Kinematic Constraint

ME 297-1
Fall 2011
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Based on Jim Burge's notes and other online resources

Theory of kinematic design

- Every rigid body has 6 degrees of freedom. 3-translational and 3-rotational
- Theory of kinematic design assumes that
  - **perfectly rigid bodies** with infinite elastic modulus have **point contacts**.
  - Rigid body has (6-N) degrees of freedom where N is the number of contact points
  - 3 points determine a plane. More that 3 contact points for a planar surface causes distortion of the surface.
  - A rigid body with **more that 6-contact points** is **overconstrained** and is likely to be distorted and uncertain in position.
- Kinematic deign
  - provides motion with complete lack of play or backlash
  - Can be built without precision manufacturing techniques.

More on kinematic Constraint

For holding a body (rigid thing) with the highest precision, we require:

- Full 6 DoF constraint
  - If 6 DoFs not fully constrained, then one is loose.
- No overconstraint. Any overconstraint can cause problems:
  - constraints can push against each other, resulting in stress and deformation.
  - constraints pushing against each other will “lurch*” when forces exceed threshold
- Kinematic constraint :
  - All DoFs are constrained, and very strictly, none are overconstrained
- Semi-Kinematic :
  - Slight overconstraint is allowed

*An act or instance of swaying abruptly

Kinematic Pairs

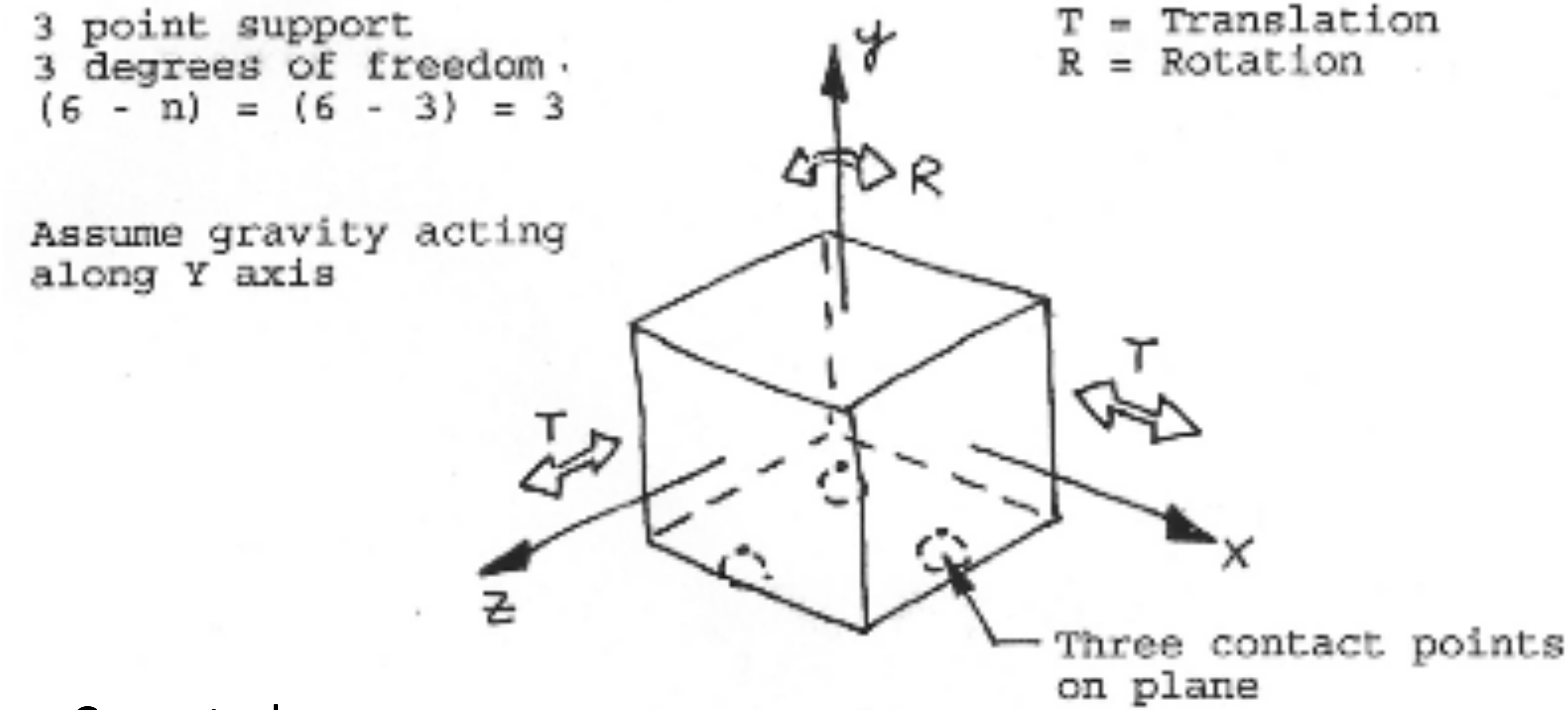
- Machines and instruments are made up of elements that are suitably arranged and many of which that are movably connected.
- Two parts that are in contact and move relative to one another are called **kinematic "pairs"** or can be thought of as being kinematically coupled.

Contact points & degrees of freedom

- The number of contact points between any two rigid bodies is equal to the number of their mutual constraints
- Examples:
  - Sphere on a plane
    - one contact point
    - Motion with respect to plane is constrained in Z

  - Sphere in a trihedral hole
    - Three points in contact
    - Three translations are constrained

3-point support
3 contacts on plane



3 point support
3 degrees of freedom
(6 - n) = (6 - 3) = 3

Assume gravity acting along Y axis

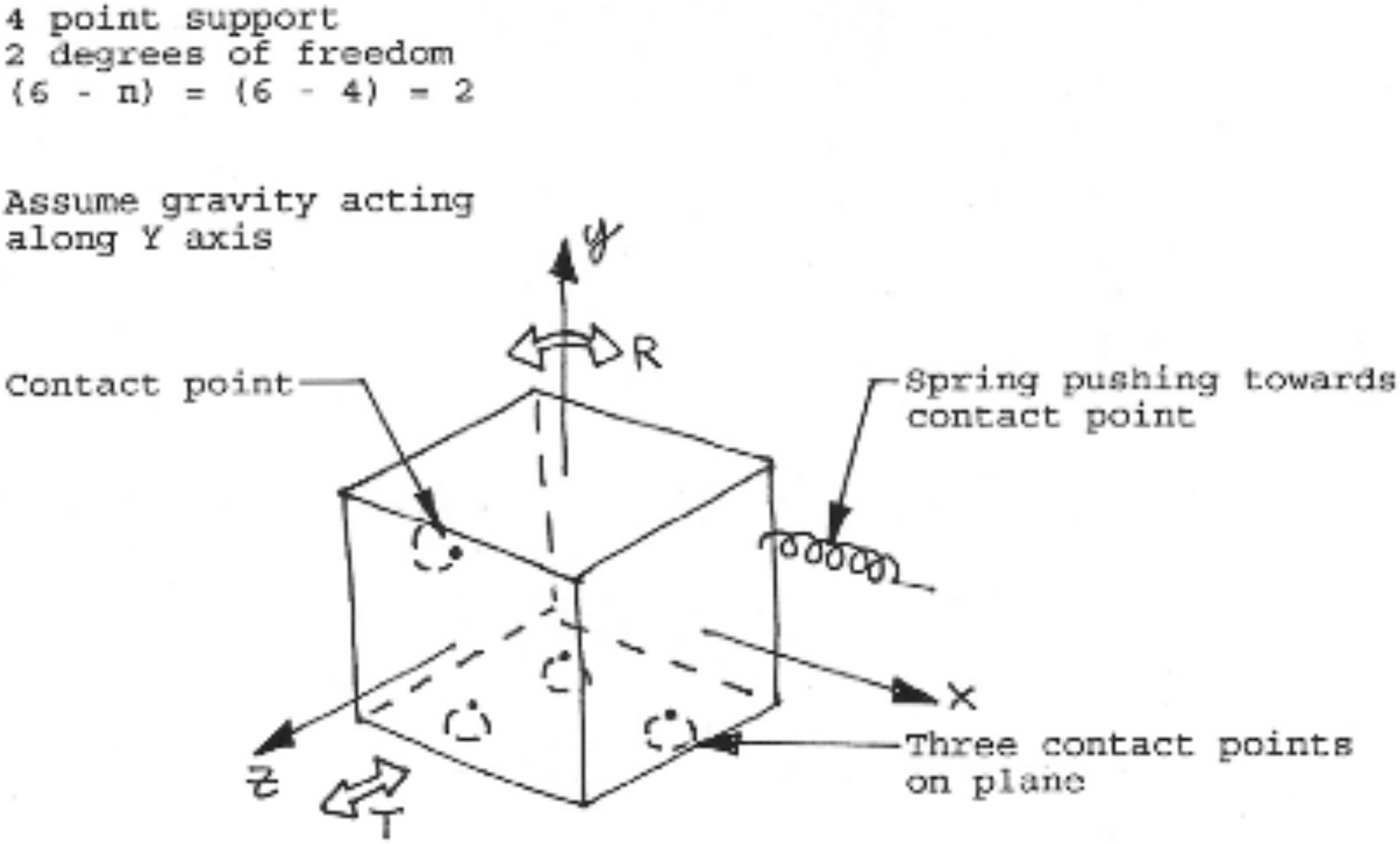
T = Translation
R = Rotation

Three contact points on plane

- Case study
    - 4 points on a plane
    - 2 points on a plane
    - 3 points on a line

