# Chapter 2
# Geometrical Optics

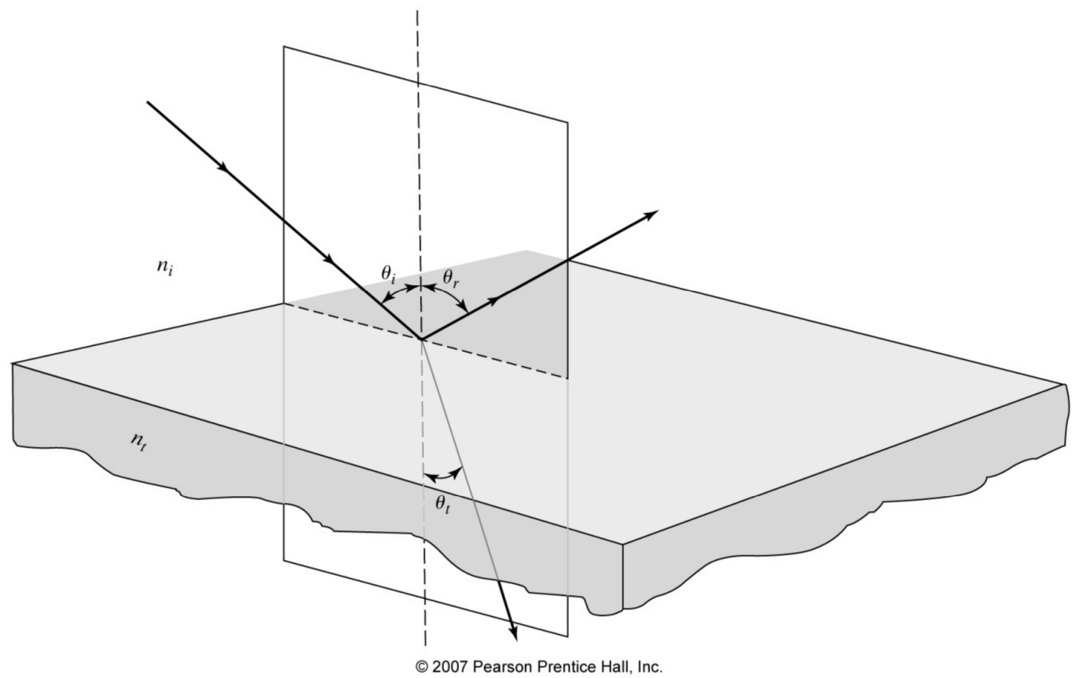
Lecture Notes for Modern Optics based on
Pedrotti & Pedrotti & Pedrotti
Instructor: Nayer Eradat
Spring 2009

## Geometrical optics approximation

We are in the realm of geometrical optics when wavelength is considered to be negligible compared to the dimensions of the relevant optical components of the system.

1. Light travels out from its source along straight lines or rays
2. The energy of light transmitted along these rays.

$$\lim_{\lambda \to 0}(physical\ \ optics) = (geometrical\ \ optics)$$




Reflection and refraction at an interface between two optical media.

## Law of reflection
## Law of refraction (Snel's Law)

**Plane of incidence:**

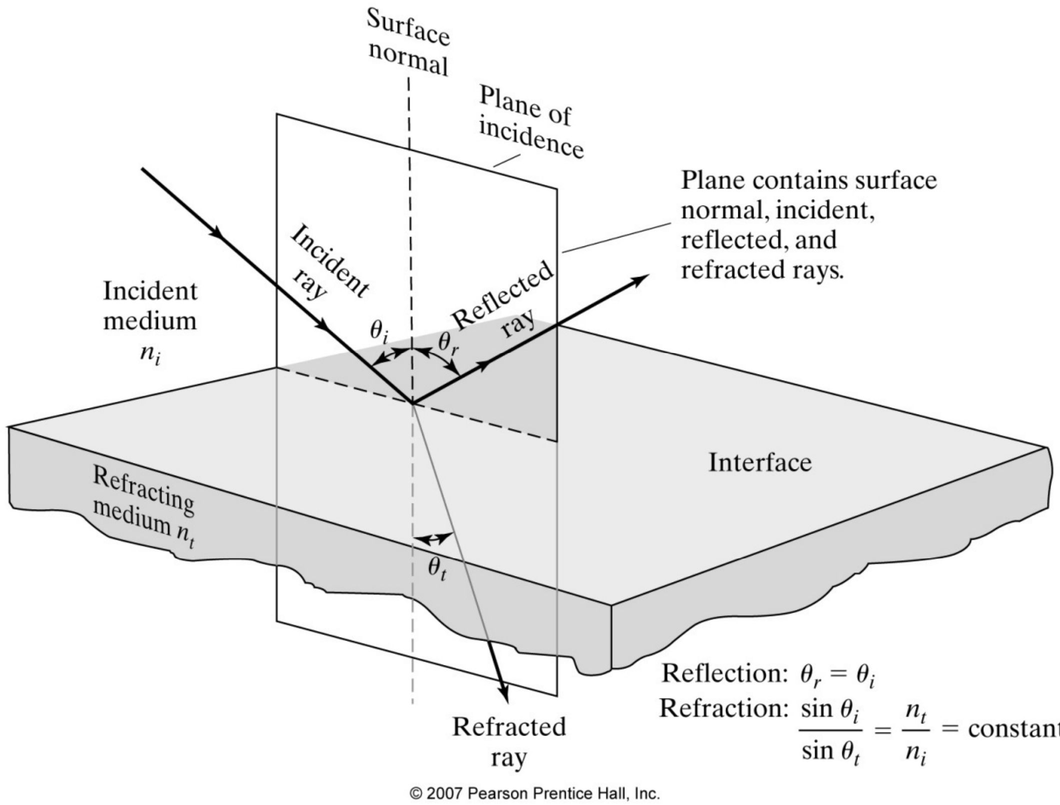
The plane defined by the incident ray and the normal to the interface is called plane of incidence.

**Law of reflection:**

When a ray of light is reflected at an interface dividing two optical media, the reflected rays remain within the plane of incidence, and the angle of reflection is equal to the angle of incidence.

**Law of refraction (Snel's law):**

When a ray of light is refracted at an interface dividing two optical media, the transmitted ray remains within the plane of incidence and sine of the angle of refraction is directly proportional to the sine of the angle of incidence.

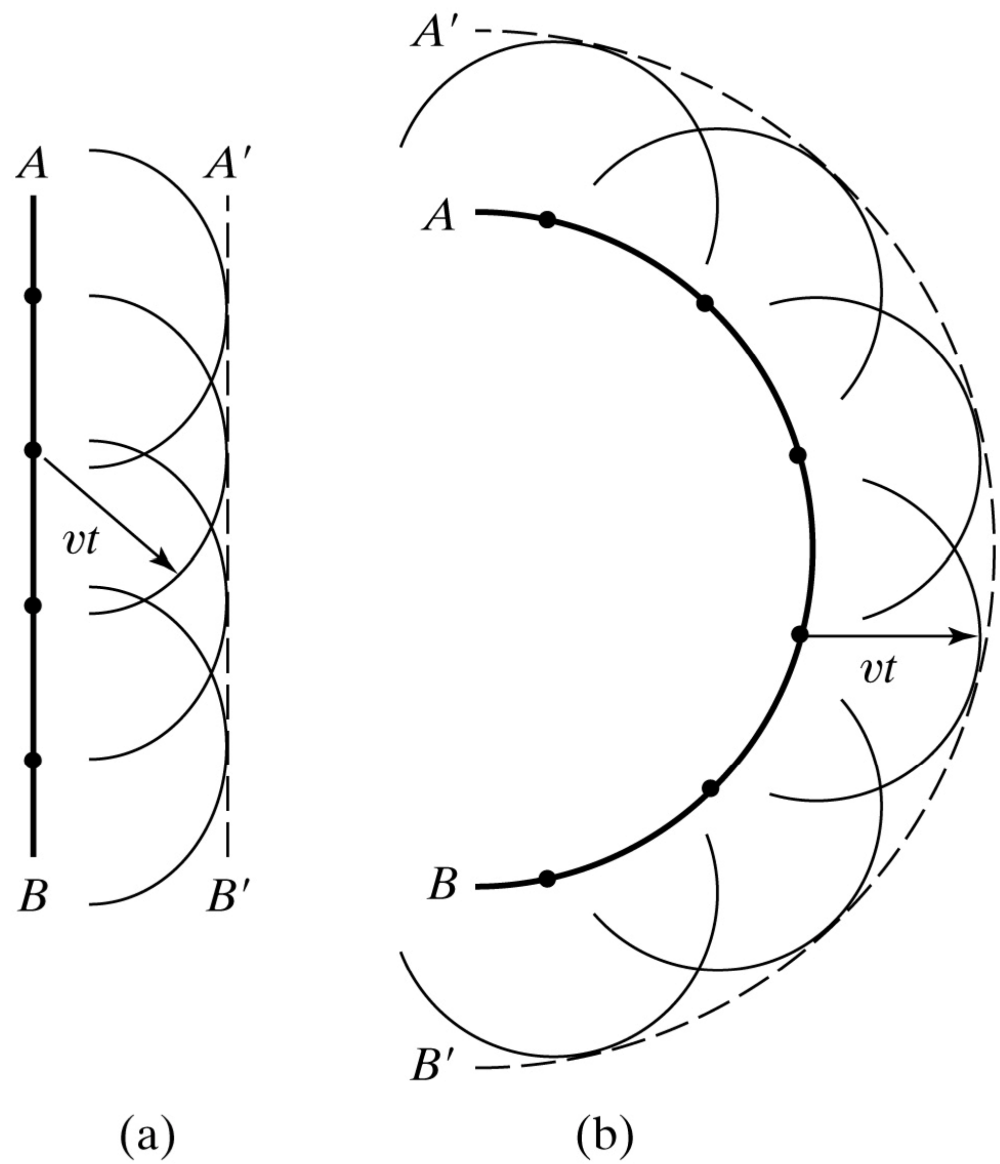



Reflection and refraction at an interface between two optical media.

