along the cable stays constant**
- **Only an approximation to real life situations.**

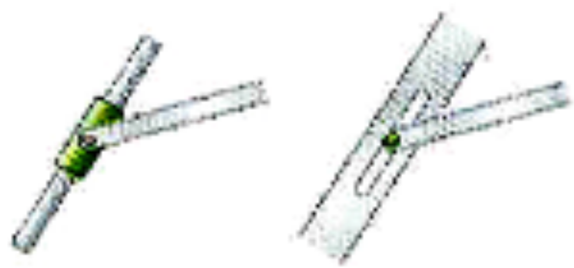
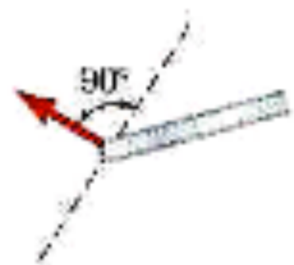
Static determinacy

- Determinacy deals with whether or not the reactions and forces in a structure can be analyzed based **solely on static equilibrium** or whether **principles from strength of materials** must be introduced.
- Determinacy deals with whether the forces in a structure can be determined knowing **only the geometry** of the structure or whether the stiffness attributes of the individual components must be known.
- A *statically determinate structure is analyzable based only on its geometry.*
- *A statically indeterminate structure is analyzable based on geometry and component stiffness.*
- *We need to be able to distinguish between determinate and indeterminate structures.*

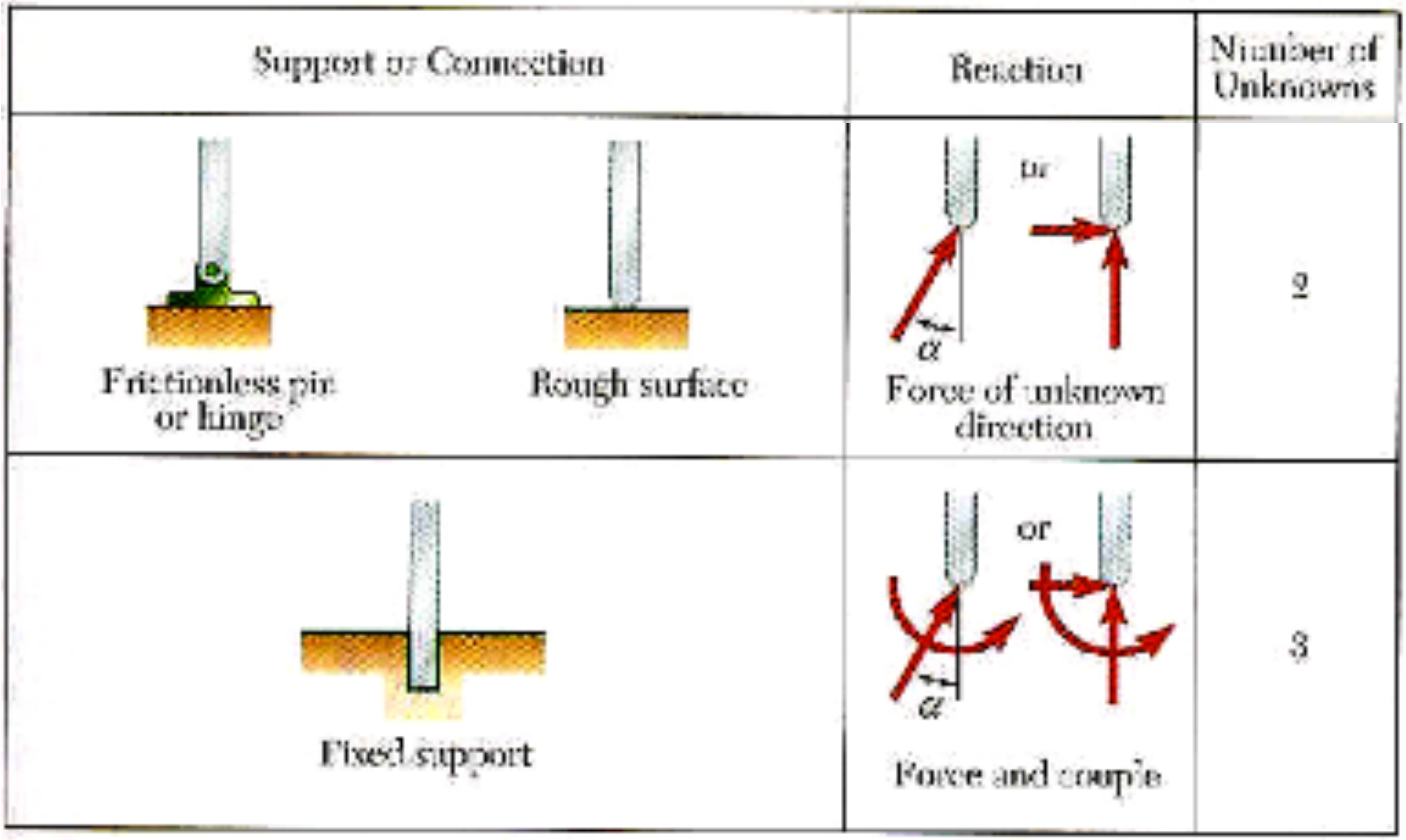
Constraints: attachment points that will maintain their position

- A degree of freedom taken away from the system.

Constraints

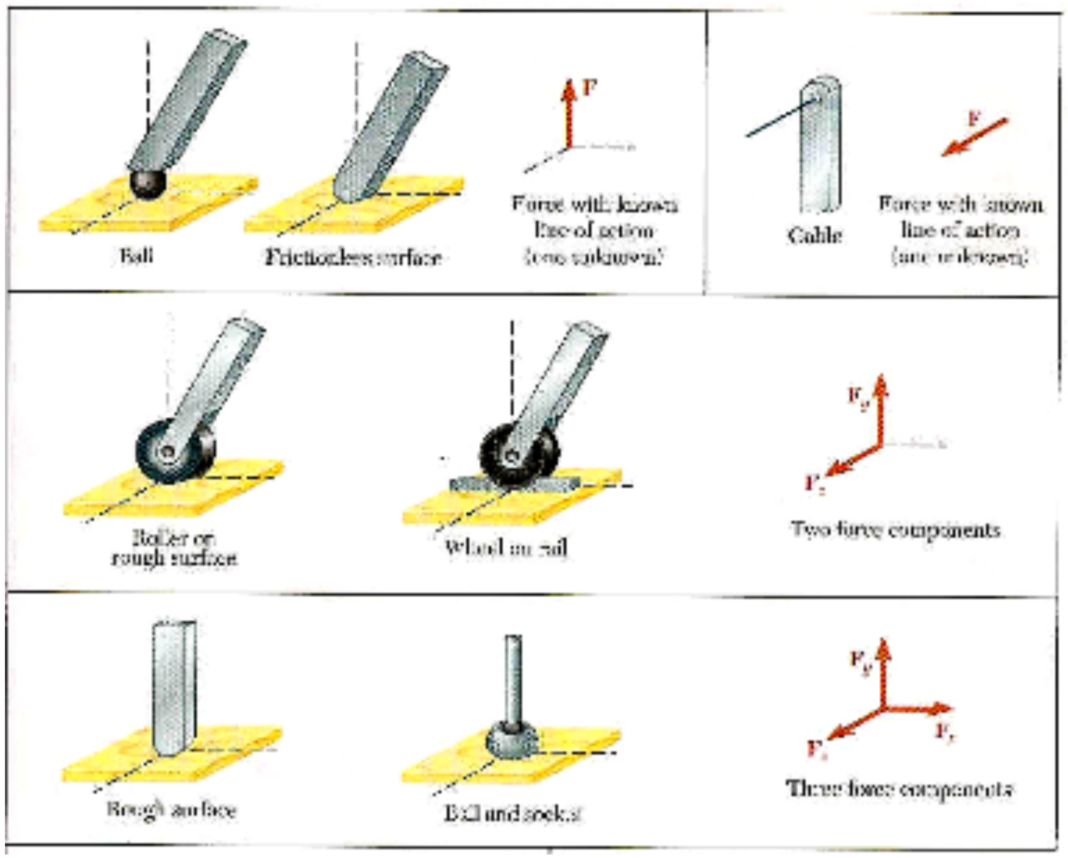
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1