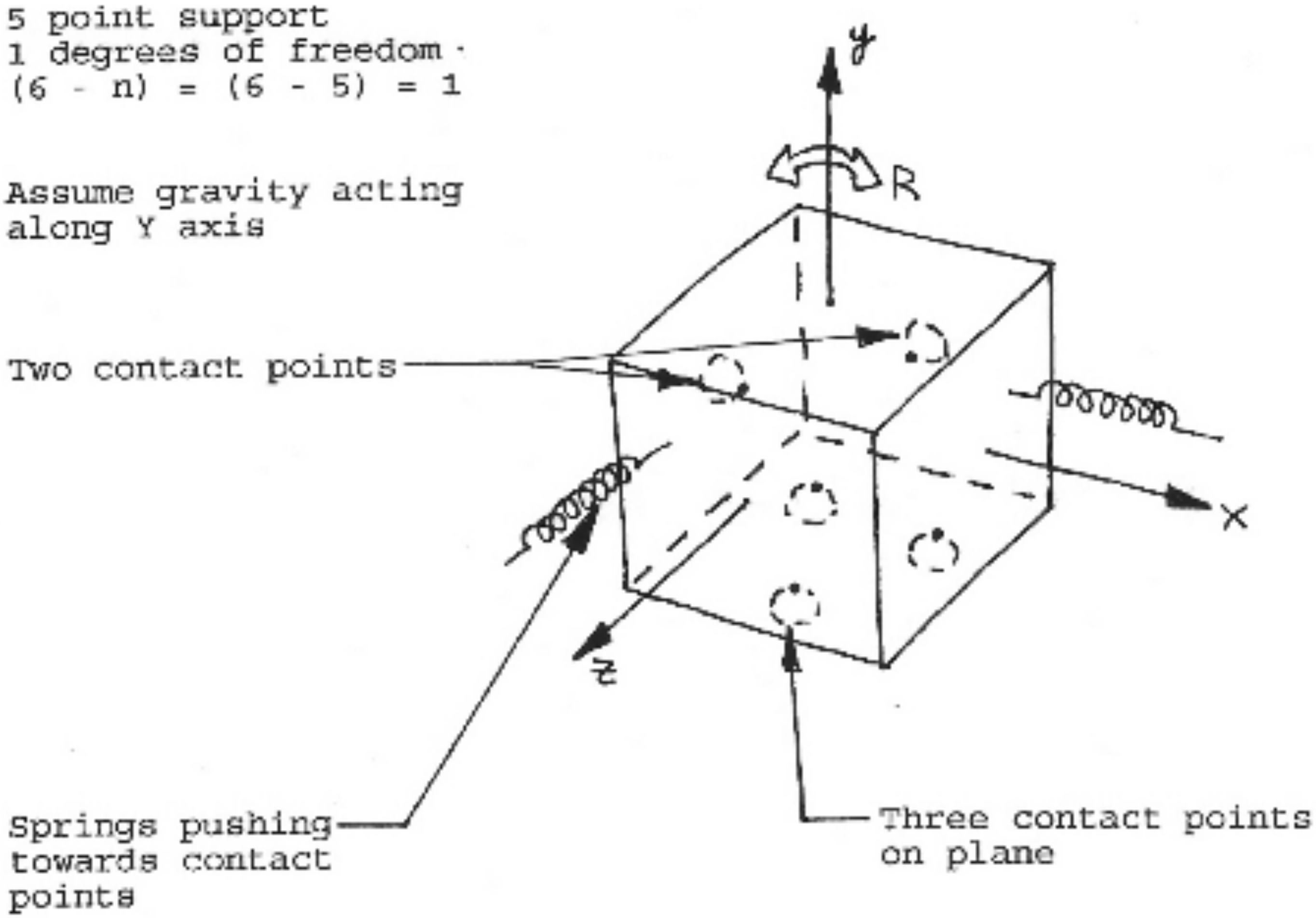
4-point support

- Balls provide position constraint.
- Springs & gravity provide preload. NO CONSTRAINT!



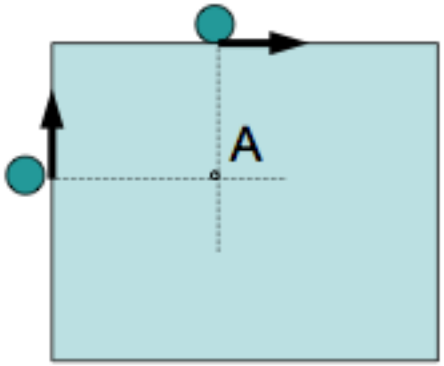
5-point constraint

- One DoF left
- Small motion : Rotation about point A



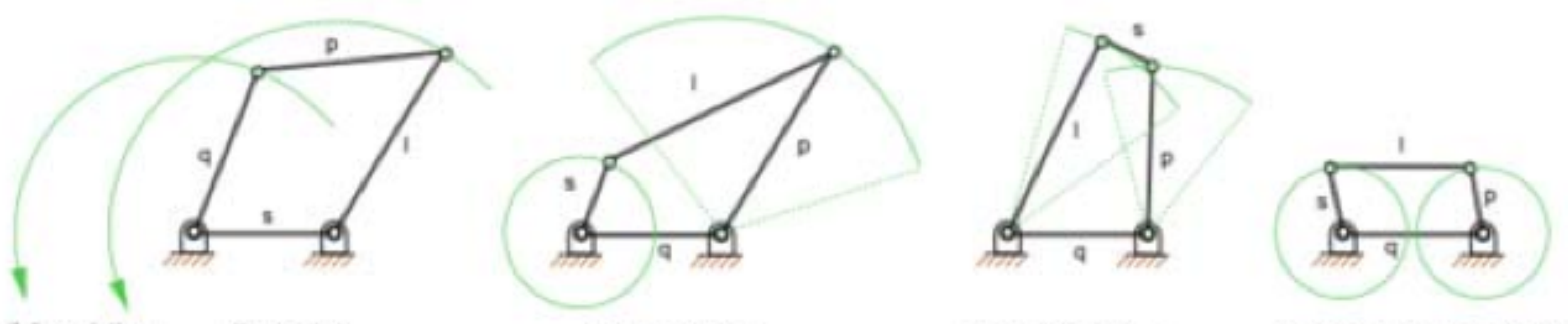
Mechanical indeterminacy example

Shows indeterminacy of upper two points.



Instantaneous center of rotation

- Instantaneous degree of freedom is rotation about a well defined point for small motions
- For large motions, the geometry changes and the position of this instantaneous center of rotation moves.



Drag-link
$s+l \ngtr p+q$
(continuous motion)

Crank-rocker
$s+l \ngtr p+q$
(continuous motion)

Double-rocker
$s+l > p+q$
(no continuous motion)

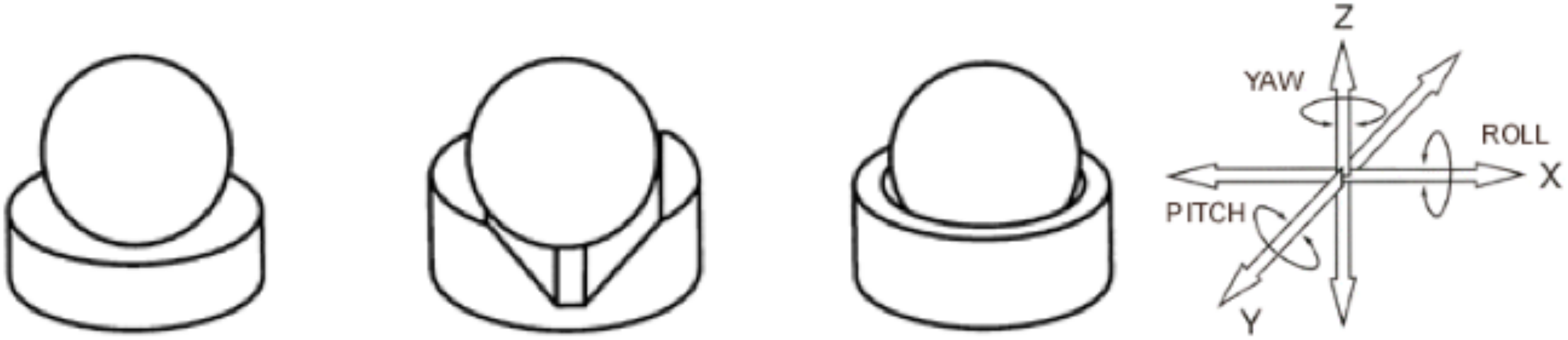
Parallelogram linkage
$s+l \ngtr p+q$
(continuous motion)

http://kmoddl.library.cornell.edu/resources.php?id=125

http://pergatory.mit.edu/2.007/lectures/2002/Lectures/Topic_04_Linkages.pdf

Use of balls

- Use symmetry of balls
- Material: stainless steel, tungsten carbide, silicon nitride, diamond
- Constrain position in 1, 2, or 3 DoF.
- Always leaves rotation about 3 axes about center of curvature, *If the ball is smooth*



Possible arrangements of constraints for degrees of freedom 0-5

- One constraint
- Five DOF
- This will prevent a translation in the direction of the force closing the constraint

- Two constraint
- Four DOF
- One of them will prevent a rotation or translation One of them will prevent a translation

