Chapter 12
The Diffraction Grating

Lecture Notes for Modern Optics based on
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Diffraction grating

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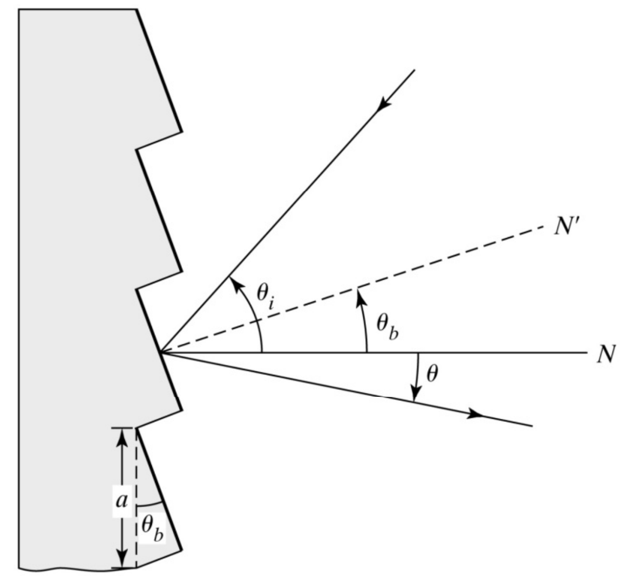


Diffraction from many slits

For N slits of width b and pitch a the diffraction pattern has the following form

$$\boxed{I = I_0 \underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{Diffraction by a single slit}} \underbrace{\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2}_{\text{Interference between N slits}}}$$

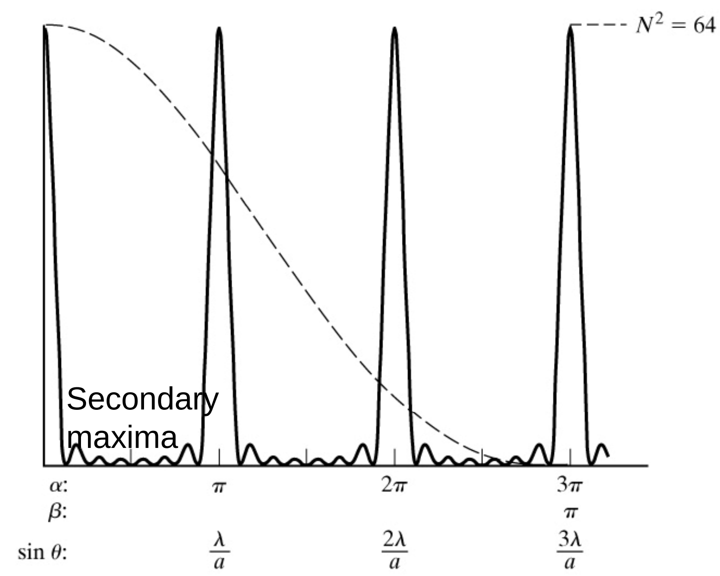
with $\boxed{\beta = (1/2)kb\sin\theta}$ and $\boxed{\alpha = (1/2)ka\sin\theta}$

The pattern consists of series of sharp maxima that we call principal maxima.

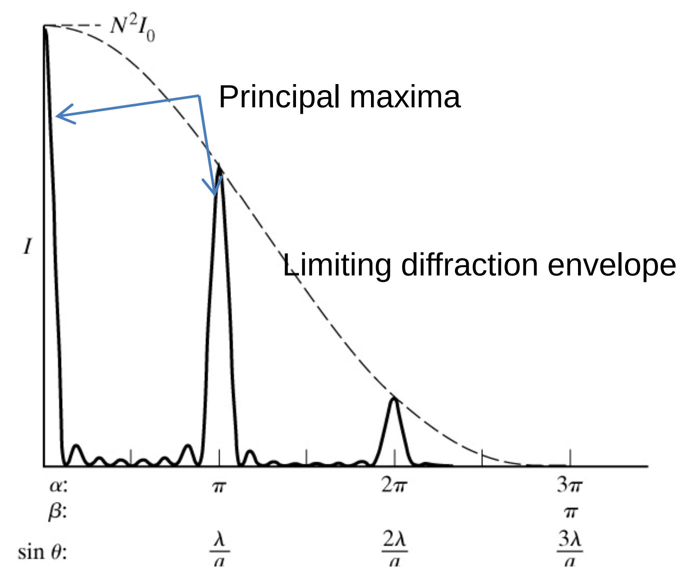
$\boxed{\mathrm{I}_{\text{principal maxima}} \propto N^2}$ cantered at values $\alpha = 0, \pm\pi, \pm 2\pi, \pm 3\pi, ...$

Between successive peaks there are $N-2$ secondary peaks.