Constraints

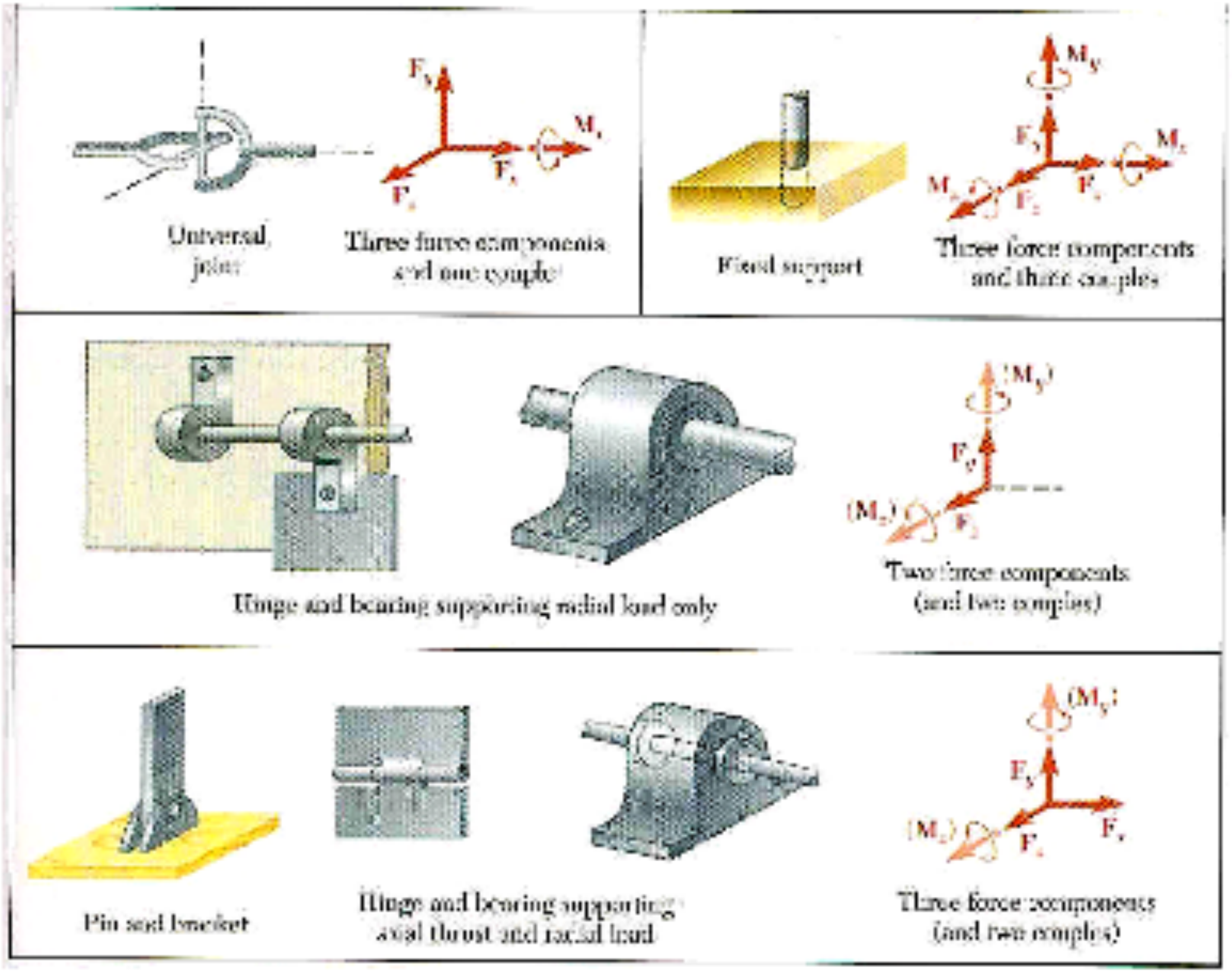


Support or Connection		Reaction	Number of Unknowns
Frictionless pin or hinge	Rough surface	Force of unknown direction	2
Fixed support		Force and couple	3