Possible arrangements of constraints for degrees of freedom 0-5

- 3 constraints
- 3 degrees of freedom

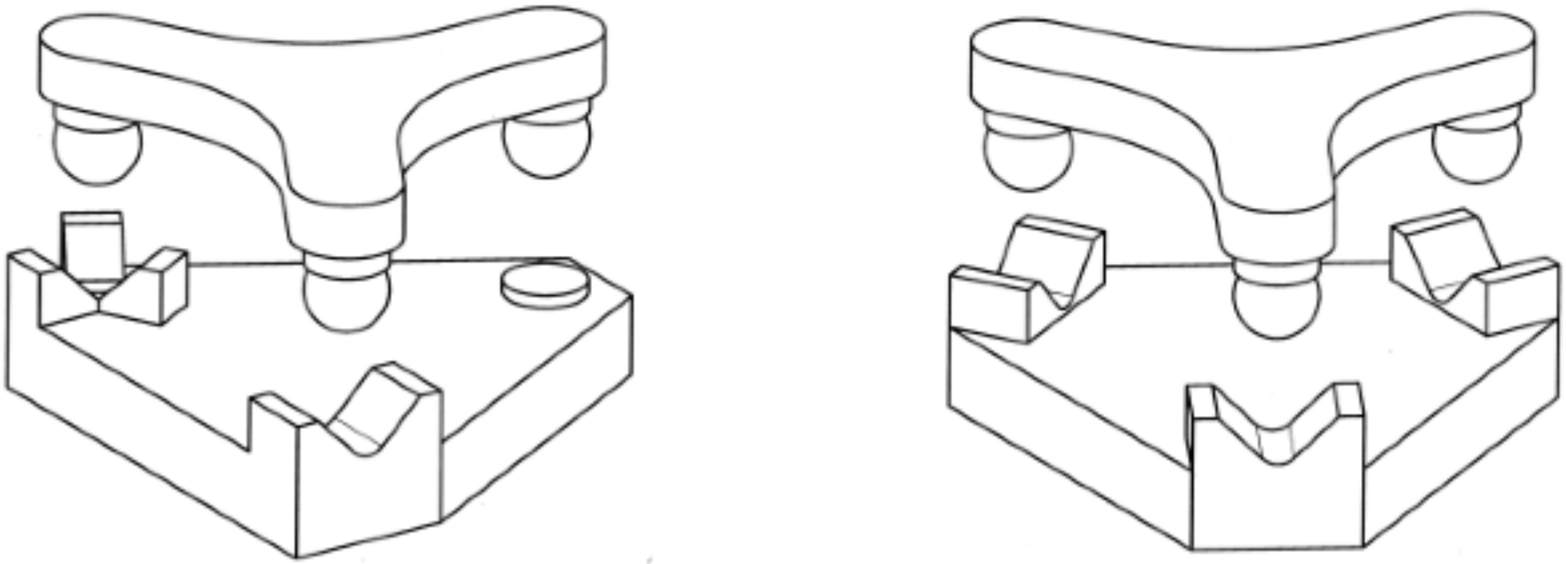
- 4 constraints
- 2 DOFs

Possible arrangements of constraints for degrees of freedom 0-5

- 5 constraints
- 1 DOF

- 6 constraints
- 6 DOF

Kinematic interface

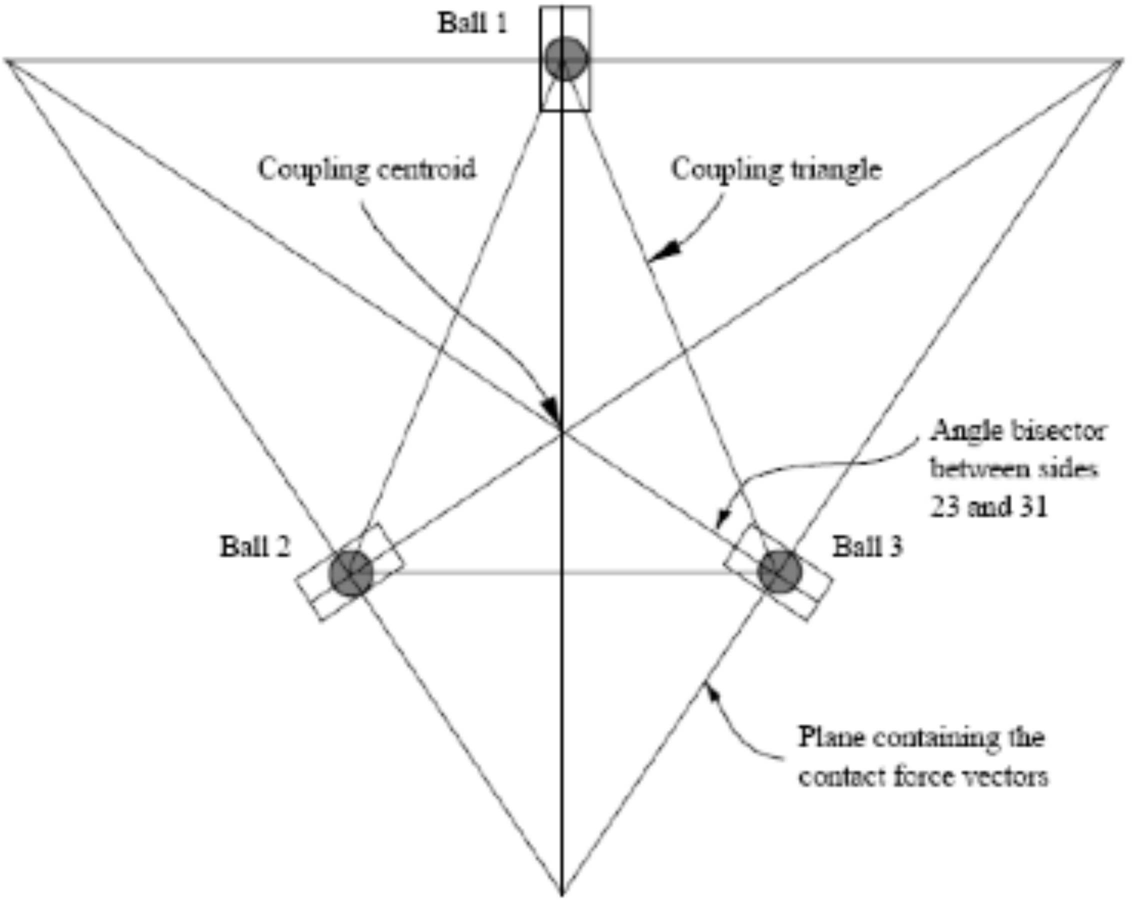


Split Vee block with
a large sphere

Small spherical bottons
of a large spherical
radius and split Vee
block

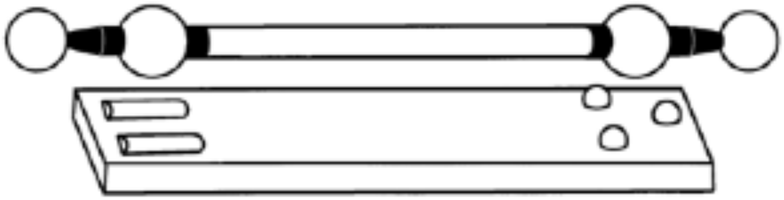
3 V geometry

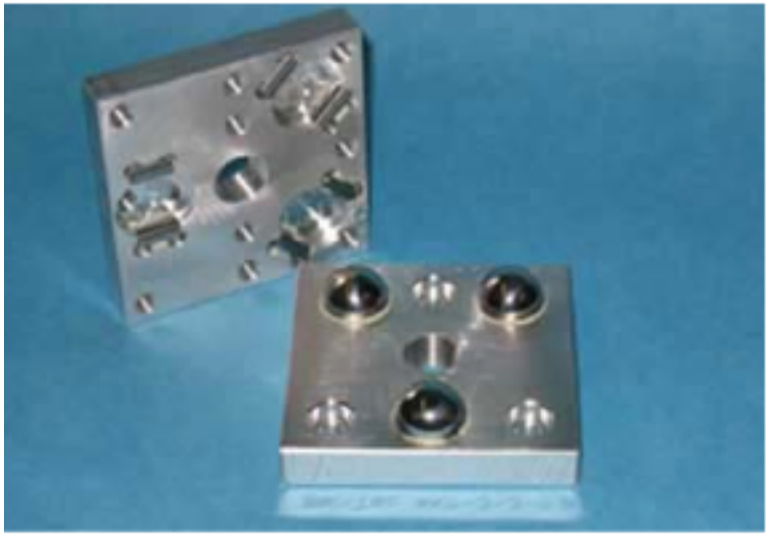
- Ideally the normals to the contact planes should bisect the coupling triangle's angles:



Solcum

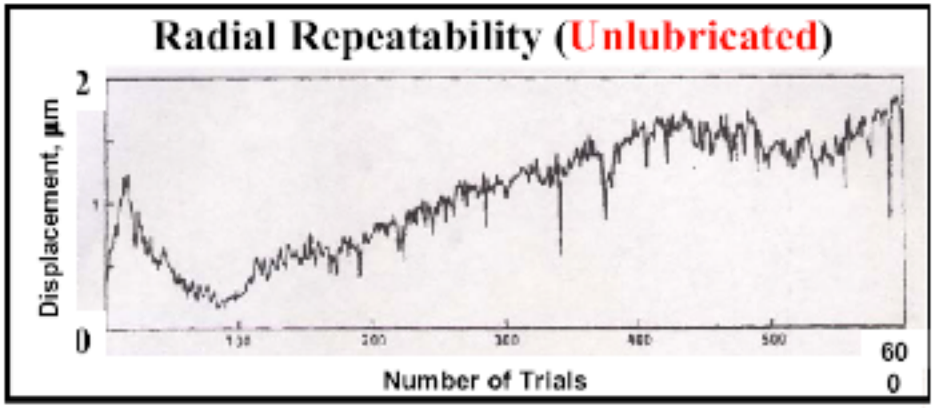
Kinematic hardware

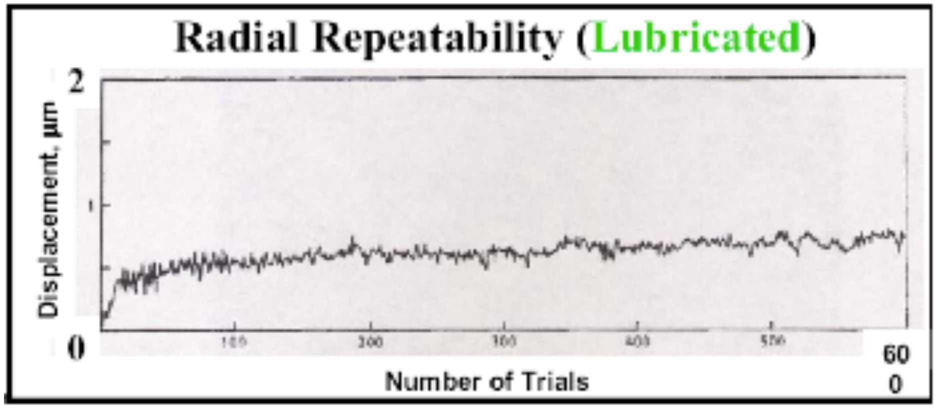




Kinematic location

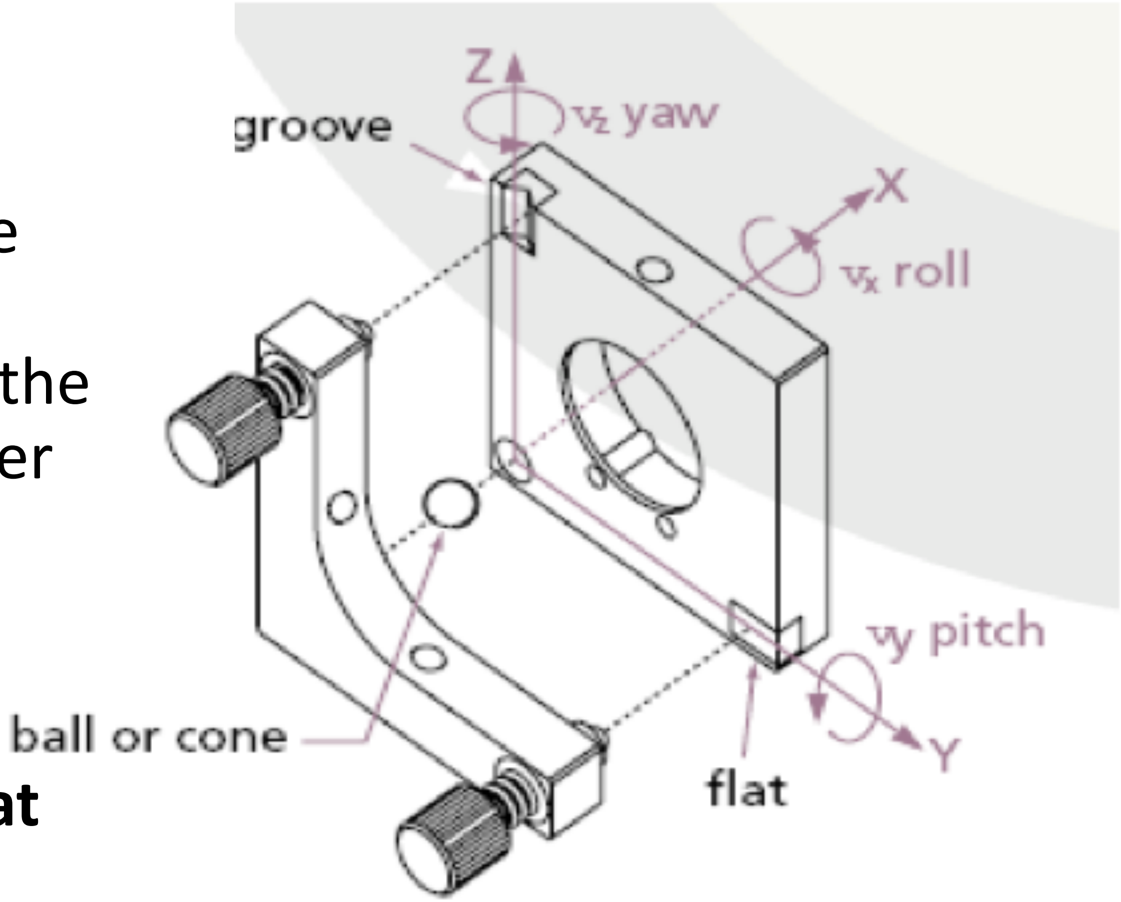
- Since the point contacts are well defined, the **location is repeatable to sub-micron.**
- Definition of the point contact depends on friction, surface finish, loads.