The full irradiance is product of the diffraction pattern and interference pattern.



(a)



The grating equation

In many-slit problem in chapter 11 the plane of the incident wavefronts was parallel to the plane of the slits. Here we assume plane of the incident wavefront makes an angle θ_i with the plane of the grating. The net path difference between the successive slits is:

$\Delta = \Delta_1 + \Delta_2 = a\sin\theta_i + a\sin\theta_m$

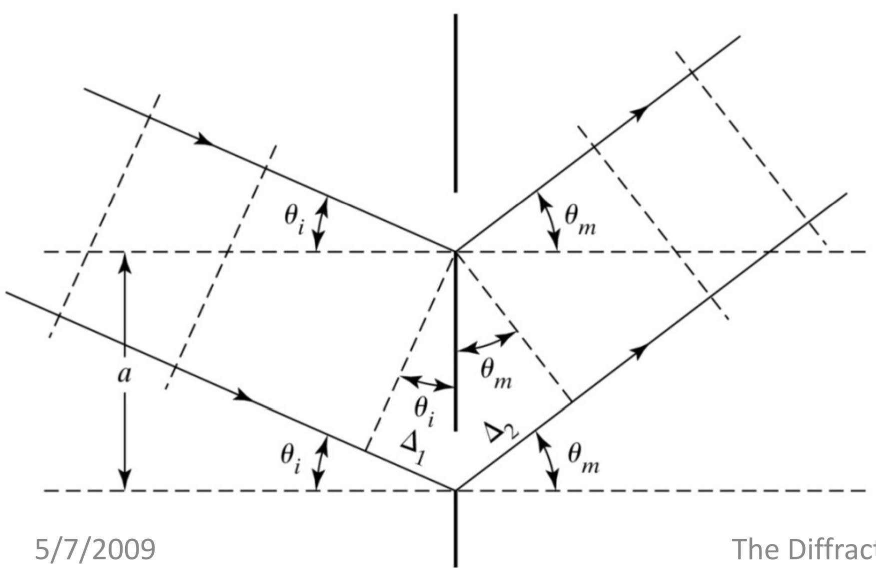
Sign convention: when θ_i and θ_m are on the same side of the normal to the grating, $\theta_m > 0$

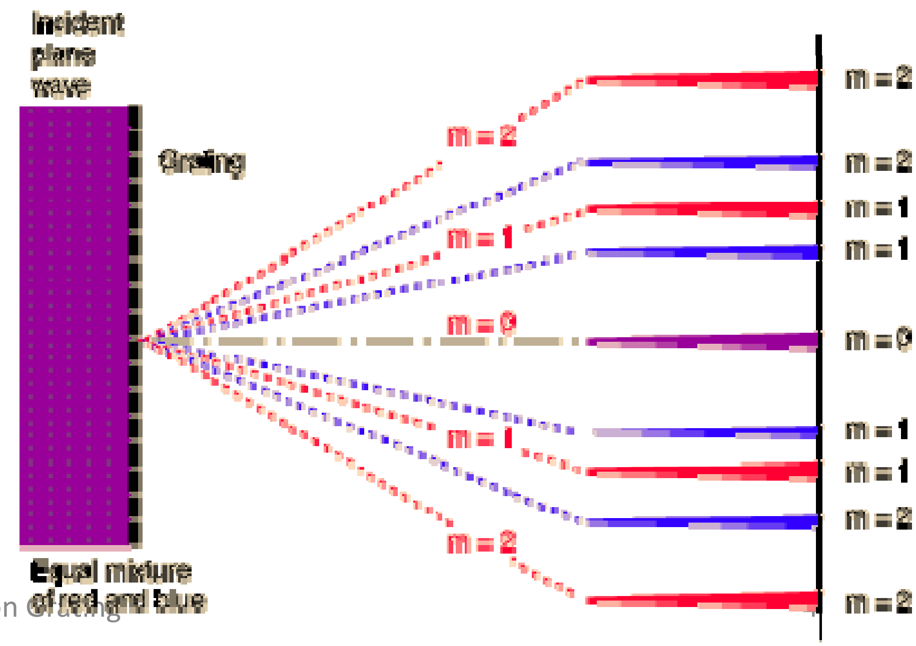
when the diffracted rays on the opposite side of the normal to the grating, $\theta_m < 0$

The grating equation is then:

$$\boxed{a\left(\sin\theta_i + \sin\theta_m\right) = m\lambda \text{ where } m = 0, \pm 1, \pm 2, \ldots}$$

For each value of the m different wavelengts will be enhanced at different angles excpt for $m = 0$ that all wavelengths will be enhanced at $\theta_m = -\theta_i$





Diffraction grating and many-slit

1) Maxima for different wavelengths occur at different positions

2) Higher number of slits result narrower principal maxima in diffraction pattern.