## Huygens' Principle

Light is composed of series of pulses emitted from each point on the luminous body.

Wave disturbance-wavefront- may be regarded as a secondary source of spherical waves (wavelets) which themselves progress with the speed of light in the medium and whose envelope at a later time constitute the new wavefront.

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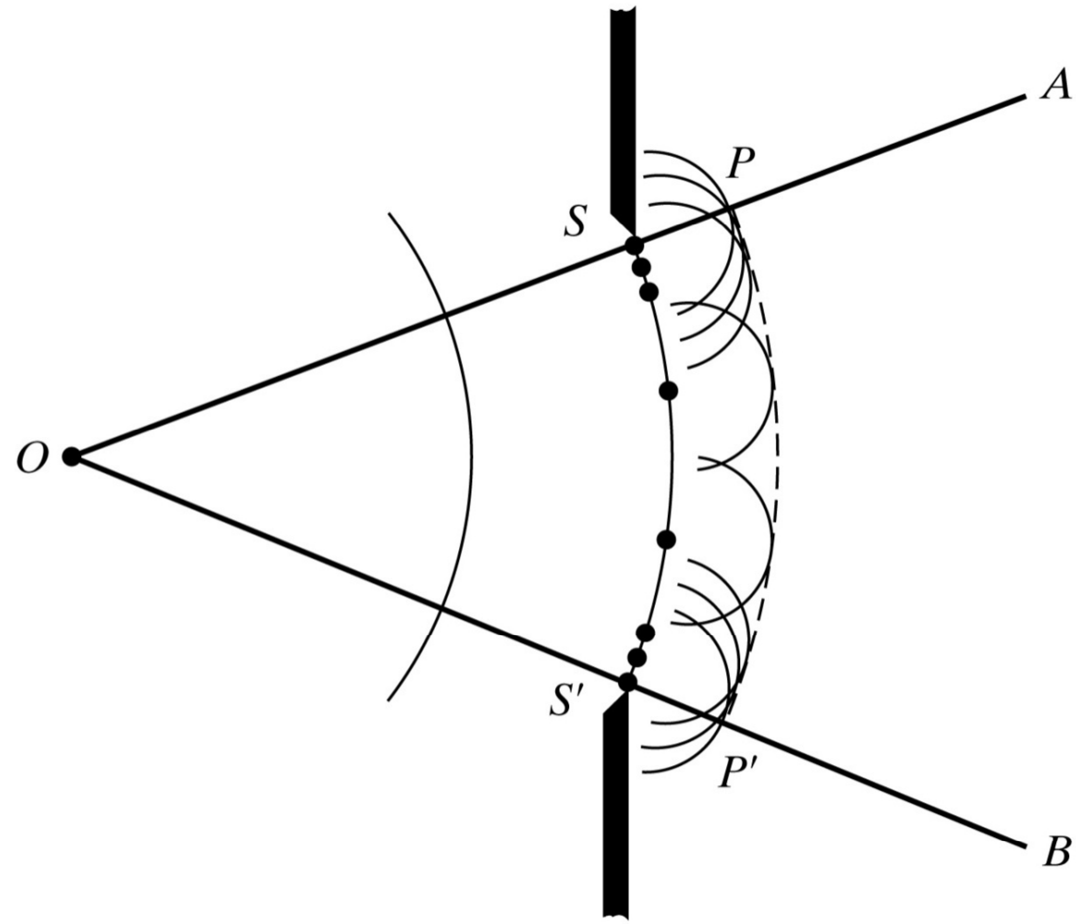


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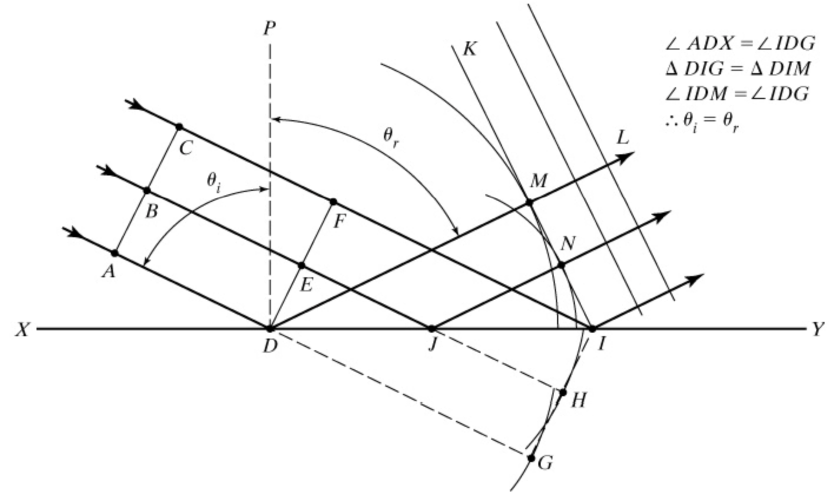
## Deduction of the laws of reflection and refraction with Huygens' principle



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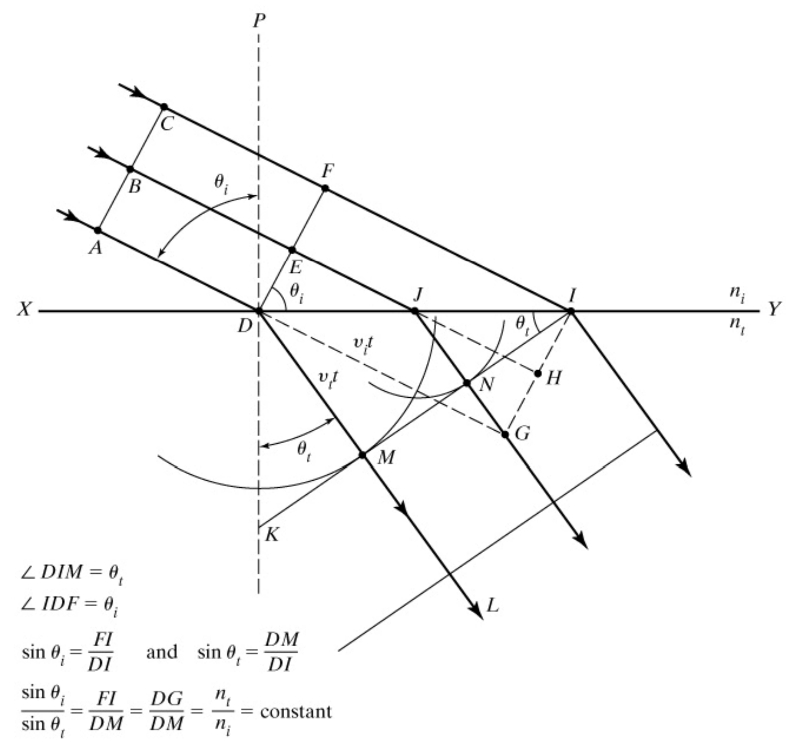

(b)


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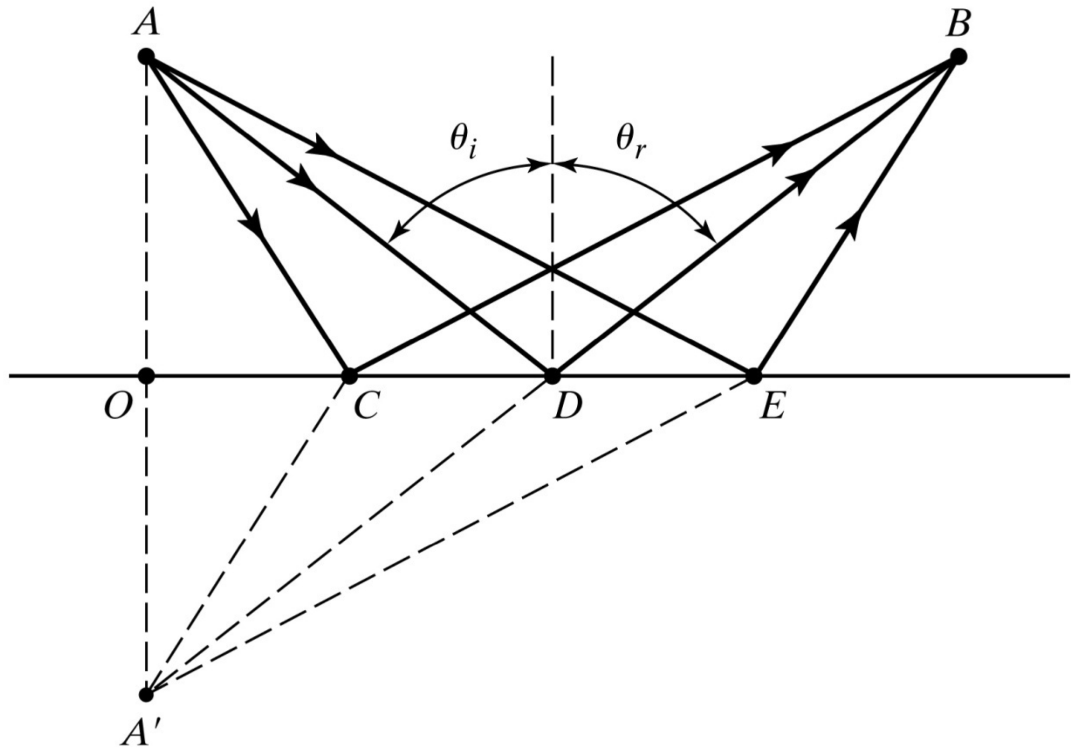
## Economy of nature

Hero of Alexandra: light propagating between two points always takes the shortest path.