Radial Repeatability (Unlubricated)


Radial Repeatability (Lubricated)

Application of kinematic constraint for precision motion

- For three balls fixed, kinematic constraint
- Move one ball at a time (with micrometer) to rotate the stage about the axis defined by the other two balls
- Very stable
- Smooth motion
- **Not shown, springs that hold this together**



Application of kinematic concepts for motion control

- 5 DoFs constrained using kinematic principles
- Remaining DoF is used for the motion

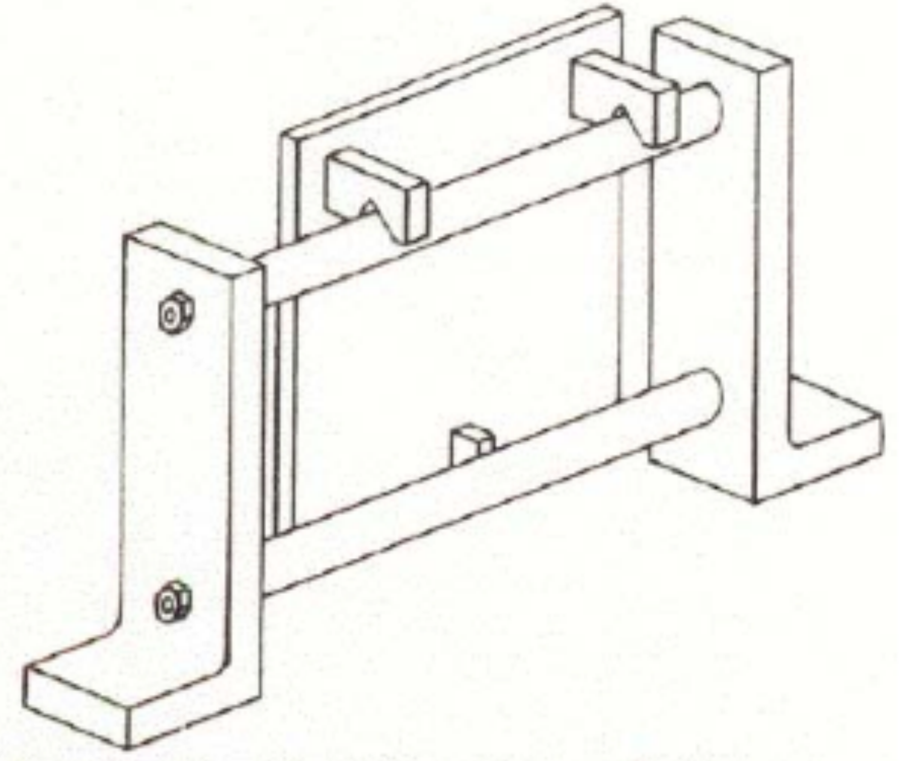
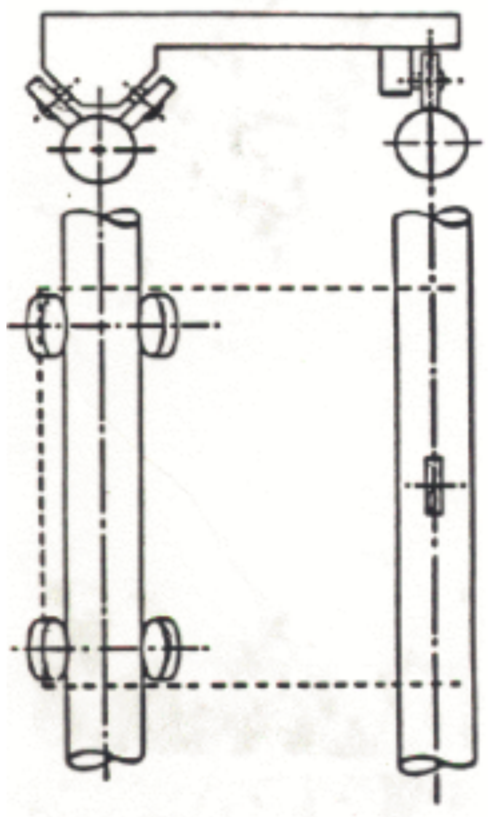
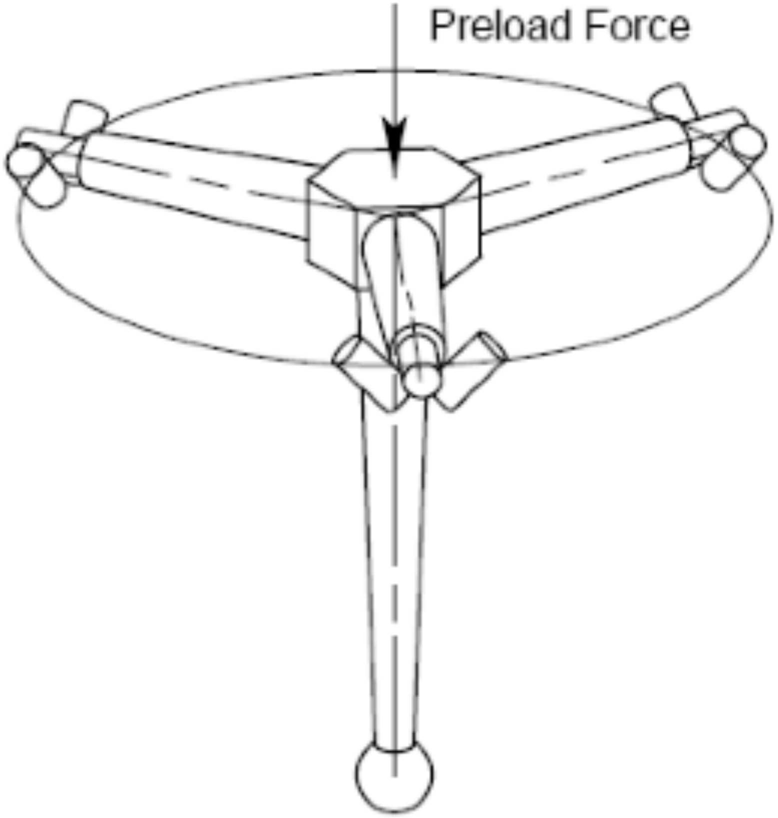


FIG. 6.2.—Kinematic design employing cylindrical surfaces as guides. Such surfaces can be accurately made.

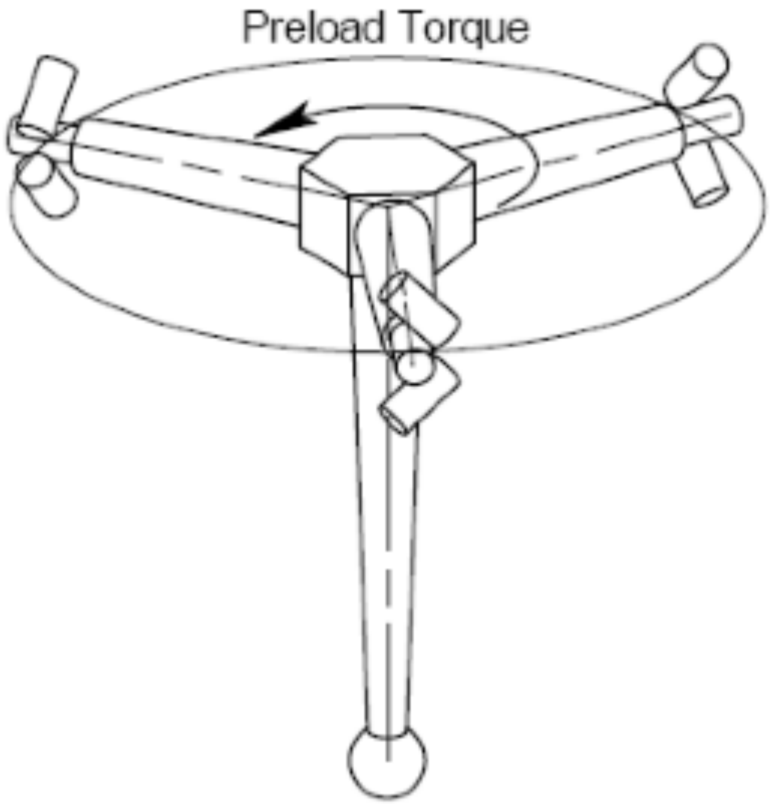


Direction of V and preload

- The moment required to unseat one vee while pivoting about the other two vees is a factor of two less than the moment required to unseat two vees while pivoting about the third vee.
- A moment applied about any axis in the plane of the vees produces equal reaction at all vees



(a)