A diffraction grating is a periodic multiple-slit device that is designed to take advantage of the sensitivity of its diffraction pattern to the wavelength of the incident light. $\boxed{\text{invented by Fraunhofer}}$

There are two kinds of grating: $\begin{cases} \text{transmission} \\ \text{reflection grating} \end{cases}$

Slits are called lines or rulings

Grating spacing is the distance between rulings

Question:

Where is the first order maxima for red compared to blue on the screen?

$a\left(\sin\theta_i + \sin\theta_m\right) = m\lambda$ where $m = 0, \pm 1, \pm 2, \ldots$

For a given m and θ_i the longest wavelength has the smallest deviation angle θ_m.

This is opposit of the prism deviation that the largest deviation occur of the largest λ.

Free spectral range of a grating

Example: Visible spectrum is spread between 400 & 700 nm. Find the angular width of the first order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating. Do the first and second order spectrum overlap? What about second and third order spectra? Does your answer depend on the grating spacing? No

$$a\left(\sin\theta_i + \sin\theta_m\right) = m\lambda \text{ where } m = 0, \pm1, \pm2, \ldots$$

$$\theta_i = 0, \quad a = 1\times10^{-3}/600 = 1.67\times10^{-6}$$

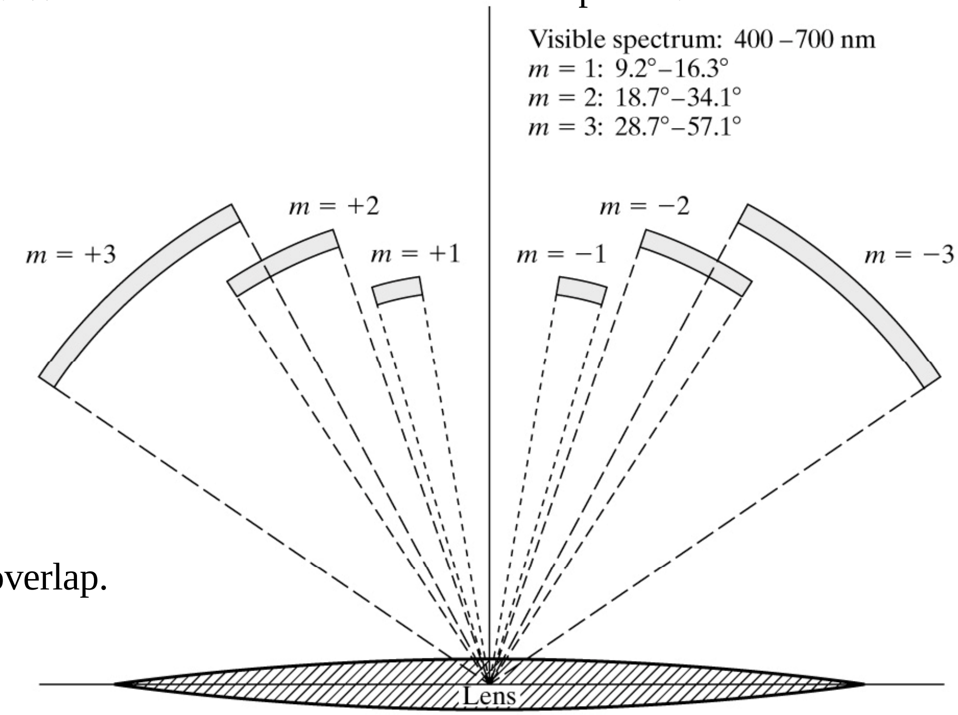
$$a\sin\theta_{1r} = \lambda_r \rightarrow \sin\theta_{1r} = \frac{700\times10^{-9}}{1.67\times10^{-6}} \rightarrow \theta_{1r} = 24.77^0$$

$$a\sin\theta_{1b} = \lambda_b \rightarrow \sin\theta_{1b} = \frac{400\times10^{-9}}{1.67\times10^{-6}} \rightarrow \theta_{1b} = 13.88^0$$

$$\Delta\theta_1 = 10.57^0$$

$$\sin\theta_{2r} = \sin\theta_{1r}/2 \rightarrow \theta_{2r} = 12.097^0$$

The 1st and 2nd order do not overlap, the 2nd and 3rd overlap.