Idealization of 3D supports



Idealization 3D supports

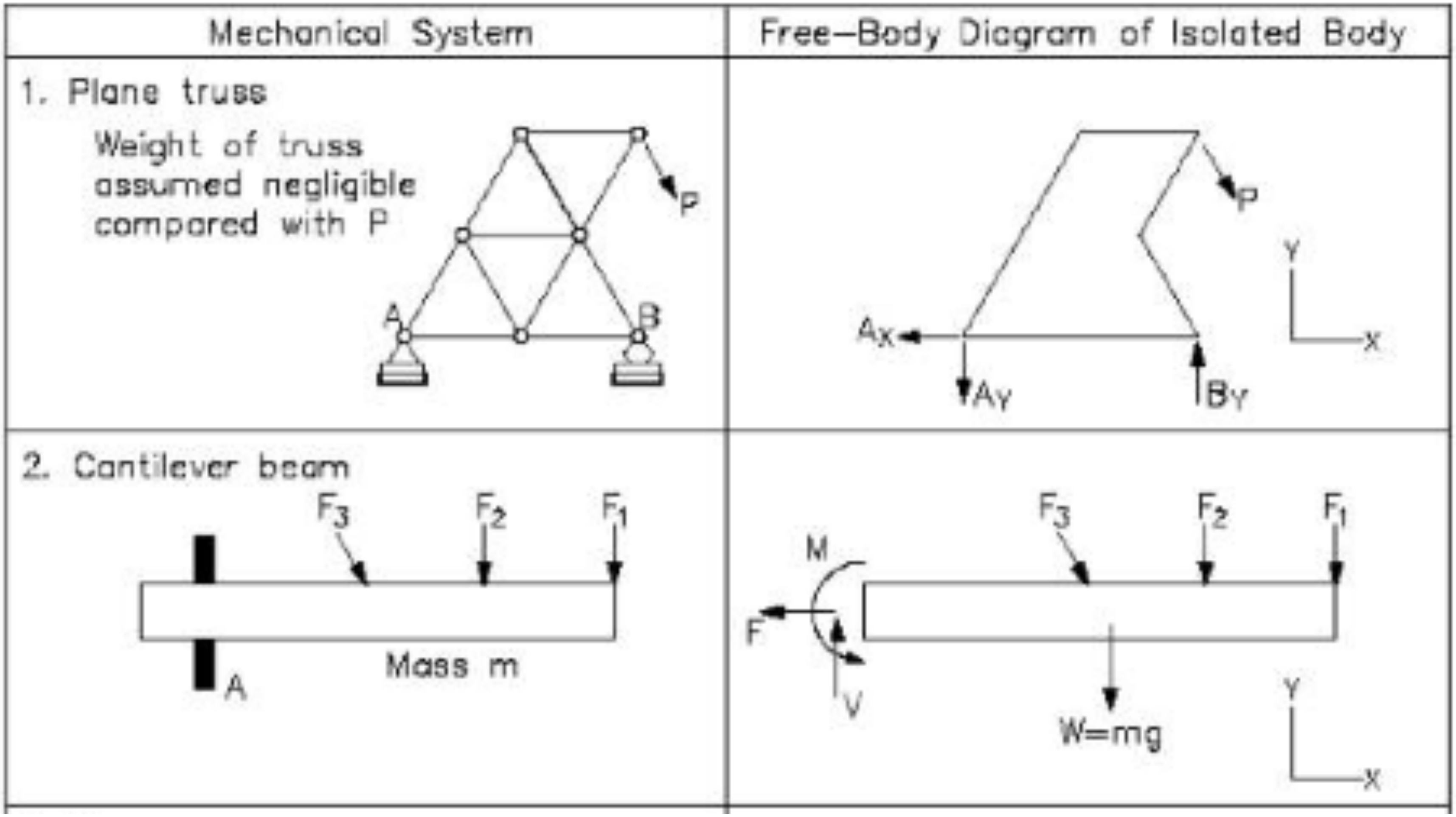


Free body diagrams: A tool to solve mechanics problems

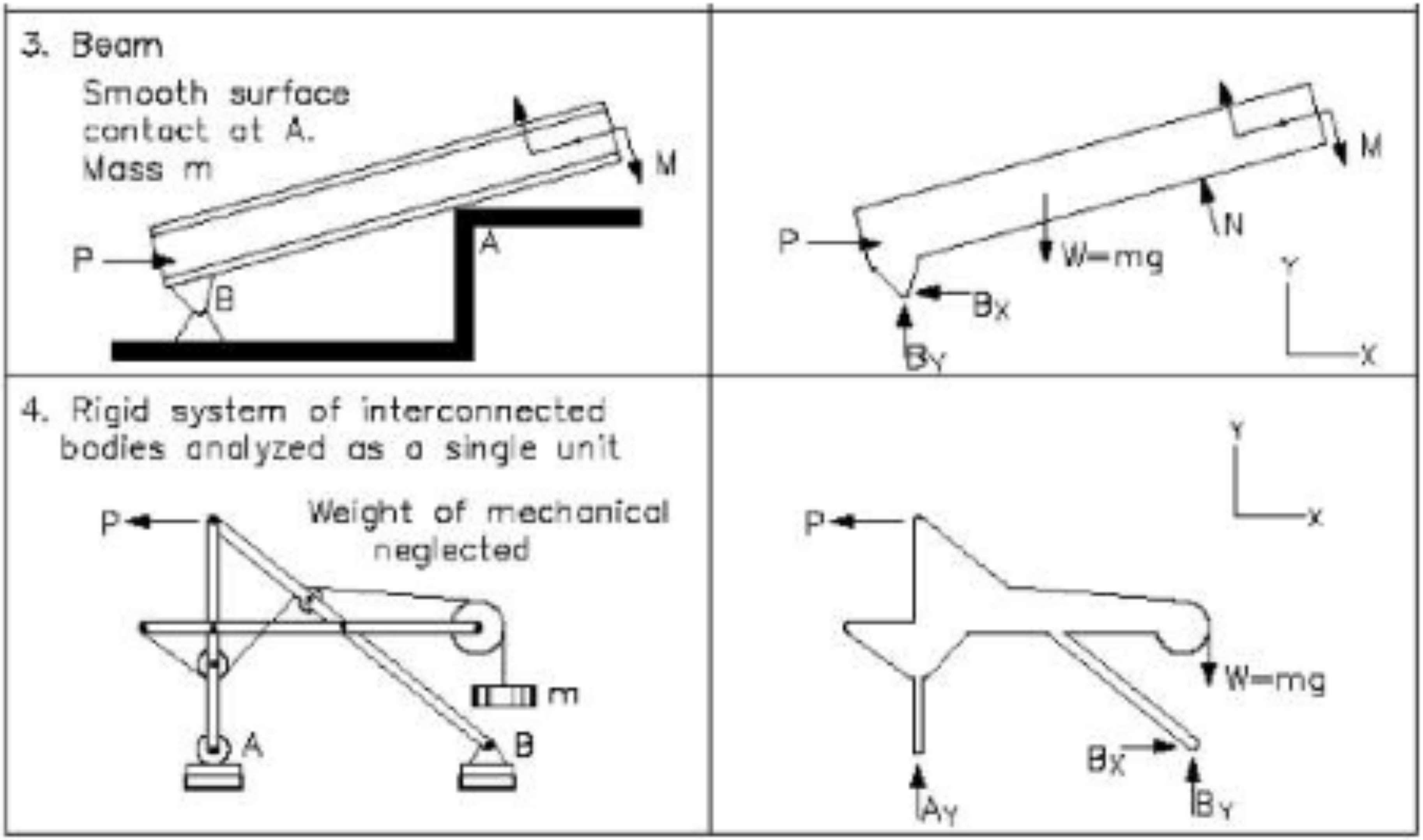
1. **Determine which body or combination of bodies is to be isolated.** The body chosen will usually involve one or more of the desired unknown quantities.
2. **Isolate the body or combination of bodies chosen with a diagram that represents its complete external boundaries.**
3. **Represent all forces that act on the isolated body (include gravity, normal forces, friction etc.)as applied by the removed contacting bodies in their proper positions in the diagram of the isolated body**. Do not show the forces that the object exerts on anything else, since these forces do not affect the object itself.
4. **Indicate the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience.**