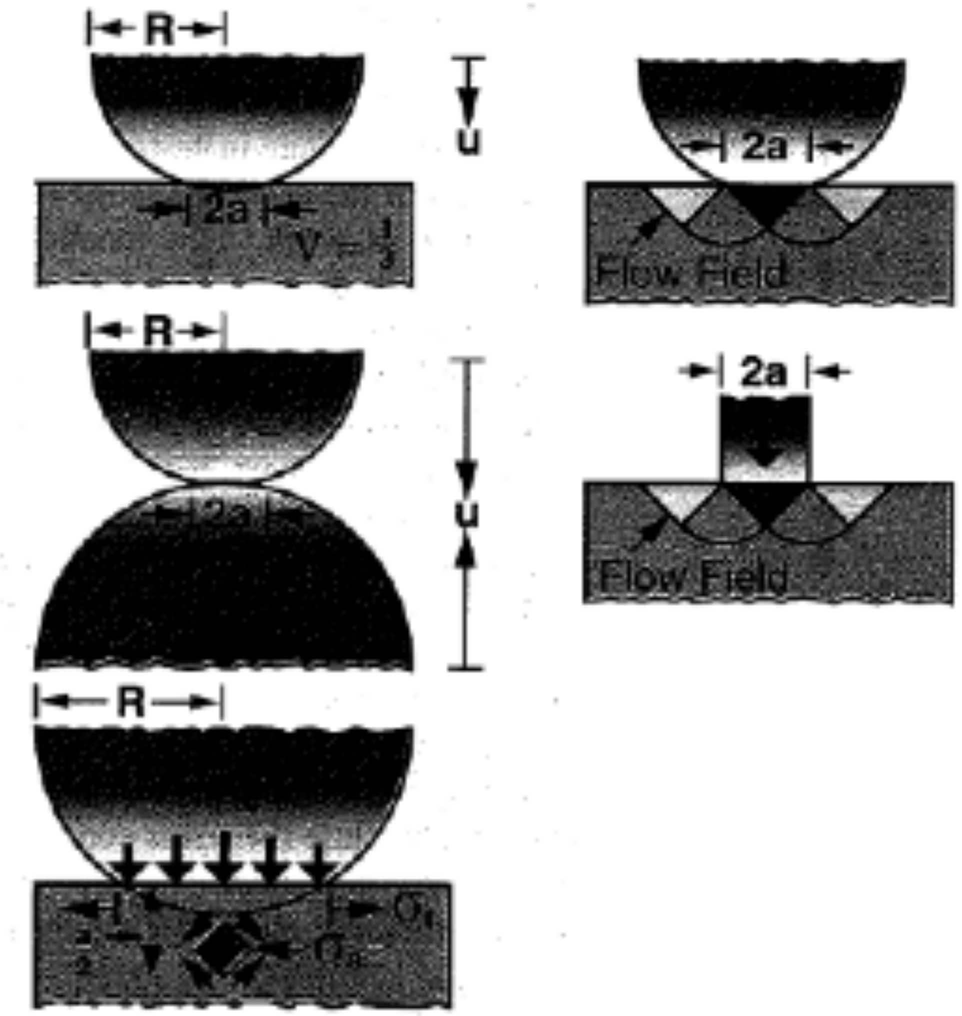


(b)

Problems with point and line contact

- Nominally, the contact area is **zero for a point or line**
- **In reality, the contact area comes from deformations and depends on the geometry and material properties.**
- **More force causes more deformation which increases the contact area.**
- **Non-point contact = not purely kinematic**
- **Stiffness of the interface = Force required for displacement is very low for the unloaded case. and very nonlinear.**
- **Preload is required to make the structure stiffer.**
- **Increased preloading makes stiffer, more stable interface in normal direction**
- **But: Stress = Force/Area becomes very high and can damage the materials**
- **Tangential effects due to friction can be large**

Contact stress



Stiffness

$$k = \frac{dF}{du} \cong \frac{3}{2}\left(E^2 R F\right)^{1/3}$$

Stress

$$(\sigma_c)_{\max} \cong \left(\frac{E^2 F}{R^2}\right)^{1/3} \cong \frac{2}{3}\frac{k}{R}$$

$$\tau_{\max} \cong \frac{(\sigma_c)_{\max}}{3}$$

$$\boxed{\left.\begin{aligned} a &= 0.7\left(\frac{F\,R}{E}\right)^{\frac{1}{3}} \\ u &= 1.0\left(\frac{F^2}{E^2\,R}\right)^{\frac{1}{3}} \end{aligned}\right\}\quad \nu = \tfrac{1}{3}}$$

$$\boxed{\begin{aligned} a &= \left(\frac{3}{4}\,\frac{F}{E^*}\,\frac{R_1\,R_2}{(R_1 + R_2)}\right)^{\frac{1}{3}} \\ u &= \left(\frac{9}{16}\,\frac{F^2}{(E^*)^2}\,\frac{(R_1 + R_2)}{R_1\,R_2}\right)^{\frac{1}{3}} \end{aligned}}$$

$$\boxed{\begin{aligned} (\sigma_c)_{max} &= \frac{3F}{2\,\pi\,a^2} \\ (\sigma_s)_{max} &= \frac{F}{2\,\pi\,a^2} \\ (\sigma_t)_{max} &= \frac{F}{6\,\pi\,a^2} \end{aligned}}$$

R_1, R_2 = Radii of spheres (m)

E_1, E_2 = Modulii of spheres (N/m^2)

ν_1, ν_2 = Poisson's ratios

F = Load (N)

a = Radius of contact (m)

u = Displacement (m)

σ = Stresses (N/m^2)

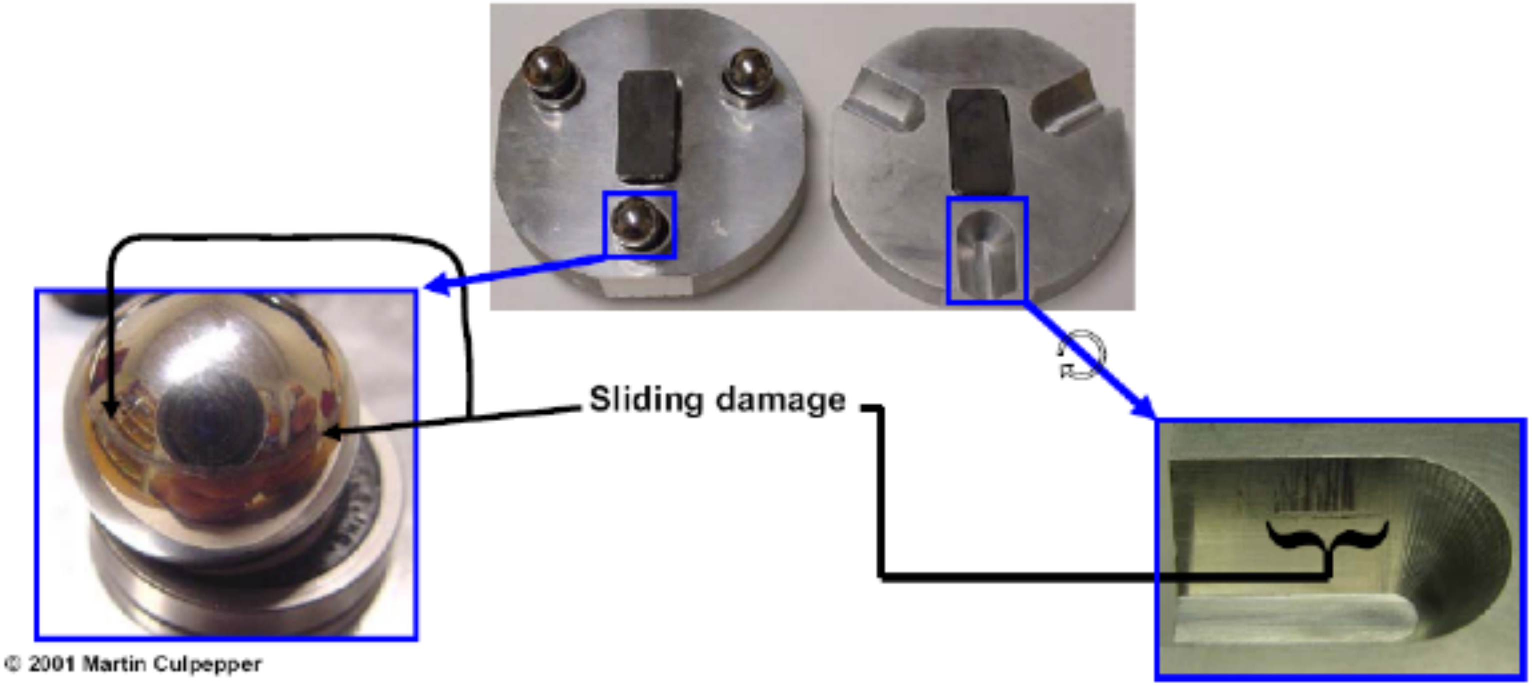
σ_y = Yield stress (N/m^2)

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}$$

$$\boxed{\frac{F}{\pi\,a^2} = 3\,\sigma_y}$$

Effect of contact stress

- Contact stress can cause fretting of the surface
- Lubrication helps.
- Aluminum is very bad.
- Different materials works best



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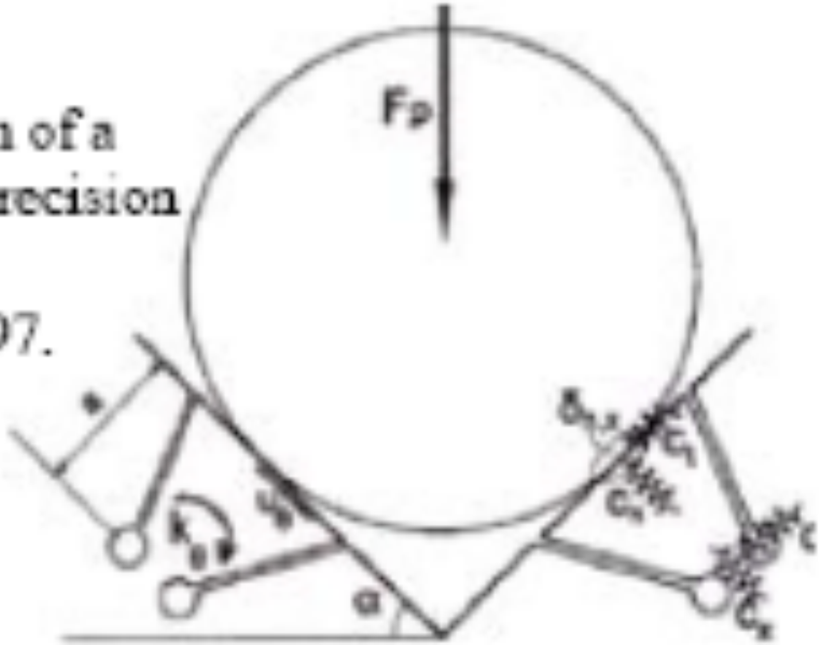
Picture from:
Schouten, et. al., "Design of a kinematic coupling for precision applications", Precision Engineering, vol. 20, 1997.