(b)

Free spectral range (λ_{fsr}) for a diffraction grating: the non-overlapping wavelength range in a particular order is called the free spectral range.

λ_1 shortest wavelength in the spectrum, and λ_2 longest wavelength

λ_{fsr} for order m is determined by coinciding the λ_2 in order m with λ_1 in order $m+1$

$$m\lambda_2 = (m+1)\lambda_1 \rightarrow \boxed{\lambda_{fsr} = \lambda_2 - \lambda_1 = \frac{\lambda_1}{m}}$$ Note λ_{fsr} is different for different orders.

$\lambda_{\text{fsr1}} = 400$, $\lambda_{\text{fsr2}} = 200$ yes there will be overlap, $\lambda_{\text{fsr3}} = 133$, only 133 nm is not overlaping.

Dispersion of a grating

Higher diffraction orders become less intense under the envelope of the single slit diffraction.

Within an order the wavelengths spead better for higher orders.

The angular dispersion defined as

$\boxed{\mathcal{D} \equiv \frac{\mathrm{d}\theta_m}{d\lambda}}$ is the angular separation per unit wavelength.

The grating equation: $a\left(\sin\theta_i + \sin\theta_m\right) = m\lambda$

For normal incidene: $a\left(\sin\theta_m\right) = m\lambda \rightarrow \cos\theta_m d\theta_m = \frac{m}{a} d\lambda$

$$\boxed{\text{Angular dispersion} \equiv \mathcal{D} \equiv \frac{\mathrm{d}\theta_m}{d\lambda} = \frac{m}{a\cos\theta_m}}$$

Linear dispersion on a screen at the focal point of a lens with focal length of f:

$$\boxed{\text{Linear dispersion} \equiv \frac{dy}{d\lambda} = f\,\frac{\mathrm{d}\theta_m}{d\lambda} = f\,\mathcal{D}}$$

Example : λ=500nm, normal incidence on a grating 5000 grooves/cm used with a lens of focal length 0.5m. What is the angular and linear dispersion in first order?

a=$2\times10^{-4}\,cm$; asinθ=mλ gives the diffraction angle m=1 and $\theta_1 = 14.5^0$; then the angular dispersion:

$\mathcal{D} = \frac{m}{a\cos\theta_m} = 5165 rad/cm = 0.0296^0/nm$; the linear dispersion f$\mathcal{D} = 0.258 mm/nm$

Resolution of a grating

High resolution with diffraction gratings is not achhieved by high dispipersion.

Like the Fabry-Perot cavity we define the resolution fo a grating as: $\boxed{\Re \equiv \frac{\lambda}{(\Delta\lambda)_{\min}}}$

with this definition each wavelength peack appear narrower for high-resolution gratings.

$\Delta\lambda_{\min}$ is the minimum wavelength interval of two spectral components that are just resolvable by Rayleigh criteron. For normal incidence the angle of $\boxed{\text{principal maximum}}$ of order m is:

$$a\sin\theta_m = m(\lambda + \Delta\lambda)$$

With Rayleigh's criterion this peak $\boxed{\text{coincides with the first minimum}}$ of the neighboring wavelength

$$a\sin\theta_m = (m + 1/N)\lambda \rightarrow m(\lambda + \Delta\lambda) = (m + 1/N)\lambda \rightarrow \boxed{\Re = mN = \left(\frac{a\sin\theta_m}{\lambda}\right)\frac{W}{a} = \frac{W\sin\theta_m}{\lambda}} \text{ where } N = \frac{W}{a}$$

TABLE 12-1 FABRY-PEROT INTERFEROMETER AND DIFFRACTION GRATING FIGURES OF MERIT

	Fabry-Perot Interferometer	Diffraction Grating
Resolving power, $\Re$	$m\mathfrak{F}$	mN
Minimum resolvable wavelength separation, $\Delta\lambda_{\min} = \lambda/\Re$	$\frac{\lambda}{m\mathfrak{F}}$	$\frac{\lambda}{mN}$
Free spectral range, λ_{fsr}	$\frac{\lambda_1}{m}$	$\frac{\lambda_1}{m}$
Meaning of parameters	m: Number of half–wavelengths in the Fabry-Perot length. $\mathfrak{F}$: Cavity finesse	m: Diffraction order N: Number of grooves in grating