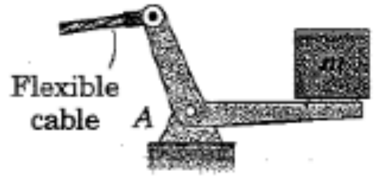
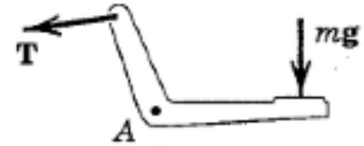
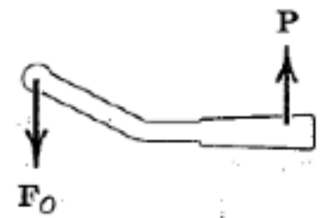
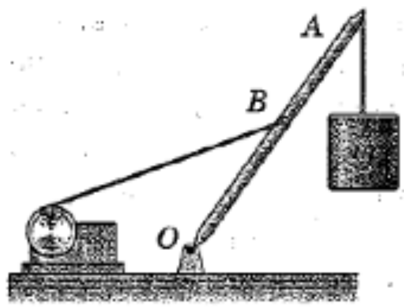
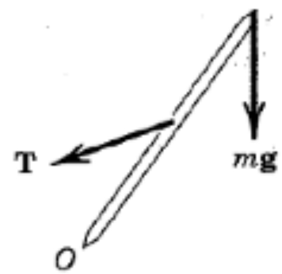
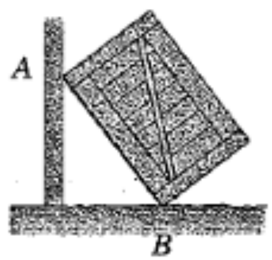
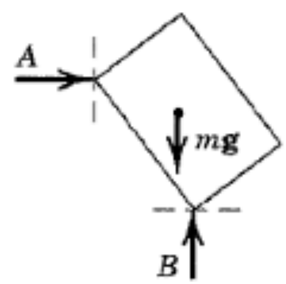
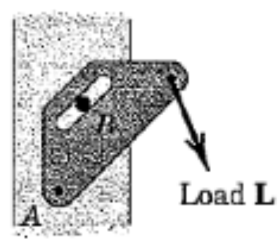
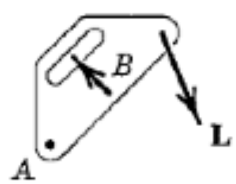
- Free-body diagram serves the purpose of focusing accurate attention on the action of the external forces (on the body of choice).
- The diagram should not be cluttered with excessive information.
- Force arrows should be clearly distinguished from other arrows to avoid confusion.

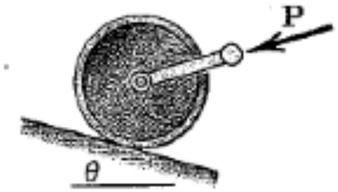
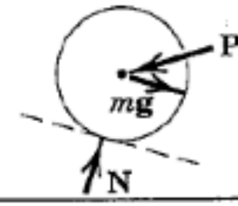
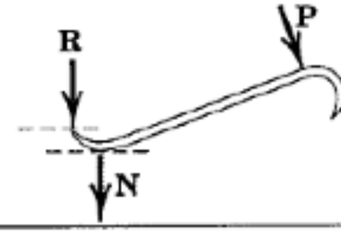
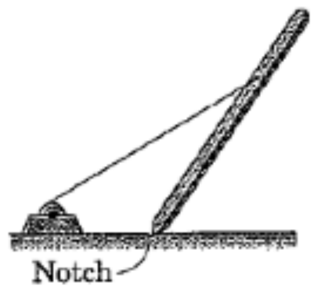
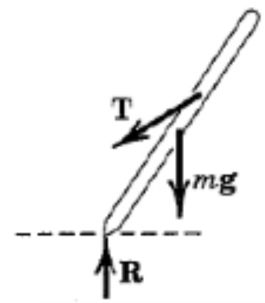
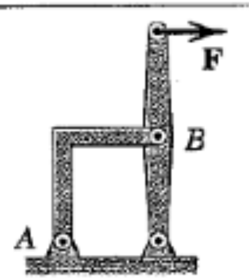
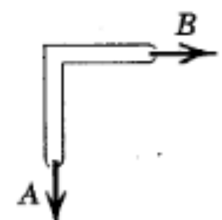
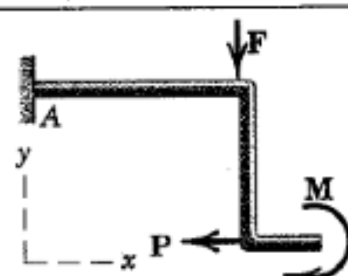
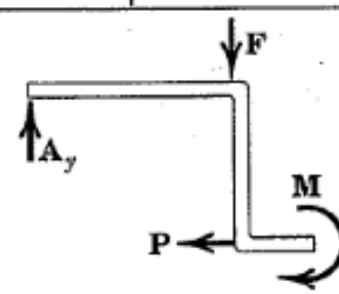
Sample FBDs


Mechanical System
Free–Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
A
B
P
Ax
Ay
By
Y
X
2. Cantilever beam
F3
F2
F1
A
Mass m
M
F
V
W=mg
Y
X

Sample FBDs

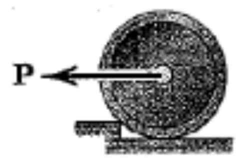

3. Beam
Smooth surface contact at A.
Mass m
M
P
A
B
M
N
P
W=mg
Y
Bx
By
X
4. Rigid system of interconnected bodies analyzed as a single unit
Weight of mechanical neglected
P
m
A
B
Y
P
X
W=mg
Bx
Ay
By

	Body	Incomplete *FBD*
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

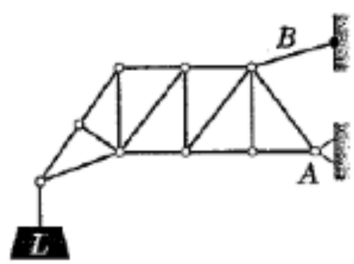
	Body	Wrong or Incomplete *FBD*
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; Pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.

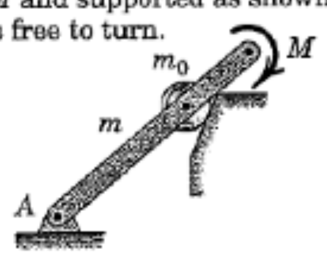
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



3. Loaded truss supported by pin joint at A and by cable at B.



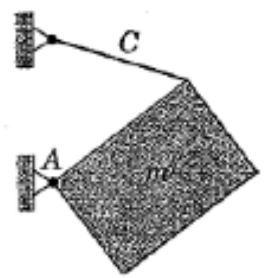
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



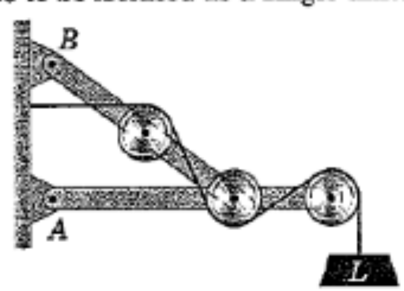
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.