Ball in V-Groove with Elastic Hinges

Repeatability as function of geometry

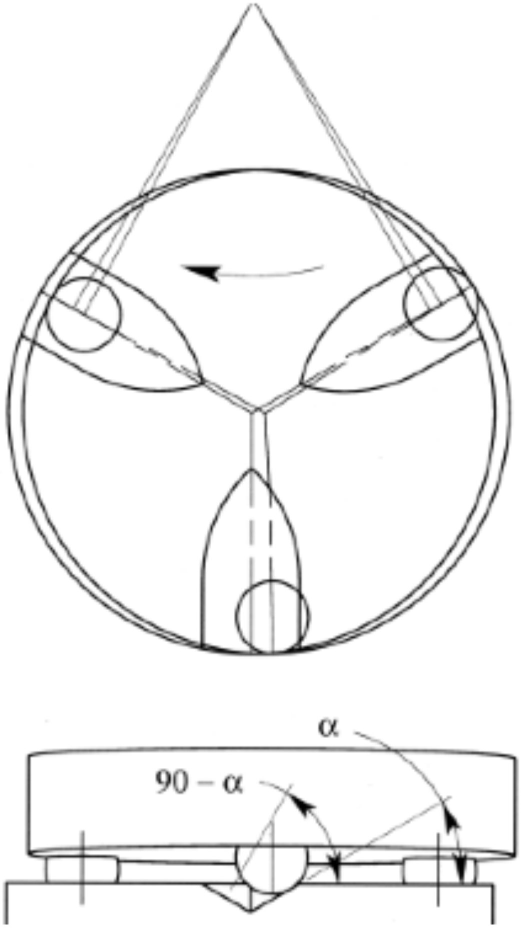


Fig. 5. The three-vee coupling slides on five constraints producing rotation about an instant center shown in the top view and also about an axis through the two seated balls.

- Non-repeatability per ball/plane interface is

$$\rho \equiv \frac{f}{k} \approx \mu \left(\frac{2}{3R}\right)^{1/3} \left(\frac{P}{E}\right)^{2/3}$$

μ = friction coefficient
R = ball radius
P = load
E = Young's modulus

- For the system (mostly horizontal):

$$\mu P$$

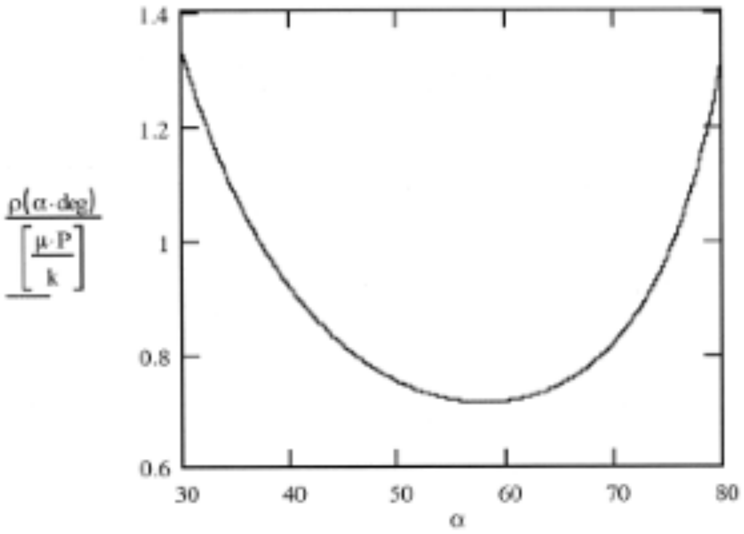
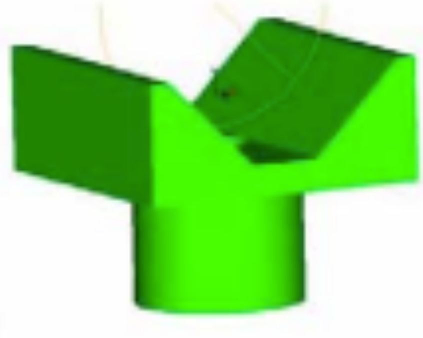
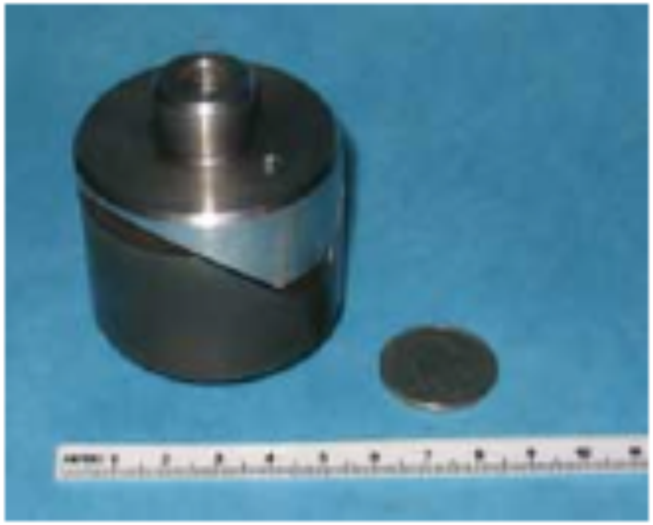


Fig. 6. The effect of the vee angle α on the repeatability of the symmetric three-vee coupling has a minimum of 0.71 at 58°.