Hero's principle is enough to prove the law of reflection but for refraction we need a more sophisticated version of economy of nature.

The shortest path in this graph is ADB that requires $\theta_i = \theta_r$

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## Fermat's principle

Light travels along the path that takes shortest time (economy of nature).

If the light slower in one medium than the other one, it takes a path that is shortest in the slow medium and longest in the fast medium. Therefore minimizing the travel time .

$$t = \frac{AO}{V_i} + \frac{OB}{V_t} = \frac{\sqrt{a^2 + x^2}}{V_i} + \frac{\sqrt{b^2 + (c-x)^2}}{V_t}$$

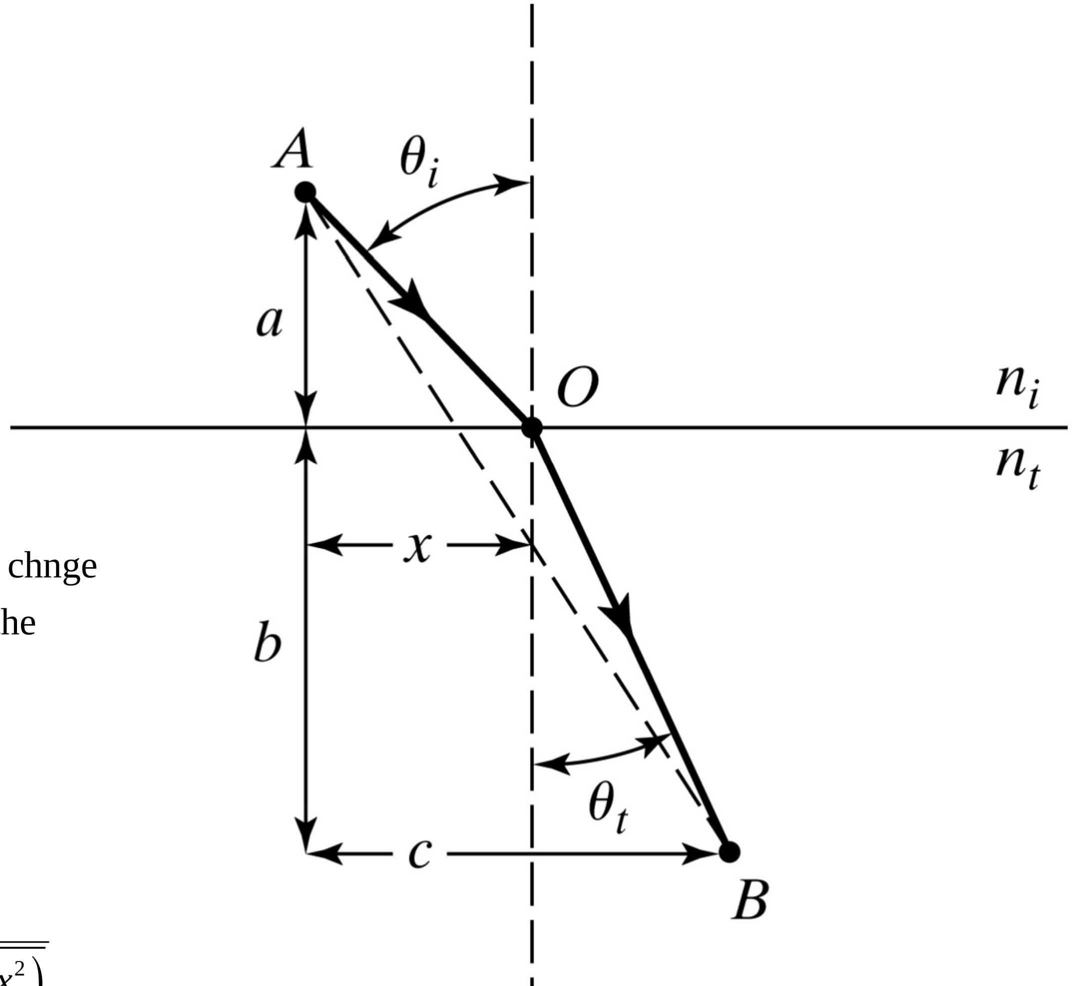
Here x is the independent variable that its chnge affects the travel time. We can minimize the travel time by setting $\frac{\mathrm{dt}}{\mathrm{dx}} = 0$

$$\frac{dt}{dx} = \frac{x}{V_i\sqrt{a^2 + x^2}} - \frac{c-x}{V_t\sqrt{b^2 + \left(c - x^2\right)}} = 0$$

$$\sin\theta_i = \frac{x}{\sqrt{a^2 + x^2}} \text{ and } \sin\theta_r = \frac{c-x}{\sqrt{b^2 + \left(c - x^2\right)}}$$

$$\frac{dt}{dx} = \frac{\sin\theta_i}{c/n_i} - \frac{\sin\theta_r}{c/n_r} = 0$$

$$n_i \sin\theta_i = n_r \sin\theta_r$$



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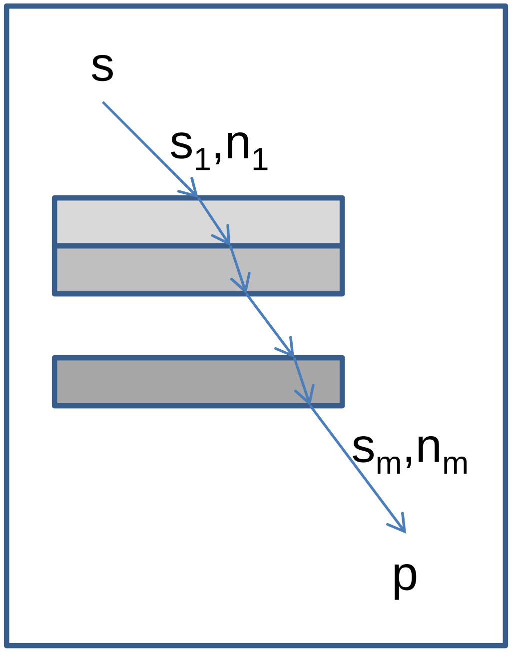
## Optical path length

Optical path length is defined as

$$OPL = \sum_{i=1}^{m} n_i s_i \text{ or } OPL = \int_s^p n(s)ds$$

in contrast to spatial path length

$$SPL = \sum_{i=1}^{m} s_i \text{ or } SPL = \int_s^p ds$$



# Modern formulation of Fermat's principle

Sometimes the shortest travel time is not unique or light may take the longest travel time. Example?

A better version:

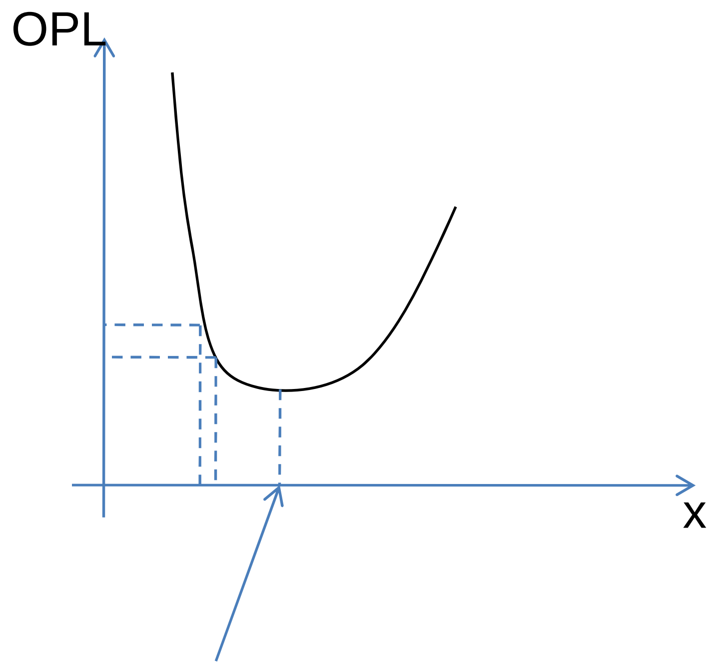
A light ray in going from one point to another must traverse an optical path that is **stationary with respect to variations of that path**.

**Variational calculus:**

A technique that determines the form of a function that minimizes a definite integral.

**In optics** the definite integral is integral of the time required for the transit of the light ray from starting to finishing point or is the optical path length.

Fermat's principle leads to **principle of reversibility.** If the source and destination switch places that light path will be the same.



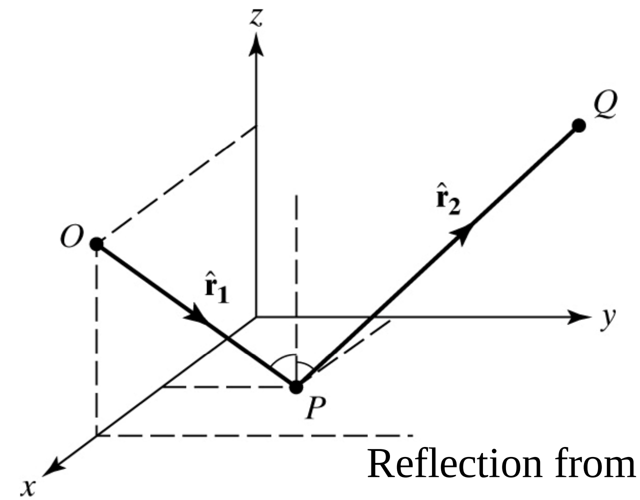

Proper value of x that minimizes the optical path length and keeps it stationary at the vicinity.