6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.

7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.



Static equilibrium

For rigid body to be at static equilibrium we must have

$$\sum \mathbf{F} = 0 \qquad \& \qquad \sum \mathbf{M} = 0$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$\sum F_x = 0 \qquad \sum M_x = 0$$

$$\sum F_y = 0 \qquad \sum M_y = 0$$

$$\sum F_z = 0 \qquad \sum M_z = 0$$

This in general is a system of 6 equations.

For systems in-plane it reduces to 3 equations.

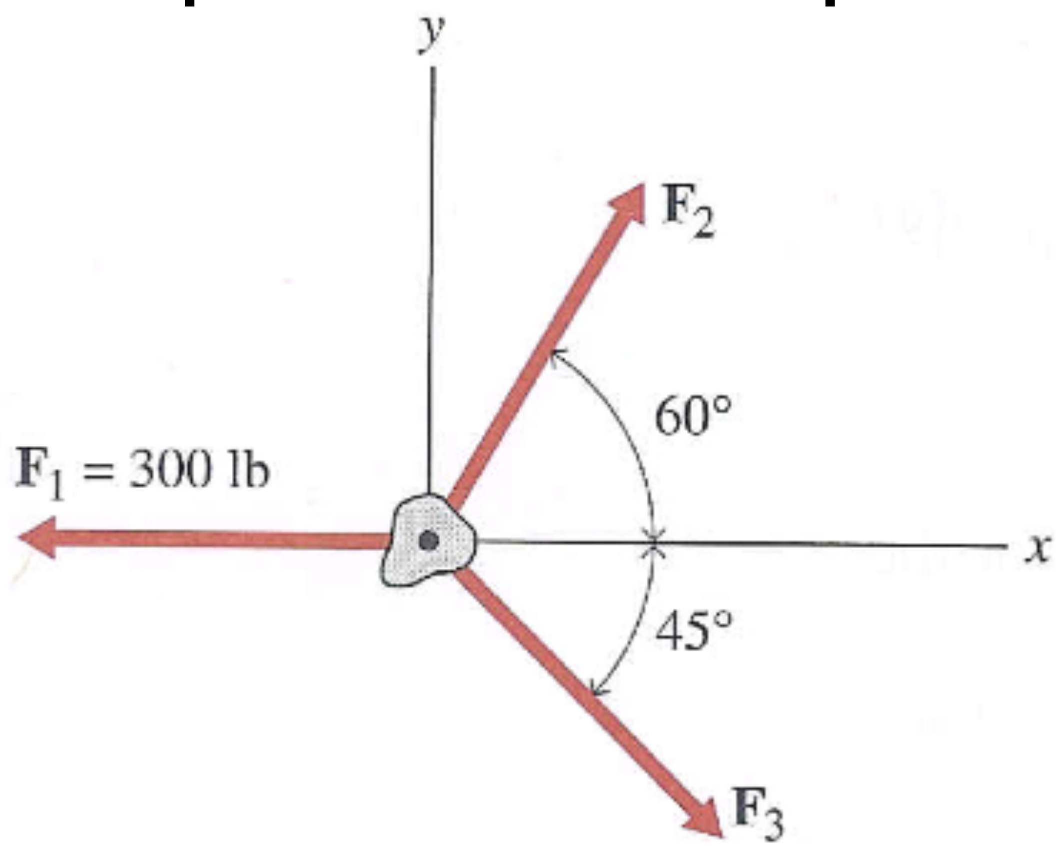
$$\sum F_x = 0 \qquad \sum M = 0$$

$$\sum F_y = 0$$

Solving static problems

1. Determine reaction forces for static equilibrium.
2. Draw Free Body Diagram
3. Decide if the problem is solvable
   a) How many unknowns?
   b) How many equations can you write?
4. Write equations to sum forces and moments to be 0.
   a) Use reaction forces as unknowns
   b) Be smart about coordinates and choice of points for summing moments
5. Solve equations for reaction forces
6. Check your answer and the direction of forces.

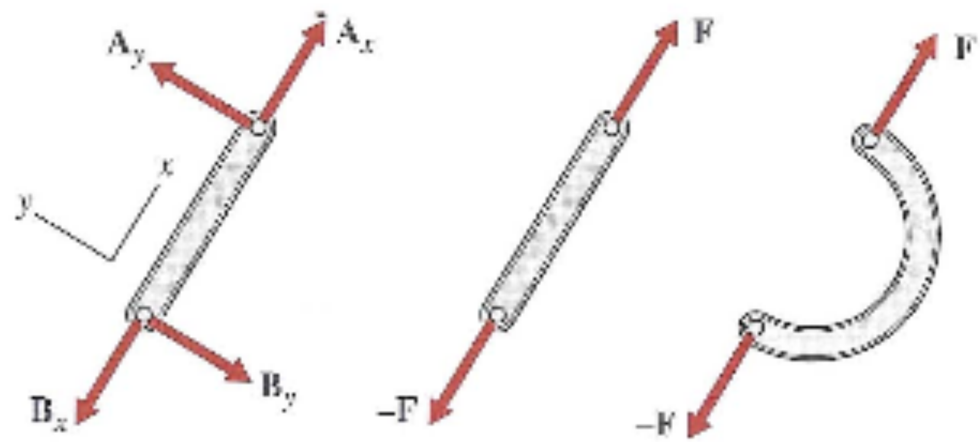
2D particle example



- **Determine magnitude of F_2 and F_3**

Link pin joints at both ends

Equilibrium requires that the forces be equal, opposite and collinear.