Use geometry to reduce contact stress

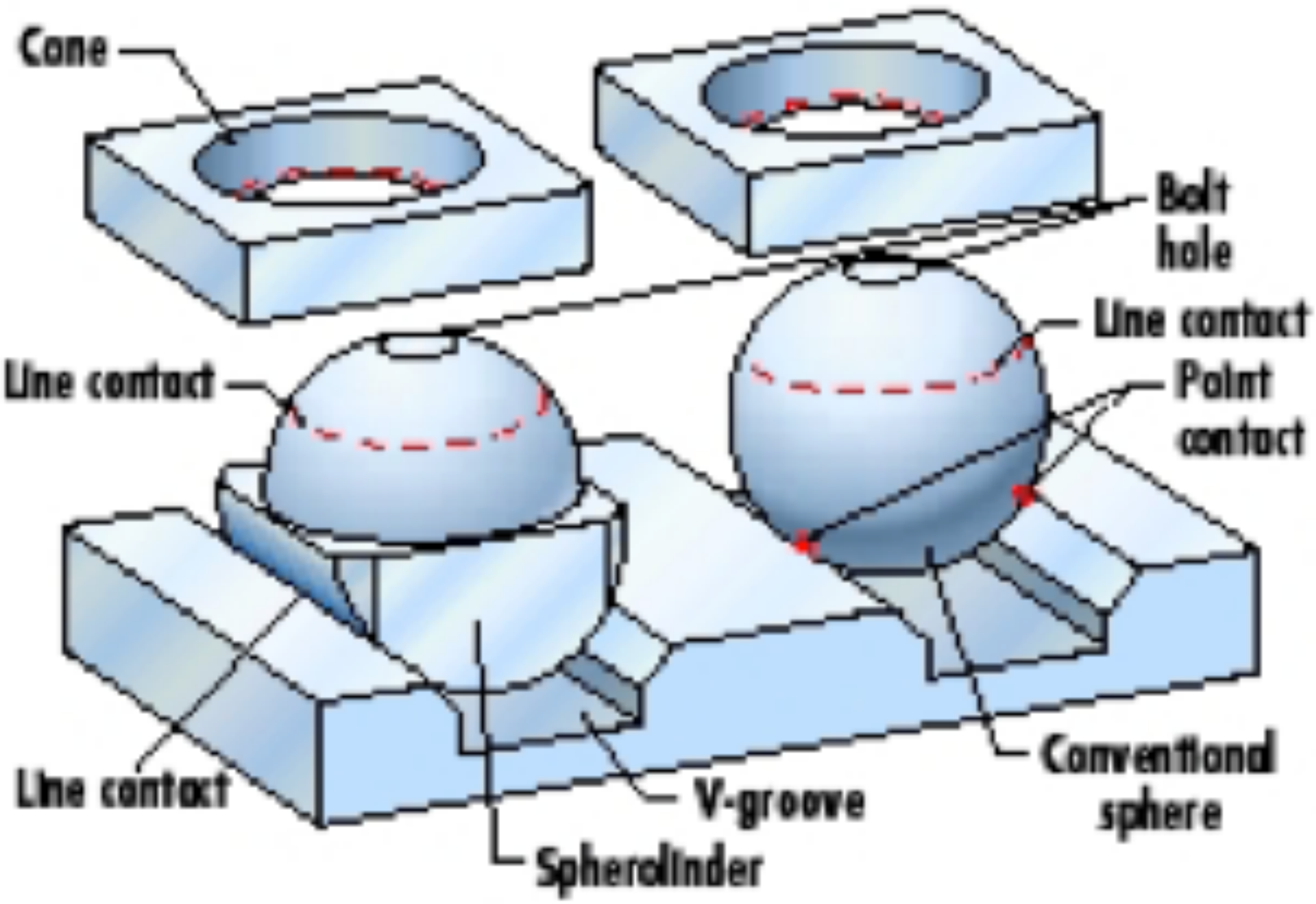
- Canoe ball, 1 meter ROC
  - Baltek





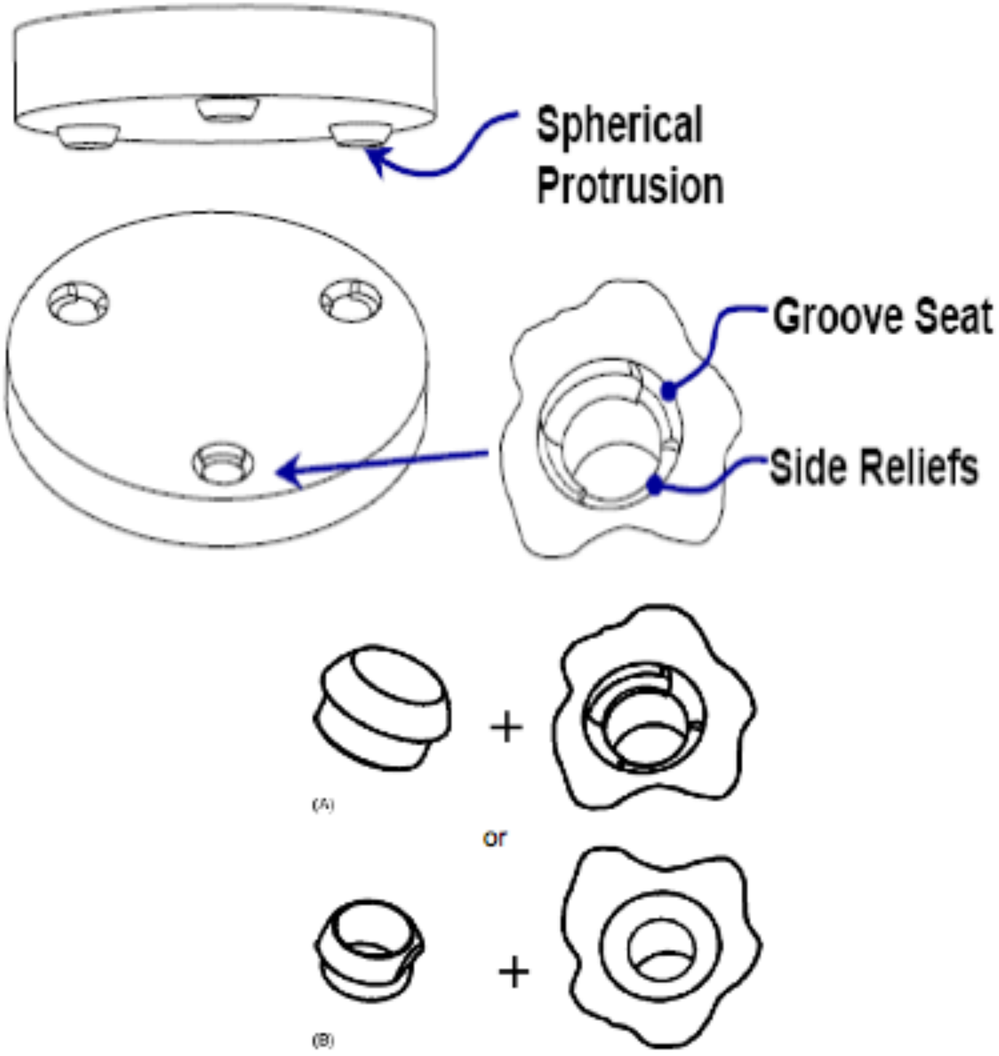
Use geometry to reduce contact stress

- Spherolinder (G2 Engineering)



Semi-kinematic design

L. Hale US Patent #6,065,898



Cylinders in V's

- Easy to make to high accuracy
- Leaves axial motion, clocking rotation unconstrained

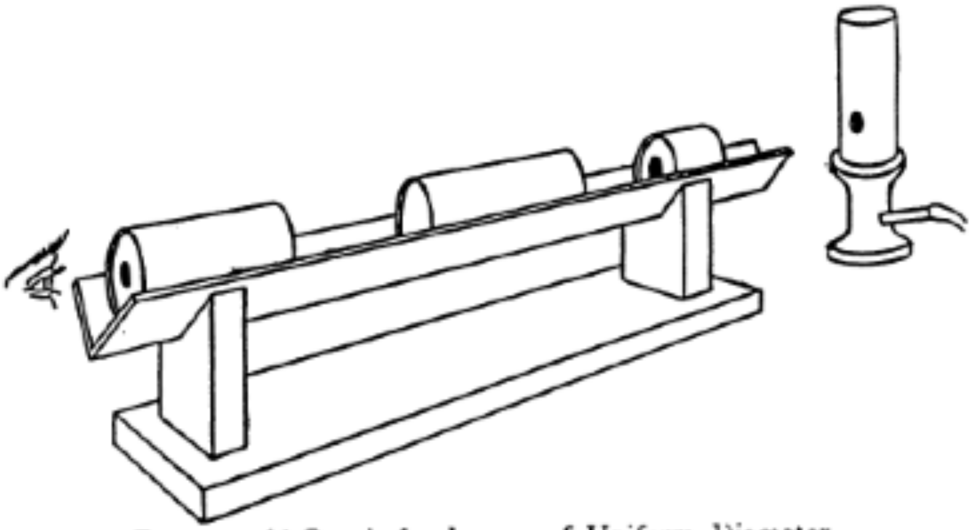
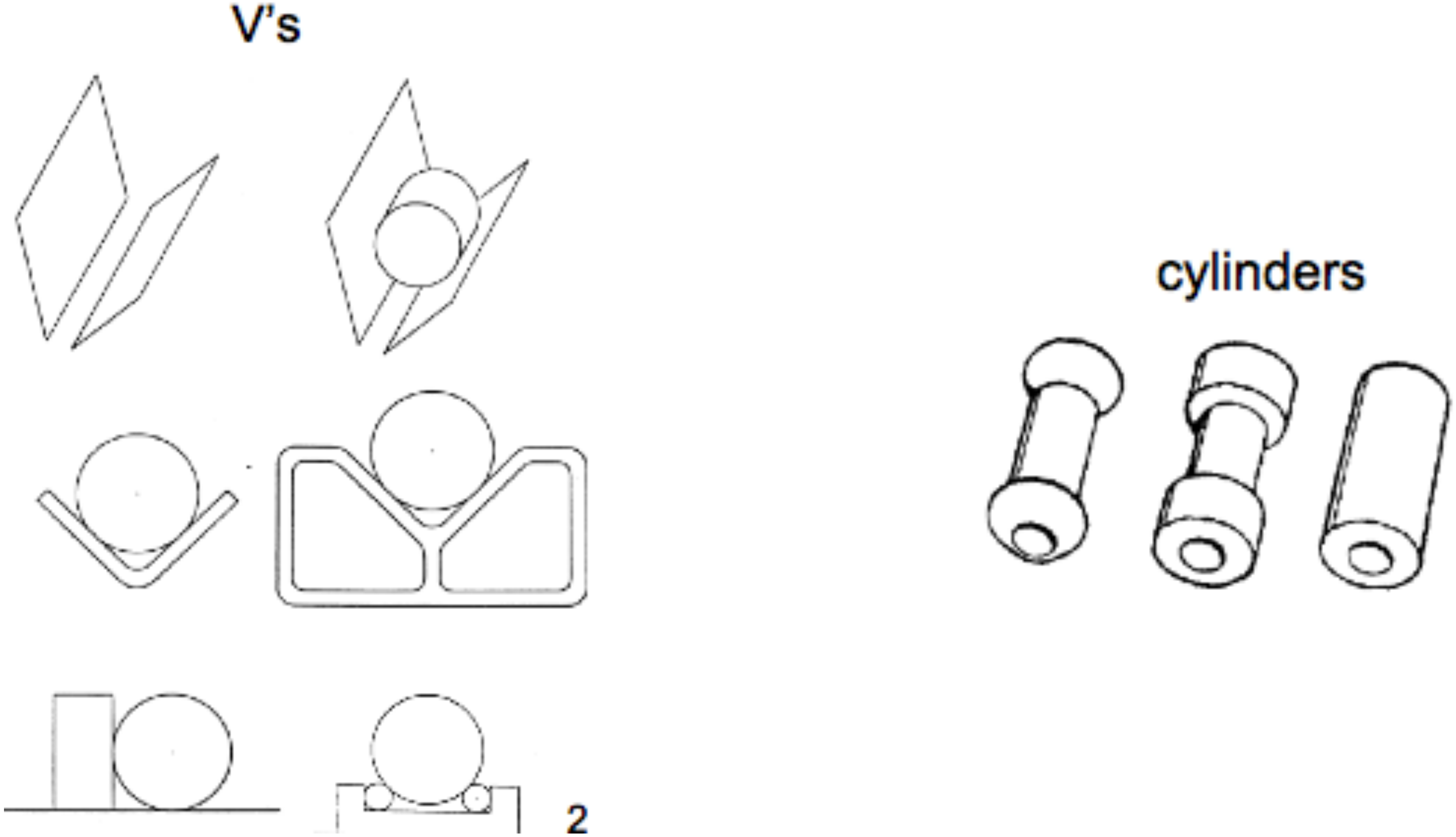


FIG. 59.—V Bench for Lenses of Uniform Diameter.

Fig 35 - From *Treatise on Practical Light*, 1911

Cylinders in V’s



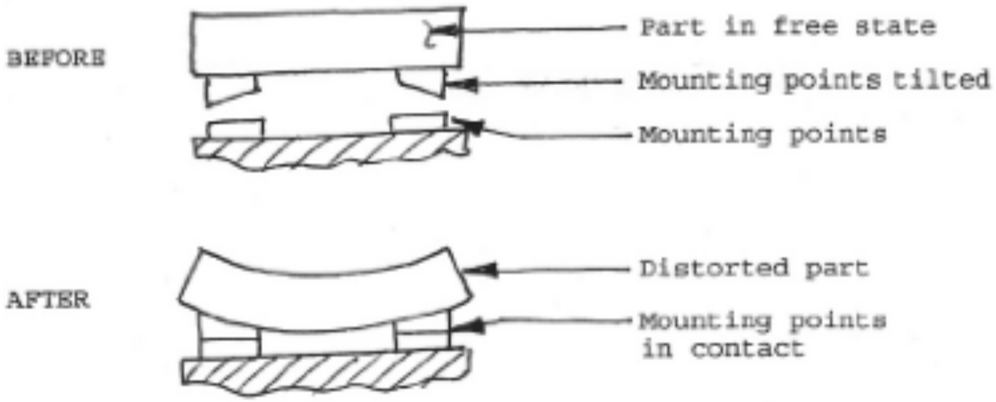
"Cylinders in Vs—An Optomechanical Methodology," Douglas S. Goodman, SPIE Proceedings 3132 *Optomechanical Design and Precision Instruments*, Santa Diego, CA, July, 1997

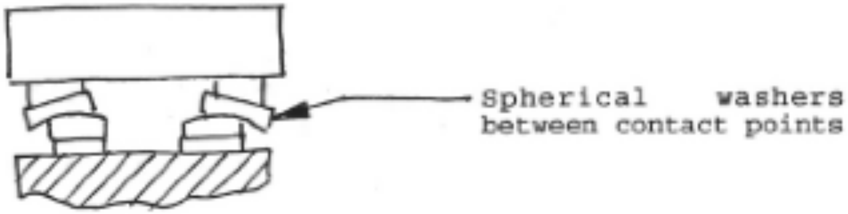
"More Cylinders in Vs," Douglas S. Goodman, SPIE Proceedings 4198, *Optomechanical Engineering*, Boston, MA, November, 2000

- A major problem with kinematic design is high stress in contact areas. Hertz contact stress theory is used to evaluate this problem.

- If stress is too high, use kinematic principles but replace point contacts with small area contacts. This is known as semi-kinematic design.

- A potential problem with semi-kinematic design is distortion of the part due to non-coplanarity of the mounting points.