## Reflection in plane mirrors

**Specular reflection** from a perfectly smooth surface: all of the rays from a parallel incident beam reflect as a parallel beam.

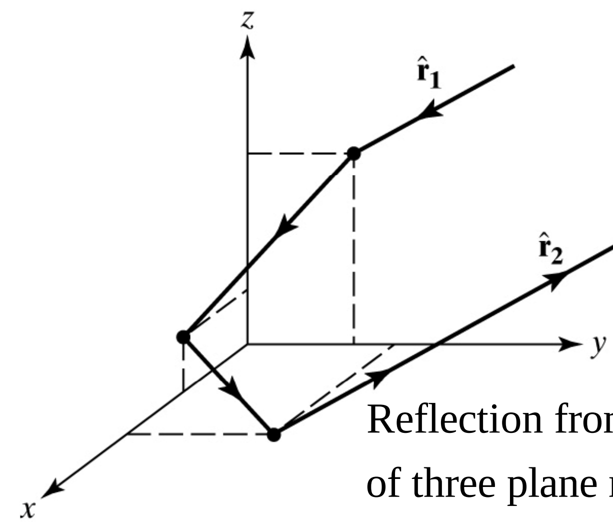
**Diffuse reflection** from a granular surface: though the law of reflection is obeyed for each ray locally microscopically granular surface results in diffusing the beam.

The **corner cube reflector** in figure (b) reflects the outgoing rays exactly parallel to the incoming rays regardless of incidence angle.




Reflection from a plane mirror

(a) $\hat{\mathbf{r}}_1 = (x, y, z) \rightarrow \hat{\mathbf{r}}_2 = (x, y, -z)$




Reflection from a corner of three plane mirrors

(b) $\hat{\mathbf{r}}_1 = (x, y, z) \rightarrow \hat{\mathbf{r}}_2 = (-x, -y, -z)$

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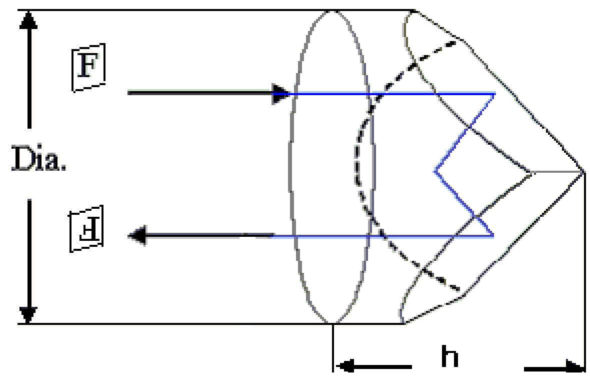
## A commercial product: Corner Cube Retroreflectors

What applications you can think for such a reflector?




| | |
|---|---|
| Material | Bk7, fine annealed (other material upon request) |
| Dimensions Tolerance | +0/-0.10 mm(±0.10mm for #PCC716-10) |
| Height Tolerance | ±0.25mm (±0.5mm for #PCC716-10) |
| Surface Flatness | <λ/4 @ 632.8 nm |
| Reflected Beam Deviation | 3", 5", 10" |
| Reflection face fitness | λ/8 @ 632.8 nm |
| Surface Quality | 40-20 Scratch & Dig |
| Clear Aperture | 90% |
| Chamfers | 0.1- 0.25 mm x 45° |
| Coating | Uncoated or silver with inconel and black overpaint |

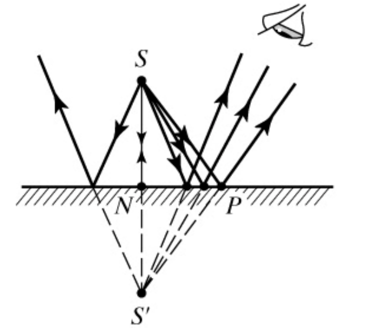
# Image formation in a plane mirror

a) Image of a point object. Following the law of reflection we see triangles SNP and S’NP are equal. All the reflected rays seems to be originating at image point S’. Image properties:
   a) Image distance = object distance
   b) Image is virtual (no actual rays intersect to build the image)
   c) Image cannot be projected on a screen.

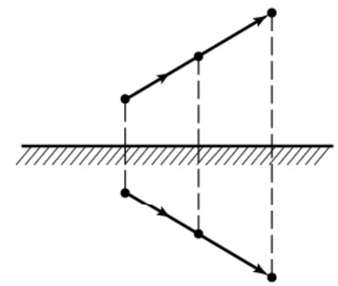
b) Image of an extended object on a plane mirror. All the properties are the same as that of the point object.
   a) Image size = object size or magnification = 1
   b) Transverse orientation of image and object are the same.
   c) A right-handed object has a left-handed image.
   d) Image location does not depend on the observer.

c) The mirror does not lie directly below the object. We can extend the mirror to construct the image.

d) Multiple images of a point object from direct reflections and multiply reflected light rays.




(a)



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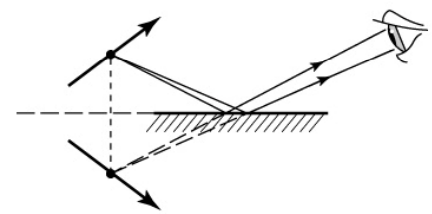

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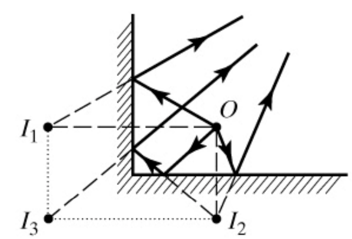


(d)


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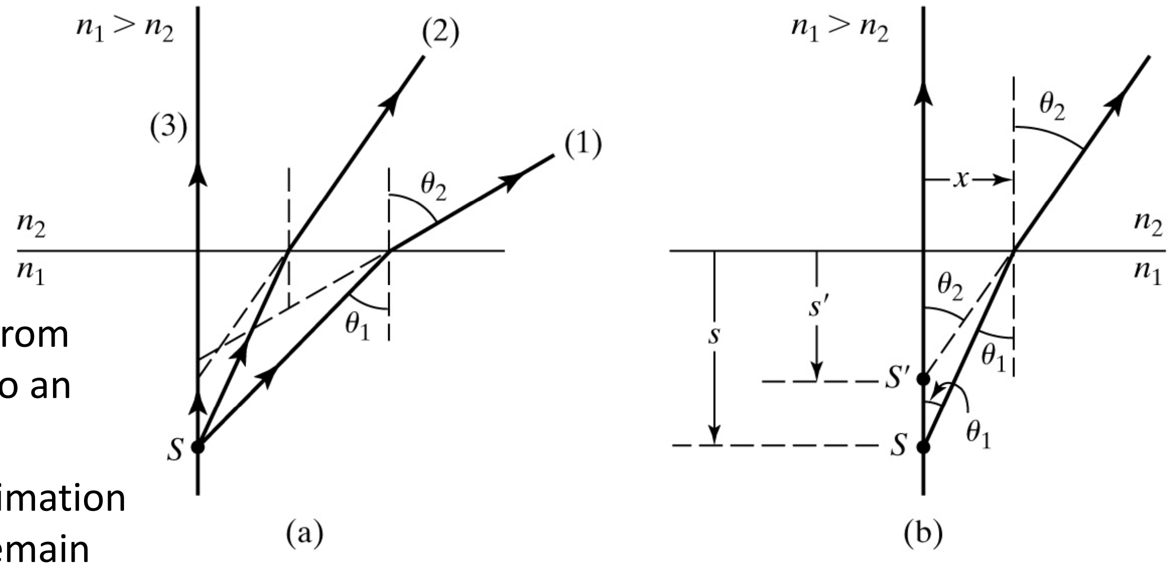
## Anouncemenbt
## Optics lab 120

Hello Ken, Peter and Nayer:

Could you please announce in your classes that there is still space in the 120C Lab.
There are still 3 slots open. We will be performing the following experiments:
Some alignment techniques
Michelson Interfereometer to measure refractive index of air
Refractive index of liquids using a hollow prism
Fabrication of a holographic grating
Small displacement measurement by laser speckle
Fourier Optics- Spatial filtering
Designing a doublet with an optical design software
Project-building a fingerprint sensor
Project-building a grating spectrometer
Ramen

Ramen Bahuguna
Professor of Physics
Director, Institute for Modern Optics
San Jose State University

## Refraction through plane surfaces




a) Extension of the rays emerging from point object S do not converge to an image point.
b) Using the paraxial optics approximation i.e. working with the rays that remain close to the central axis i.e. sin θ ≈ tan θ ≈ θ ≈$6^0$ ≈ 0.1 radians, we can observe an image of S at S’.

From Snel's law for paraxial rays we can write:

$$n_1 \tan\theta_1 \cong n_2 \tan\theta_2 \rightarrow \mathrm{n}_1 \frac{x}{s} \cong n_2 \frac{x}{s'}$$

Location of the image point: $s' = \dfrac{n_2}{n_1} s$

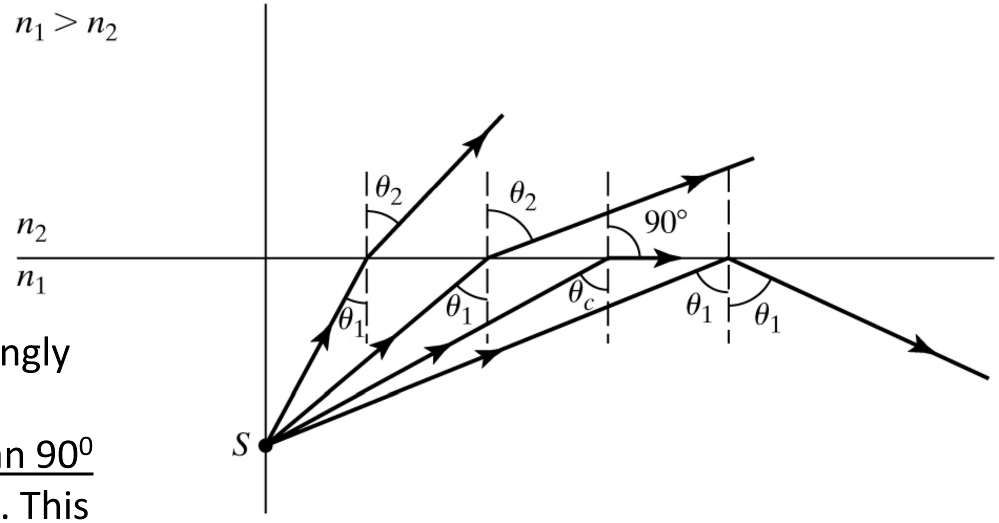
If $n_2 > n_1$ then $s' > s$ and the image forms below the object point.