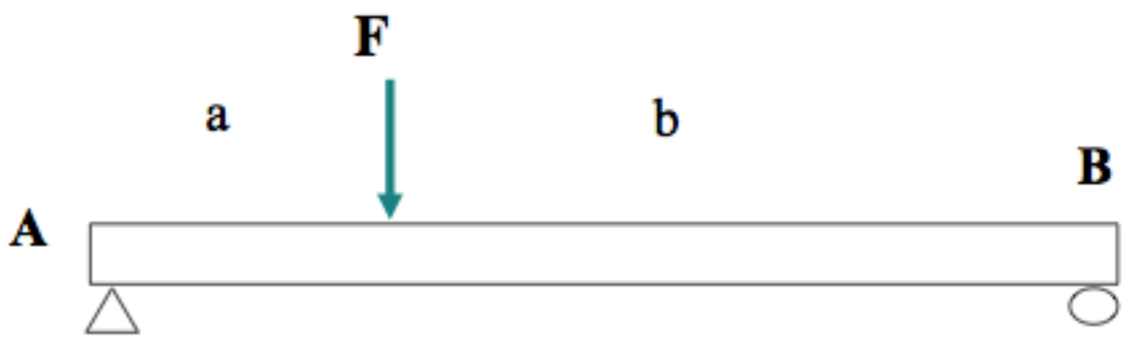


Therefore, for this member $A_y = B_y = 0$

Pin joint will not transmit a moment

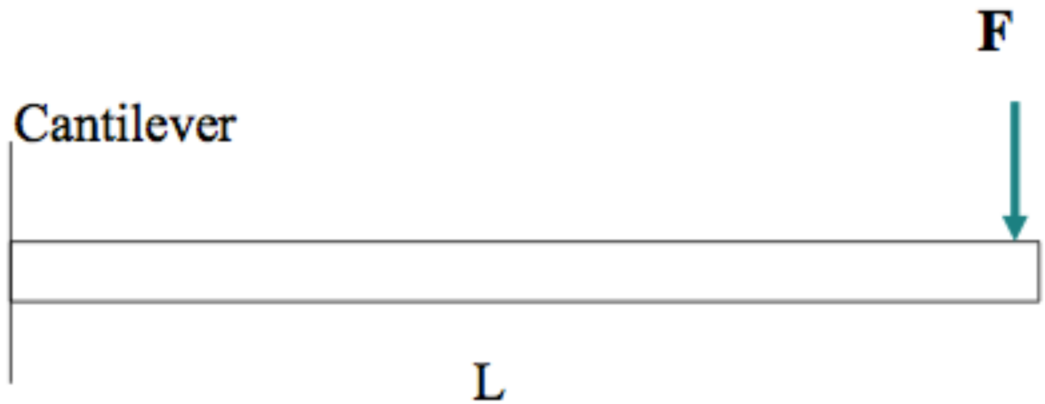
Simple support

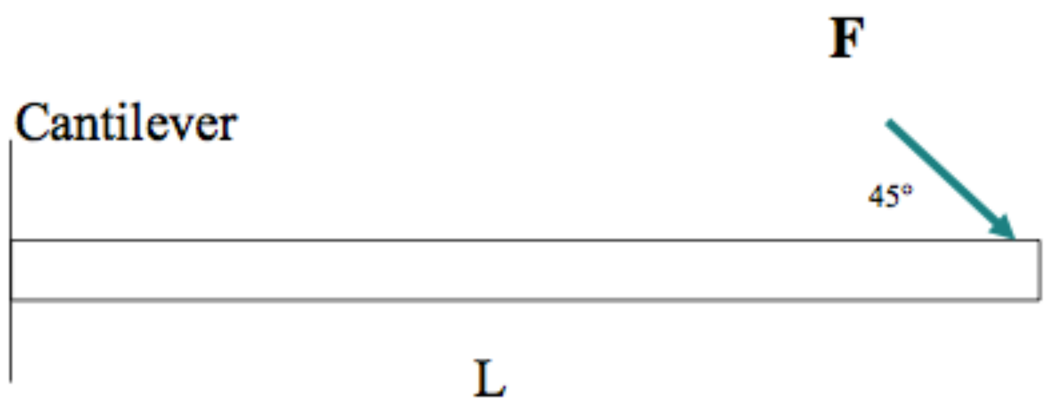
- Determine the reaction forces and moments.