☐ Assembly procedures for semi-kinematic design are usually critical if part distortion is to be avoided.

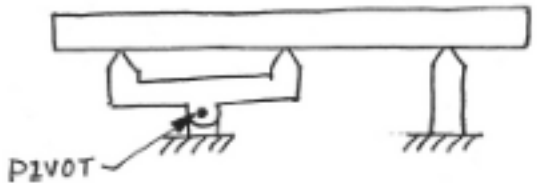
(Vukobratovich p. 76)

- Kinematic mounted parts may have excessive self-weight induced deflection between contact points. This requires additional support which nullifies kinematic design.

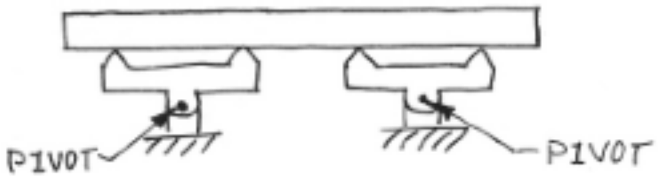
- One solution to the multi-point problem is a whiffle tree. This is a cascaded system of support where each level of support is kinematic.

- Consider a simple beam:

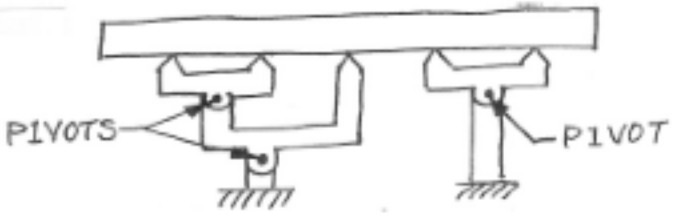
3 point support

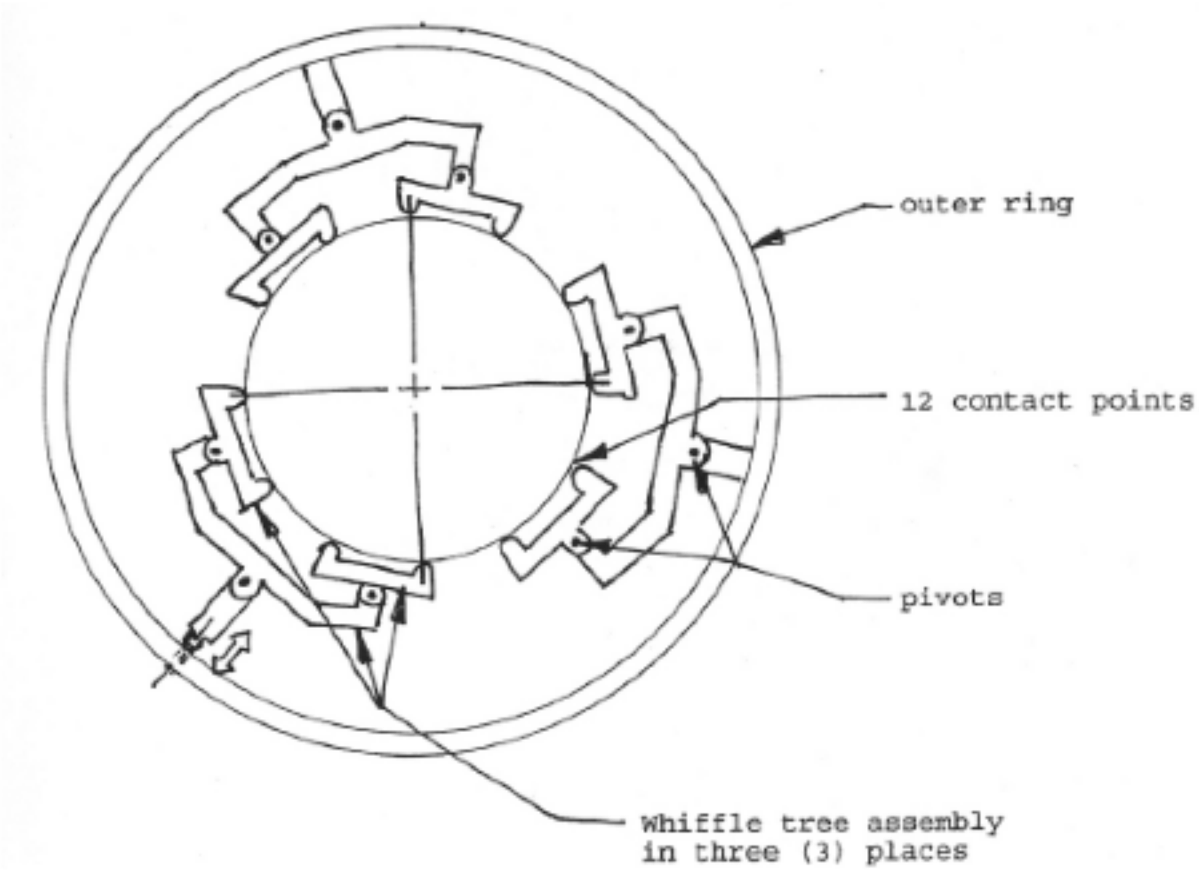


4 point support



5 point support

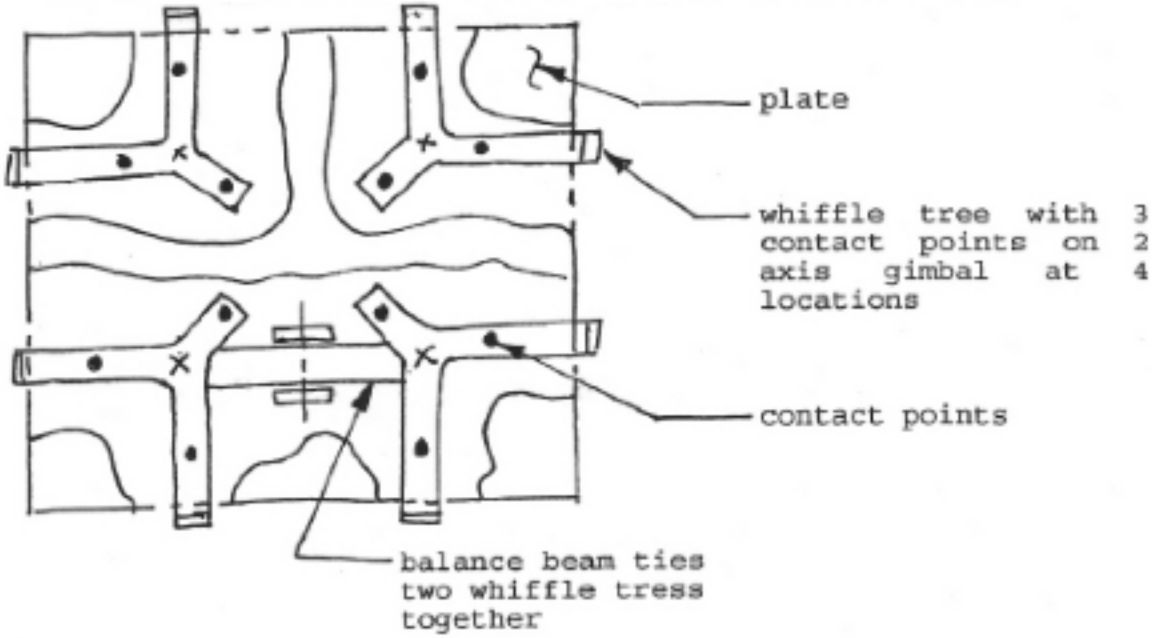




- ☐ The same approach can be used to support a disk around its edge.
- ☐ This approach gives an even distribution of forces yet preserves a three (3) point kinematic location system.

(Vukobratovich p. 79)

- The same approach also “floats” plates on whiffle trees.



plate

whiffle tree with 3 contact points on 2 axis gimbal at 4 locations

contact points

balance beam ties two whiffle tress together

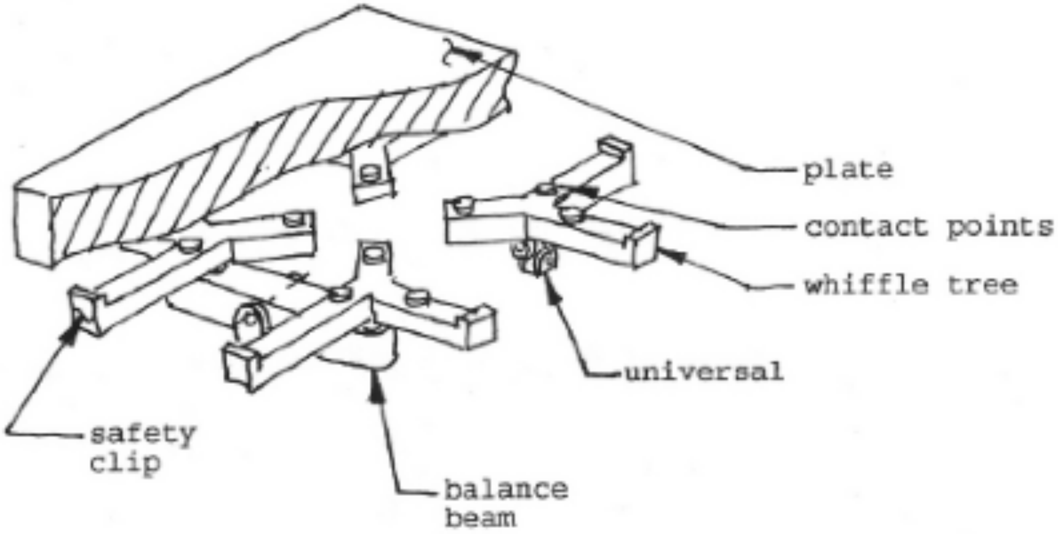


plate
contact points
whiffle tree
universal
safety clip
balance beam

Exact constraint design I

Blanding (1999) has developed a comprehensive approach to kinematic design. The basic concepts are:

1. Points on the object along the constraint line can move only at right angles to the constraint line, not along it
   - The constraint (link) is “rigid”, so it cannot stretch or shorten.
   - The component of motion must be perpendicular to the constraint.
2. An unconstrained 2-D object has 3 DOF (two translations, one rotation).
   - One constraint will eliminate 1 degree of translational freedom.
   - As a result, the object will have two independent degrees of freedom: a rotational degree of freedom that intersects the constraint line (anywhere), and a translational degree of freedom perpendicular to the constraint line.
3. Try to avoid overconstraint!
   - Parts will not fit properly
   - Assembly will either be too loose or too tight
   - Internal stresses will buildup or will be transferred
   - Warping, bowing, non-repeatable behavior due to stick-slip, premature failure