<u>Apparent depth is larger than the reality</u> (seeing objects from a pool) .

If $n_2 < n_1$ then $s' < s$ and the image forms aove the object point.

<u>Apparent depth is smaller than the reality</u> (looking into water).

## Critical angle
## Total internal reflection (TIR)

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Rays arriving at the interface with increasingly large angles must refract according to the Snell's law. For <u>refraction angles larger than 90⁰</u> the rays reflect back into the first medium. This is called **total internal reflection (TIR).**

The angle at which total internal reflection starts is called the **critical angle.**

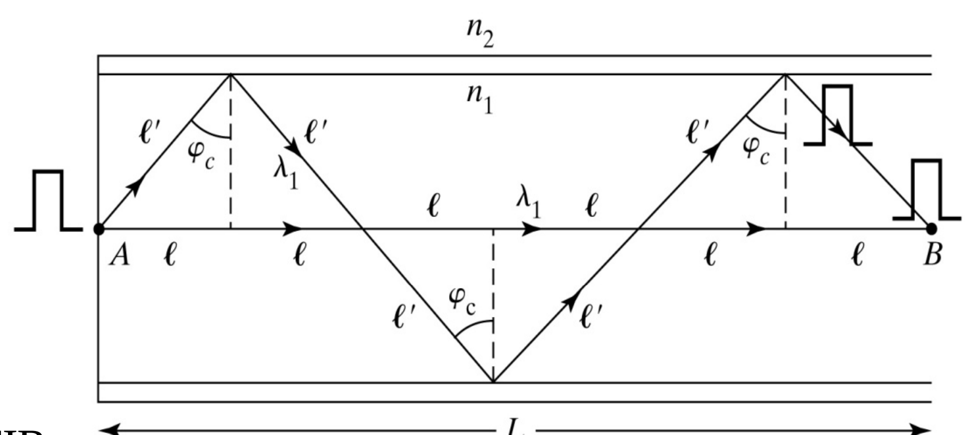
The critical angle:

$$\sin\theta_c = \frac{n_2}{n_1}\sin 90 = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This only happens if $n_1 > n_2$

Name an instrument that works based ot TIR

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## Some definitions
## Regarding imaging

**Optical system**: any number of reflecting and/or refracting surfaces that may alter direction of the rays leaving an object point.

**Object point:** location of object

**Object space:** the space between the object and front surface of the optical system.

**Real object:** is defined by real rays leaving it.

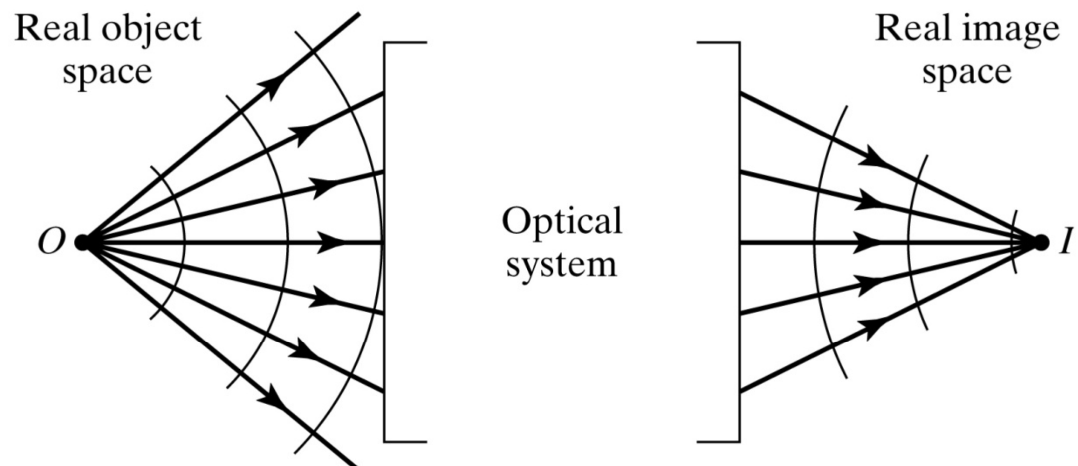
**Virtual object:** is defined by the extension of the rays or virtual rays.

**Image point:** location of the image.

**Image space:** the space between the back surface of the optical system and the image.

**Real image:** is defined by real rays intersecting

**Virtual image:** is defined by the extension of the rays or virtual rays intersecting..

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## Some assumptions and more definitions

Each individual medium in the optical system is **homogeneous** and **isotropic.**

**Homogeneous:** single phase material, no inclusions or bubbles. No severe scattering or diffusion of light inside the material

**Isotropic:** same optical properties across the media means **constant index of refraction**.

**Isochronous:** rays with the same transit time of light from the source.

**Concave surfaces:** center of the curvature is on the reflecting side or source side (light hits the cave).

**Convex surfaces:** center of curvature is on the opaque side or propagation side (light hits the cone).

## Properties and definition of the wavefront:

**Wavefronts:** the family of spherical surfaces normal to the rays that are leaving the object point.

**Wavefronts** are locus of points such that each ray contacting them represents same transit time of light from the source.

Points on a wavefront have **same optical path length from the source**. For points on a wavefront: *OPL=ct=xn=constant*

## Imaging

**Fermat's principle:** All the rays leaving an object point and arriving at the image point of the same object point via an optical system have to be **isochronous.** Otherwise we cant process the image and see it.

**Principle of reversibility:** if object and image points switch places light rays will go through the same path only in opposite direction.

**Conjugate points** for **an optical system** are images of one another.

**Ideal optical system:** all the rays –and only those rays - intersecting an optical system participate in image formation.

To reconstruct an **actual image** of an object, the optical system must generate an ideal image for each point on the object. Actual images are **blurred** because of **scattering, aberrations, diffraction** leading to reflection losses at refracting surfaces, diffuse reflection from reflecting surfaces and scattering by inhomogenities of the material.

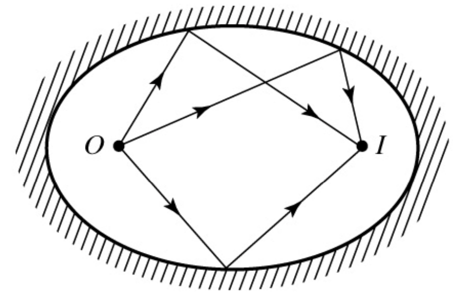
**Aberrations** occur when the **system fails to produce one-to one image** of the all points on the object.

**Diffraction limited image:** for an otherwise perfect image, the effect of **using limited portion of the wavefront** to construct an image leads to diffraction and blurred images. Diffraction poses limitation on the perfect focusing.

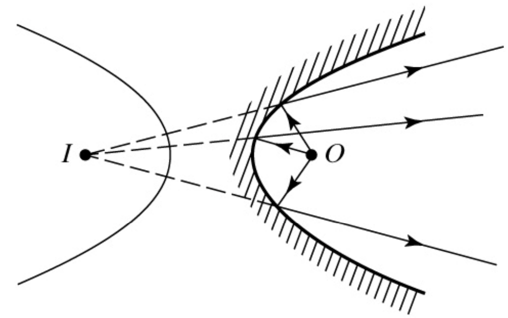
## Cartesian reflecting surfaces

**Cartesian surfaces:** reflecting / refracting surfaces that produce perfect images.

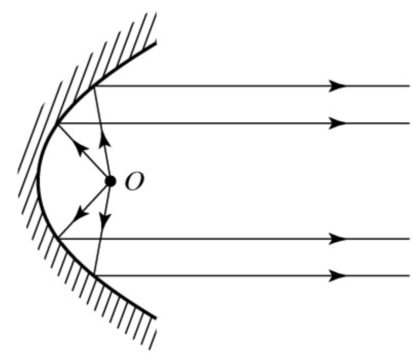
In case of reflection perfect mirrors are **conic sections.**