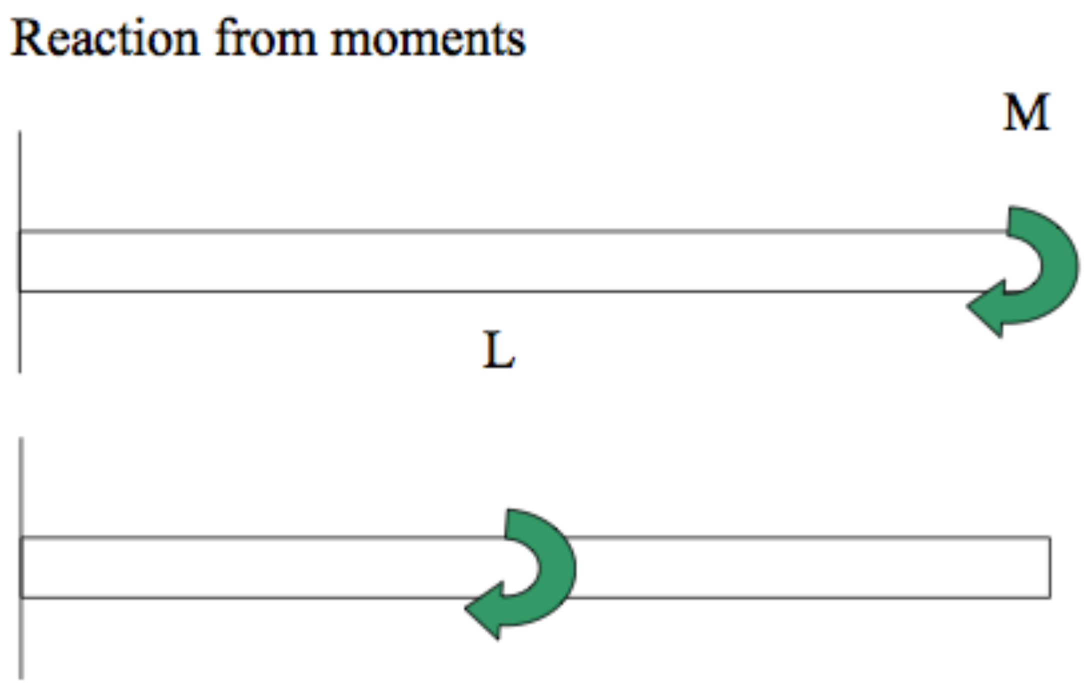
Cantilever

- Determine the reaction forces and moments

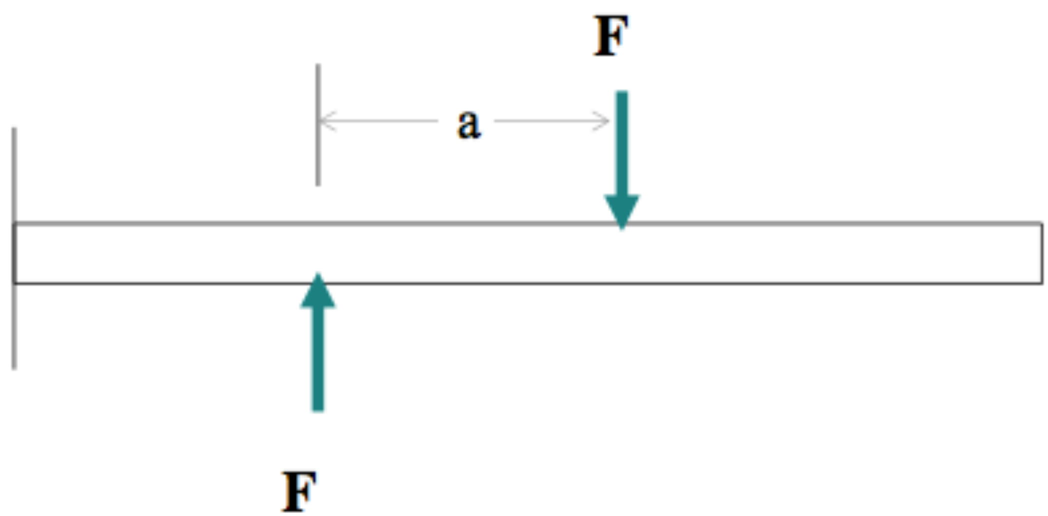


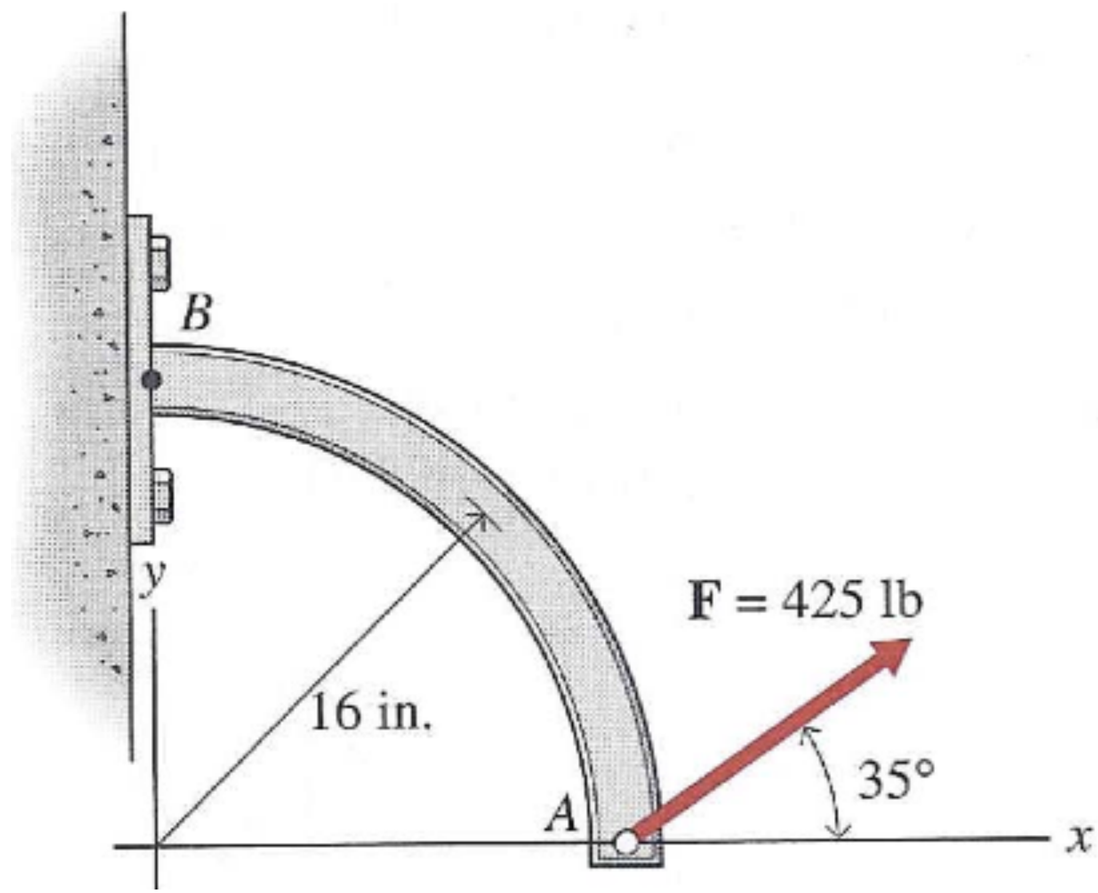


Reaction from moment



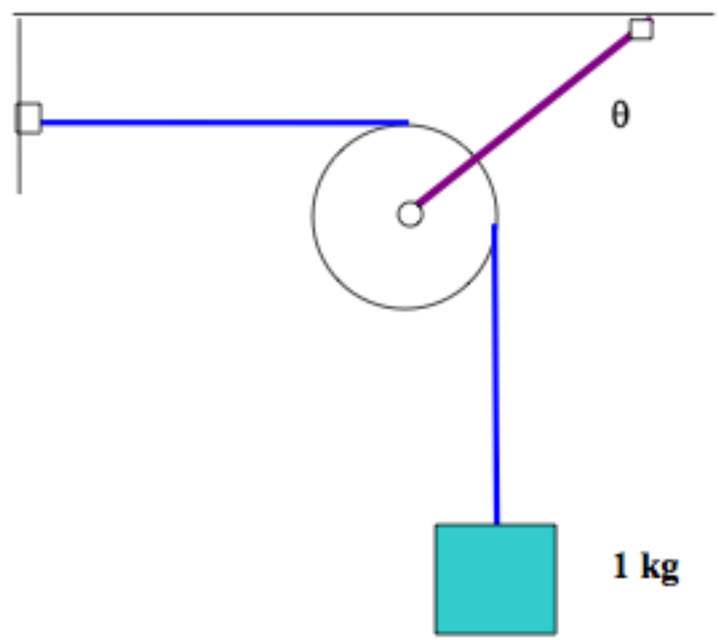
Force couples





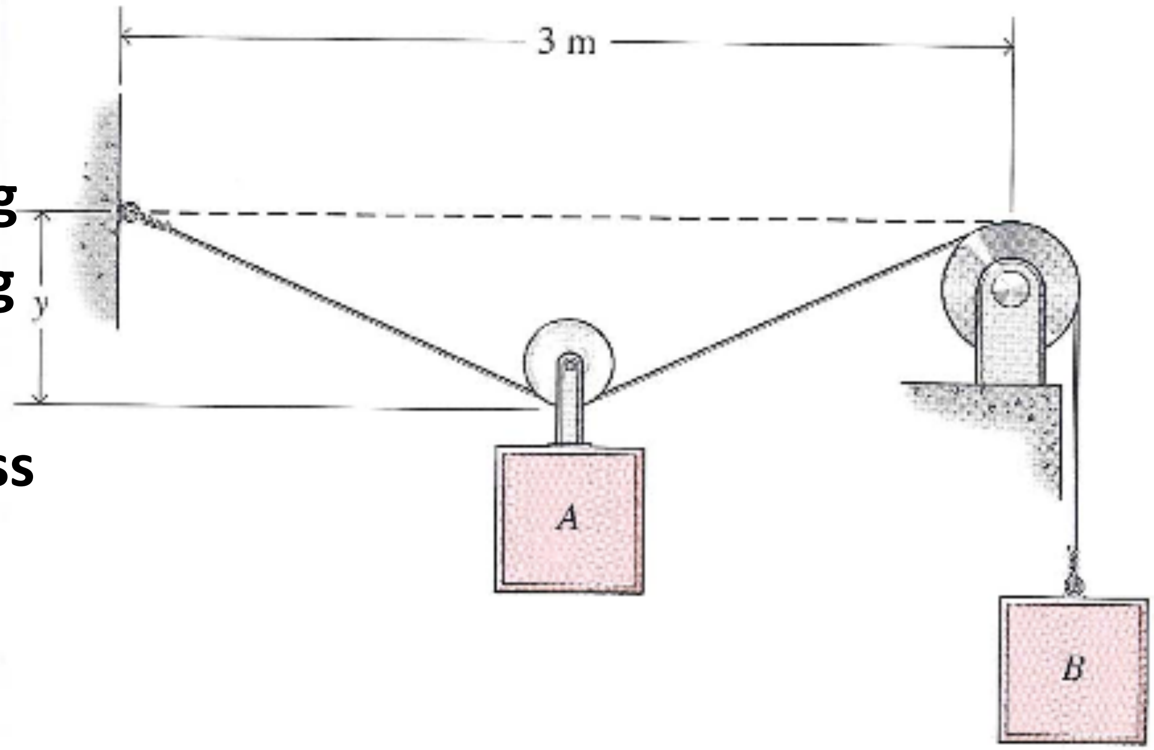
B
y
16 in.
F = 425 lb
35°
A
x

Hanging mass using a pulley



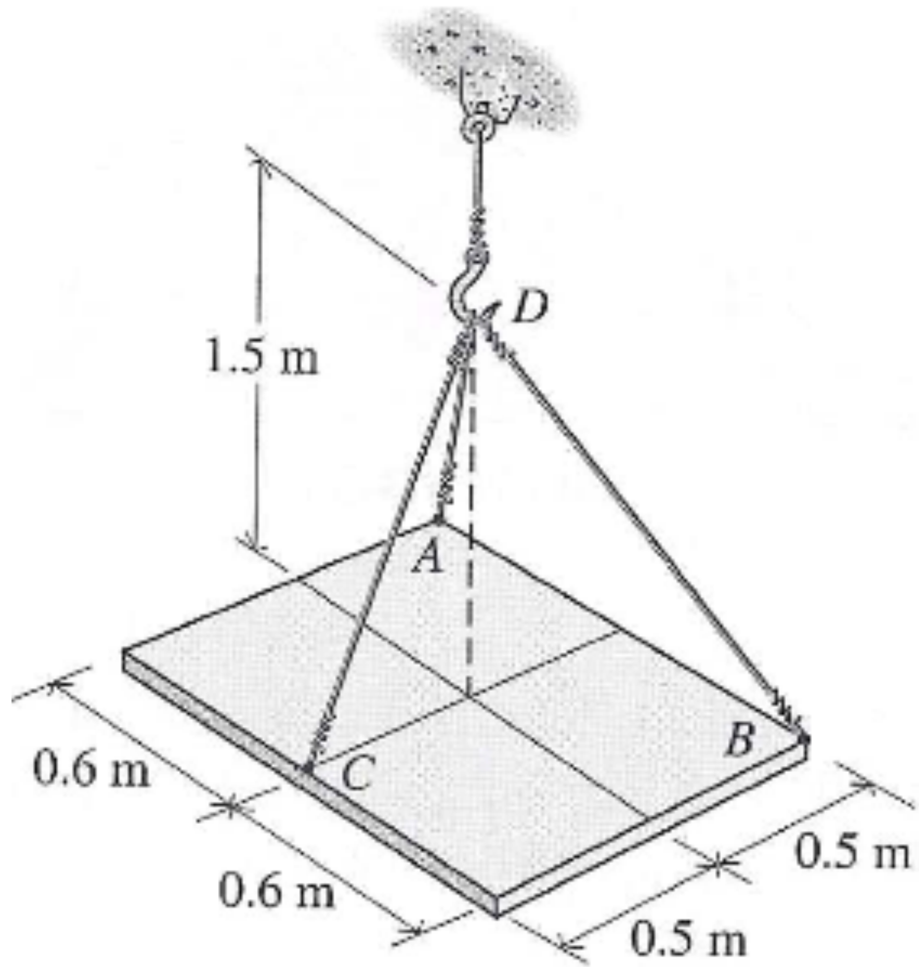
2D Pulley example

- **Specifications**
  - **Mass of block A=22kg**
  - **Mass of block B=34kg**
- **Assumptions**
  - **Pulleys are frictionless**
  - **Block A is free to roll**
  - **Cable system is continuous**
- **Determine**
  - **Displacement "y" for equilibrium**



3D cable system

- **Specifications:**
  - **Weight of plate = 250 lb**
- **Assumptions:**
  - **Plate is homogeneous**
- **Determine:**
  - **Force in each supporting cable**
- **Use direction cosines**



Rigid body equations

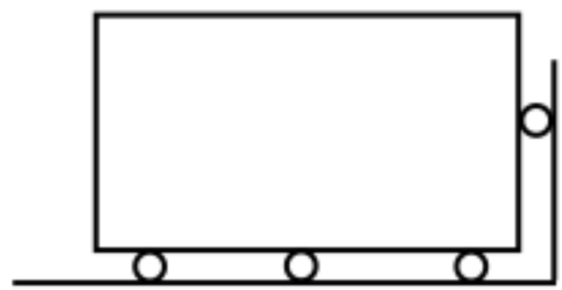
- Each body has a total of 6 degrees of freedom that define its position such as x, y, z, θ_x, θ_y, θ_z
- These lead to 6 equations that can be used to solve for reaction forces:
- $\sum Fx = 0$; $\sum Mx = 0$
- $\sum Fy = 0$; $\sum My = 0$