(a) Ellipsoid



(b) Hyperboloid



(c) Paraboloid

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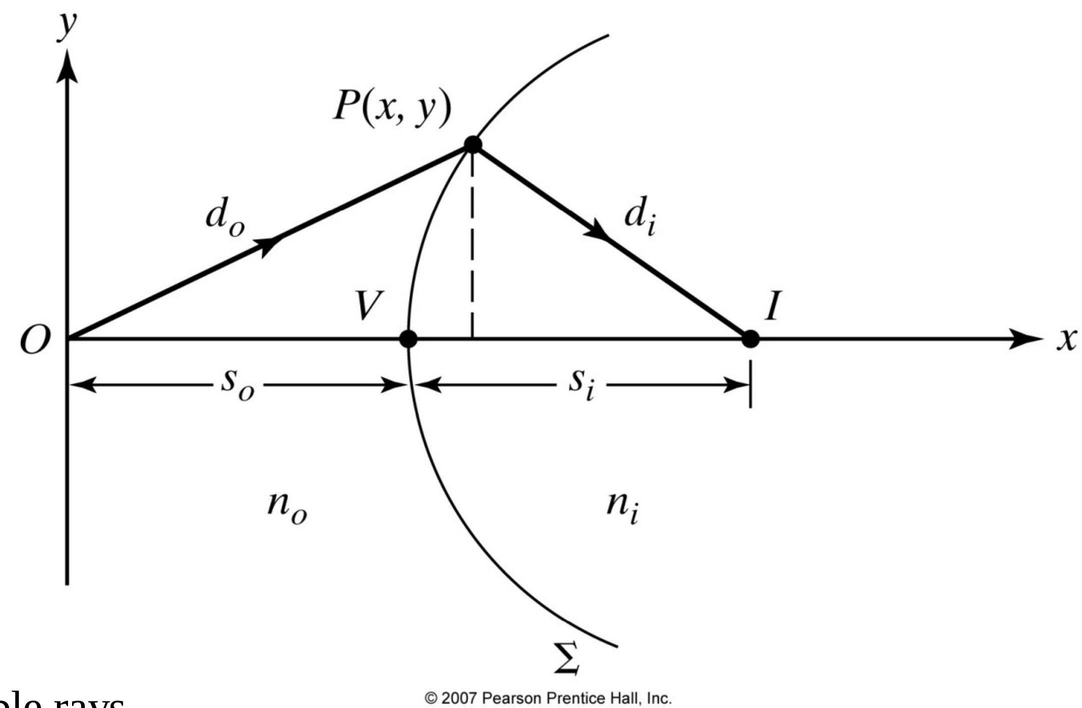
## Finding an appropriate surface that images point O at point I with refraction




Goal: write equation of a surface that creates a point image at I for the object point at O.

Solution: we require every ray refracts at the surface and reaches the image point.

For example OPI and OVI are two possible rays.

By Fermat's principle all these rays are isochronous. So we have:

$$n_o d_o + n_i d_i = n_o s_o + n_i s_i = \text{constant}$$

$$n_o \left(x^2 + y^2\right)^{1/2} + n_i \left(\left(s_o + s_i - x\right)^2 + y^2\right)^{1/2} = n_o s_o + n_i s_i = \text{constant}$$

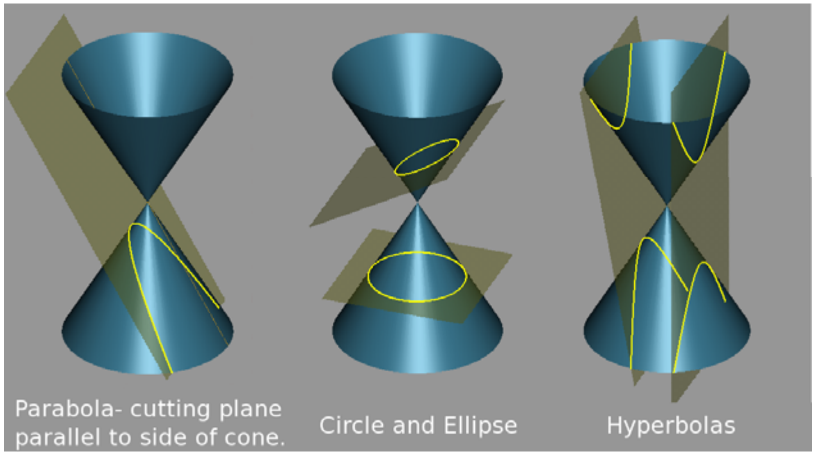
For a specific system $n_o s_o + n_i s_i$ is given and is constant so we have the equation for a conic section.

$$n_o \left(x^2 + y^2\right)^{1/2} + n_i \left(\left(s_o + s_i - x\right)^2 + y^2\right)^{1/2} = \text{constant}$$

This is an equation of a Cartesian ovoid of revolution.

We will come back to this equation. For now we limit our study to paraxial approximation.

## Conic sections (review)




Conic sections are created by intersection of a plane and a cone. The following equations express the graphs on the left side.

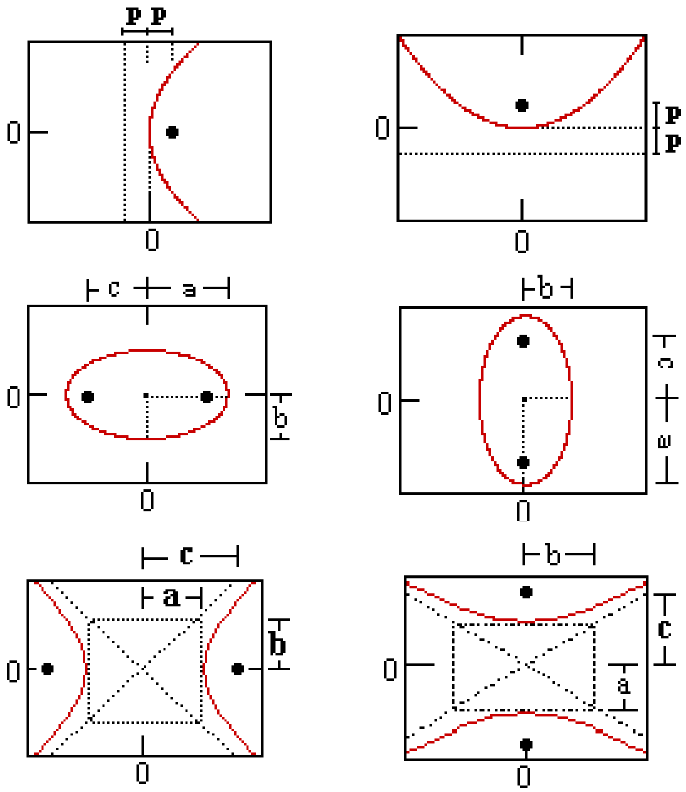
Write equation for the ones on the right saide.

Parabola: $y = x^2 / 4p$ with focal point at $(0, p)$ and directrix $y = -p$

Ellipse: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with foci at $(\pm c, 0)$ where $c^2 = a^2 - b^2$, and vertices $(\pm a, 0)$, and $a \geq b > 0$

Circle: $x^2 + y^2 = R^2$ a special case of the ellipse with $a = b = R$ and $c = 0$.

Hyperbola: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ with foci at $(\pm c, 0)$ where $c^2 = a^2 + b^2$, and vertices $(\pm a, 0)$, and asymptotes $y = \pm \dfrac{b}{a} x$



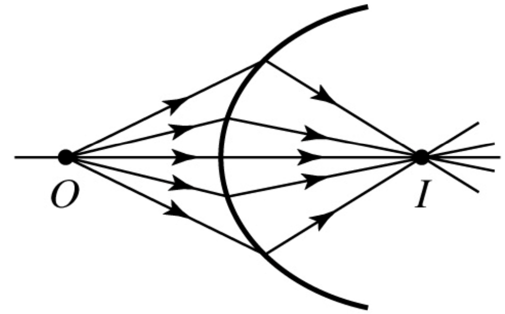
## Cartesian refracting surfaces

a) Image of O at I by refraction from a Cartesian ovoid .

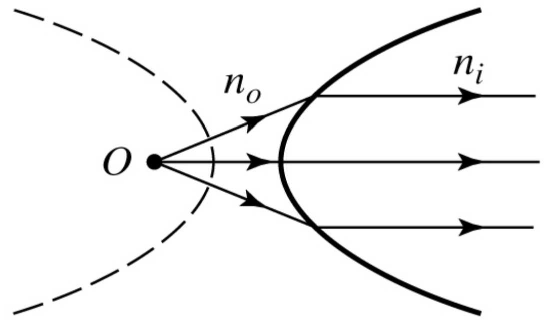
b) Hyperbolic surface images an object at O to a point at infinity. O is at one focus and $n_i > n_o$

c) Ellipse surface images an object at O to a point at infinity. O is at one focus and $n_o > n_i$

Using b and c we can make lenses.




(a)

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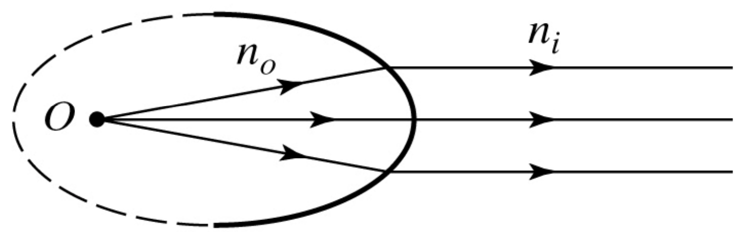


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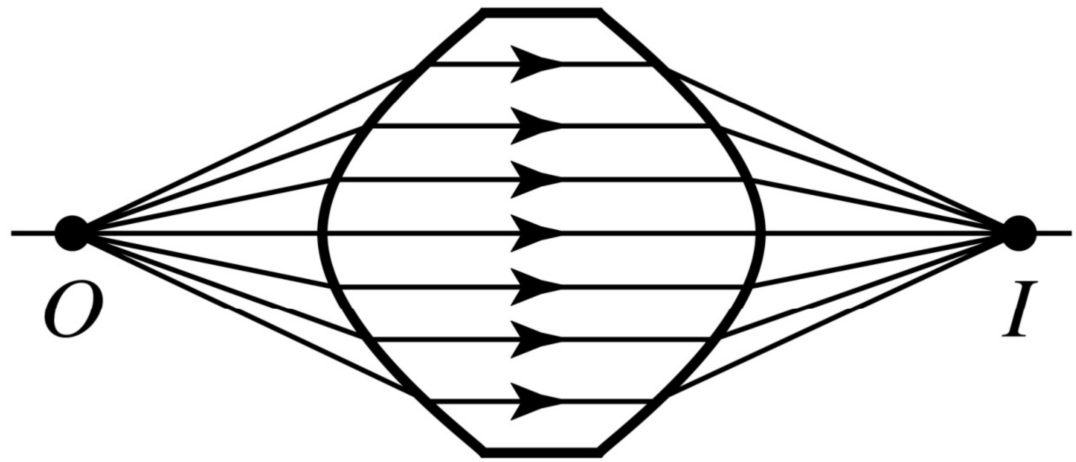
## A conic section lens

A double hyperbolic lens.

This lens is perfect but only for points at O and I.

For extended objects this is a very bad design.

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## Reflection at a spherical surface: first order approximation (paraxial)

Goal: a relationship between s and s' that only depends on R. We draw two rays one hits the vertex, and the other one hits an arbitrary point on the optical system. Using the law of reflection we can trace the two rays hitting V and P. For the covex mirror shown in the figure the image is virtual.

From the figure we can write: $\theta = \alpha + \phi;\ 2\theta = \alpha + \alpha'$

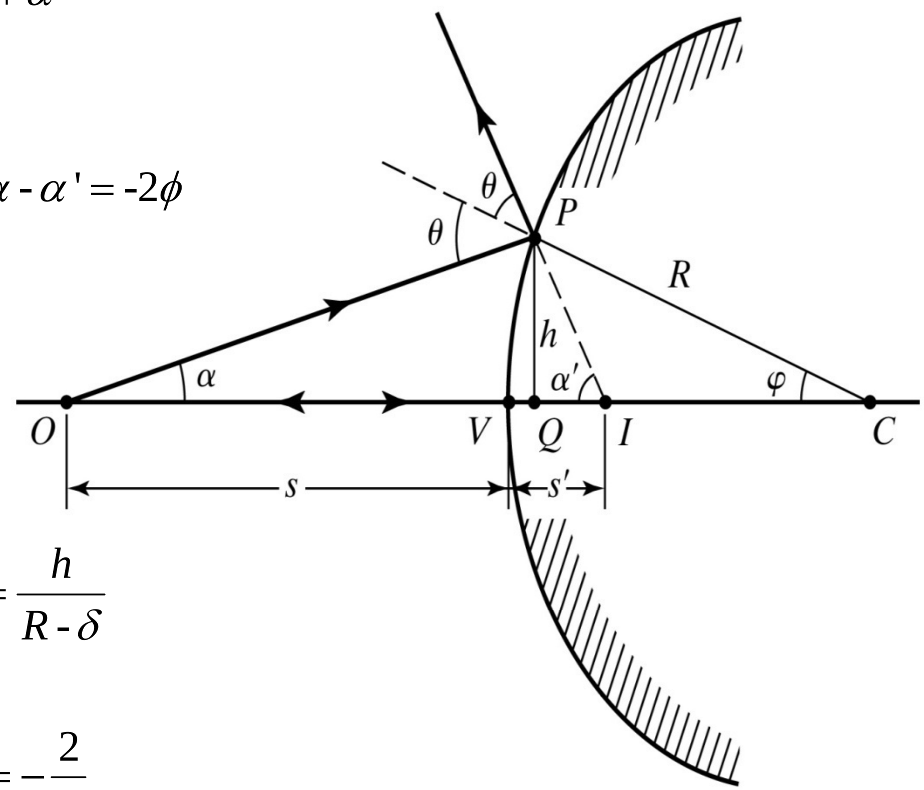
The relationship has to be independent of the ray we choose to trace so we have to eliminate $\theta$ between the two equations. We get: $\alpha - \alpha' = -2\phi$

Now applying the **paraxial approximation** :

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \cdots \xrightarrow{\text{First order}} \sin\phi \cong \phi$$

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots \xrightarrow{\text{First order}} \cos\phi \cong 1$$

$$\alpha \cong \sin\alpha = \frac{h}{s - \delta};\quad \alpha' \cong \sin\alpha' = \frac{h}{s' + \delta};\quad \phi \cong \sin\phi = \frac{h}{R - \delta}$$

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For the paraxial approximation the axial distance

$VQ = \delta << R, s, s'$ substituting the values: $\dfrac{1}{s} - \dfrac{1}{s'} = -\dfrac{2}{R}$

For the concave mirror we get: $\dfrac{1}{s} + \dfrac{1}{s'} = -\dfrac{2}{R}$

## Sign convention and focal lengths for the spherical mirrors

Light propagates from left to right.

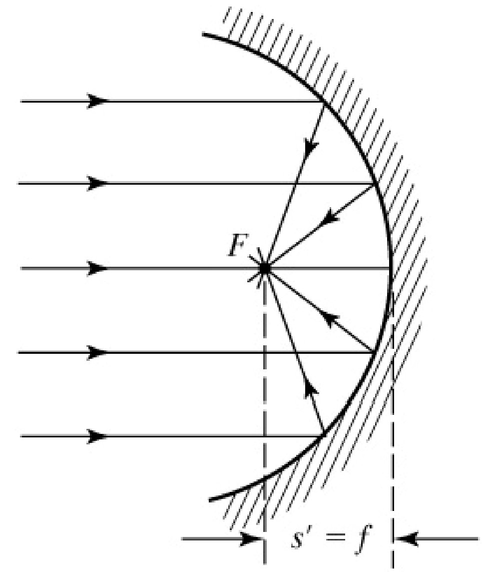
For real objects and images the distances are positive.

For the virtual objects and images the distances are negative.

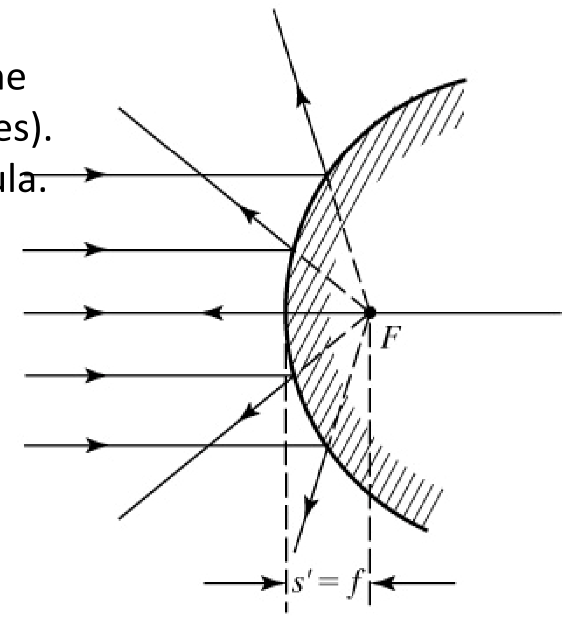
Or: **real positive, virtual negative**

Radii of curvature are **positive** when C, center of the sphere, is to the right of V, vertex equivalent to **convex mirror** and are **negative** when C is to the left of V equivalent to the **concave mirror.**

**Focal lengths of the spherical mirrors** are the image distance for the object at infinity and object distance for the image at infinity (figures). Notice **symmetric** appearance of the **conjugate points** in the formula.




(a)




(b)

The general equation for the spherical mirrors:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \text{and } f=-\frac{R}{2}\begin{cases} >0 \text{ concave mirror} \\ <0 \text{ convex mirror} \end{cases}$$

Signs of s and s' are determined from the convention.

For flat mirror R $\rightarrow \infty$ and $s = -s'$ a virtual image

## Magnification of a spherical mirror

Sign convention for magnification:

**(+)** when image and object have the **same orientation.**

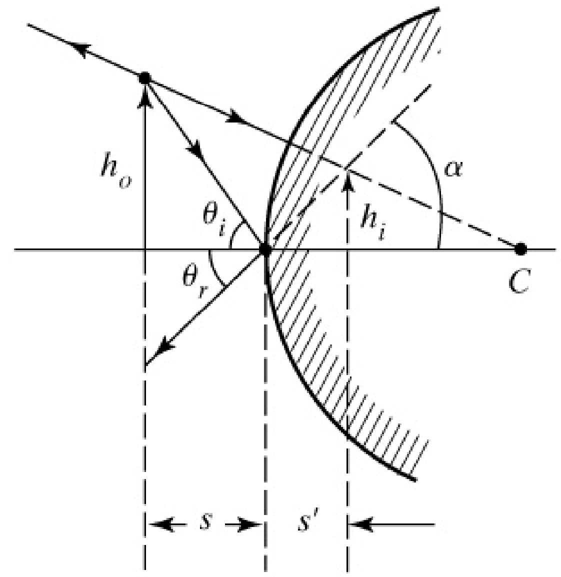
**(-)** when image and object have **opposite orientation.**

Goal: find a relationship between the image height to the object height as a function of the object and image location.

We build the construction and conclude using the law of reflection that: $\frac{h_o}{s} = \frac{h_i}{s'}$

We define the lateral magnification: $|m| = \frac{h_i}{h_o} = -\frac{s'}{s}$

since the image is virtual its distance will be negative, we need a negative sign in the formula.



(c)

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## Example 2.1

An object 3 cm height is placed 20 cm from (a) a convex (b) a concave spherical mirror, each of 10 cm focal length. Determine the position and nature of the image in each case without ray tracing. With ray tracing.

**Solution**

a) Convex mirror:

focal length is virtual so $f = -10cm$. Image distance is real so $s = +20cm$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \rightarrow s'=\frac{fs}{s-f}=\frac{(-10)(20)}{(20)-(-10)}=-6.67cm$$ the image is virtual.

$$m=-\frac{s'}{s}=-\frac{(-6.67)}{(20)}=+0.333=\frac{1}{3}$$ the image is up right because m is positive and is one third of the object in size.

b) Concave mirror:

focal length is real so $f = +10cm$. Image distance is real so $s = +20cm$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \rightarrow s'=\frac{fs}{s-f}=\frac{(10)(20)}{(20)-(10)}=+20cm$$ the image is real.

$$m=-\frac{s'}{s}=-\frac{(20)}{(20)}=-1$$ the image is inverted because m is negative and is the same size as object.