- $\sum Fz = 0$; $\sum Mz = 0$

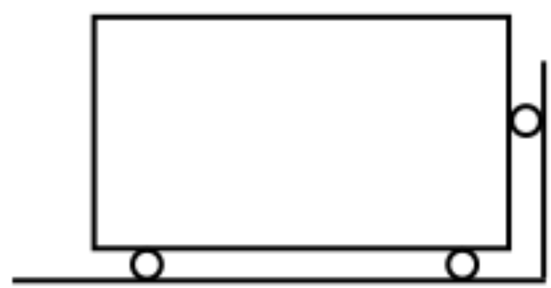
Under-constraint & Over-constraint

- If the mechanical constraints provide an attachment so that **one or more degrees of freedom are free**, the body is **under-constrained**

- If the mechanical constraints provide an attachment so that **there is no unique solution for the reaction forces**, the body is **over-constrained**

- **A body that is neither over-constrained nor under-constrained is called static determinant**





How to determine status of a mechanical system?

- If you are not sure, then try solving for the reaction forces and moments using 6 static equations and unknowns (or three for in-plane systems).
- If you have a unique solution the system is static determinant.
- If you have **multiple solutions** (more unknowns than equations) system is **over-constrained**. Reaction forces can be pushing against each other.
- If you have **more equations than unknowns** system is **under-constrained**. **Some degree of freedom is not constrained and could move**. Try to figure out what degree of freedom has not been constrained.
- You can be over-constrained and under-constrained at the same time!.