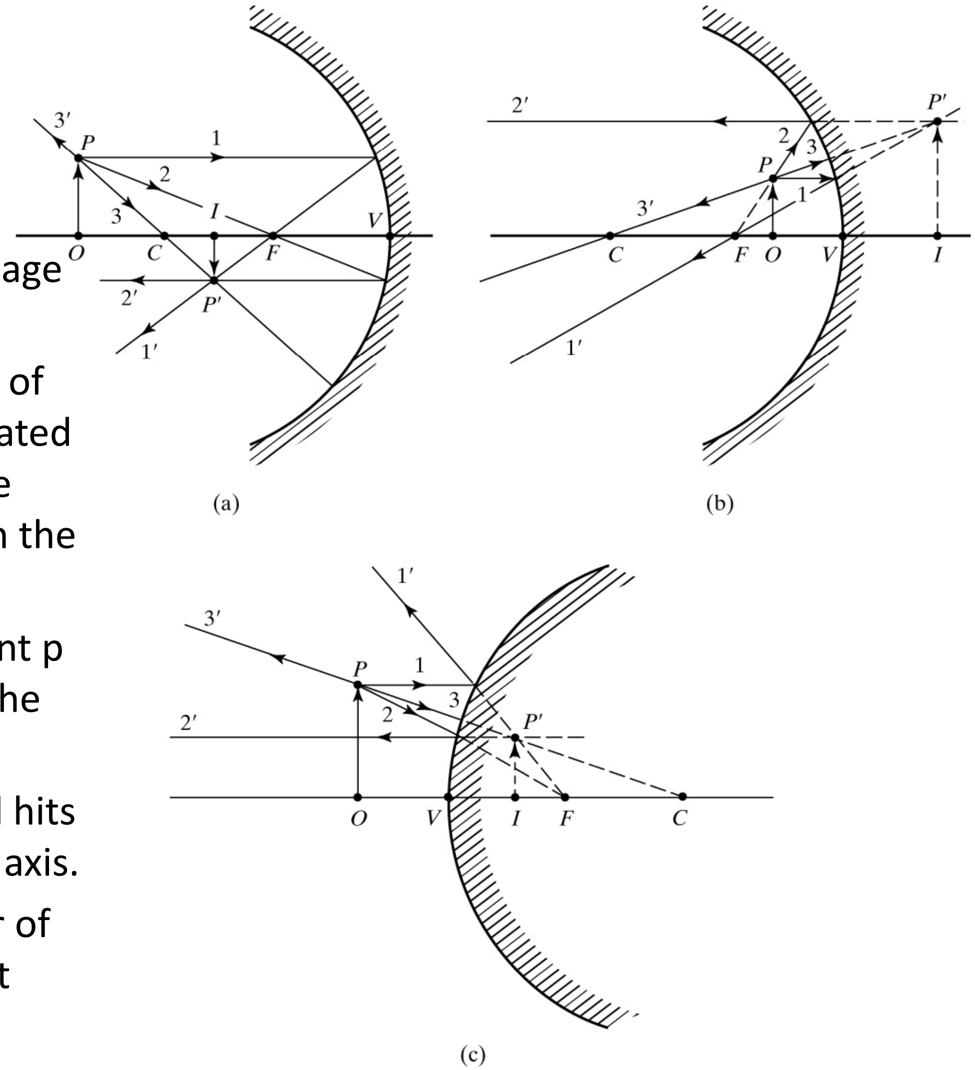
## Graphical ray tracing

Goal: to determine location and nature of the image formed by a mirror by graphical techniques.

We can use **three key rays** to determine location of the image of an arbitrary point on the object created by the mirror. For paraxial optics two of the three rays are enough to find conjugate of any point on the object. **Three key rays:**

1) A ray parallel to the optical axis from the point p hits the mirror, reflects and passes through the focal point.
2) A ray from point p passes the focal point and hits the mirror, is reflected parallel to the optical axis.
3) A ray from point p passes through the center of curvature, hits the mirror, is reflected back at itself.

Our eyes generate real images from the diverging rays arriving at their lens on the retina. We can't see the real images unless they are projected on a screen.

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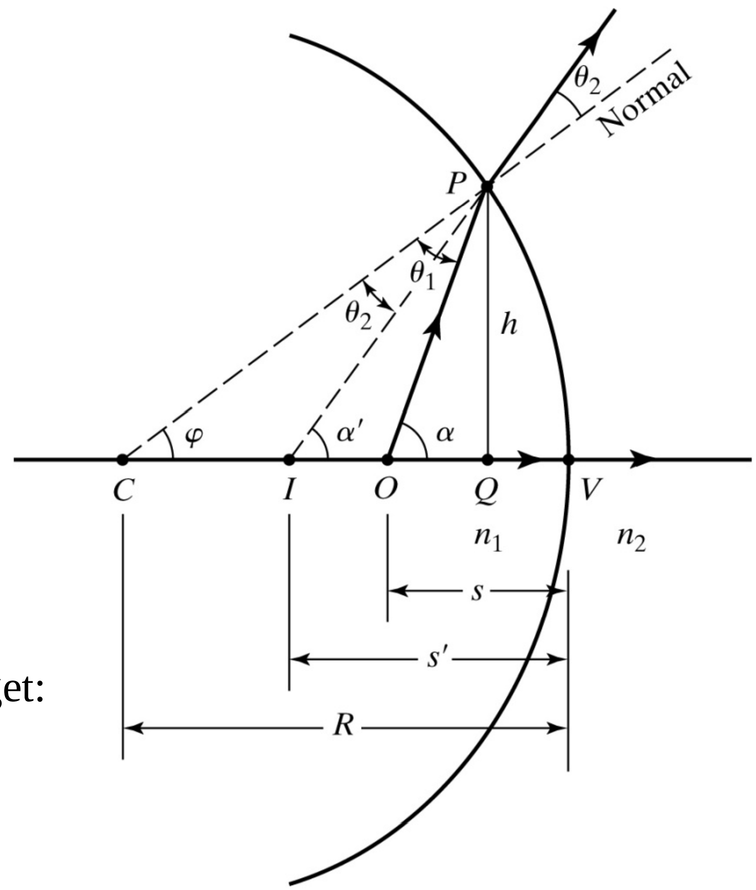
## Refraction at a spherical surface



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Goal: finding a formula for location of the image created by a refractive surface as a function of the system pasameters. Again this formula should be independent of the angle of incidence of a ray. Useng the Snell's law we have

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

In the triangle CPO: $\alpha = \theta_1 + \phi \to \theta_1 = \alpha - \phi$

In the triangle CPI: $\alpha' = \theta_2 + \phi \to \theta_2 = \alpha' - \phi$

Using the paraxial approximation and replacing the sines in the Snell's law with the value of the angles in radian we get:

$$\mathrm{n}_1\left(\alpha - \phi\right) = n_2\left(\alpha' - \phi\right)$$

Replacing the angles with their tan and assuming QV the axial distance is much less than s,s',R we find:

$$\mathrm{n}_1\left(\frac{h}{s} - \frac{h}{R}\right) = n_2\left(\frac{h}{s'} - \frac{h}{R}\right) \to \frac{n_1}{s} - \frac{n_2}{s'} = \frac{n_1 - n_2}{R}$$

With the sign convention real positive, virtual negative, and radii of curvature positive when C is to the right of V, and negative when C is to the left of V, the general formula is: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

When R $\to \infty$ the spherical surface becomes a plane refracting surface: $s' = -\left(\frac{n_2}{n_1}\right)s$

## Lateral magnification of a spherical refracting surface

Snell's law for the ray refracting at the vertex:

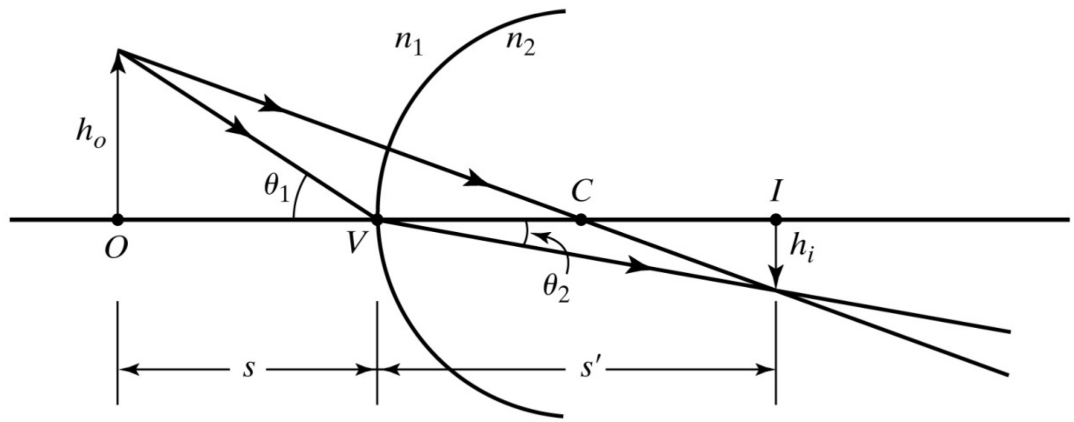
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

With small angle approximation:

$$n_1\theta_1 = n_2\theta_2 \rightarrow n_1\left(\frac{h_o}{s}\right) = n_2\left(\frac{h_i}{s'}\right)$$

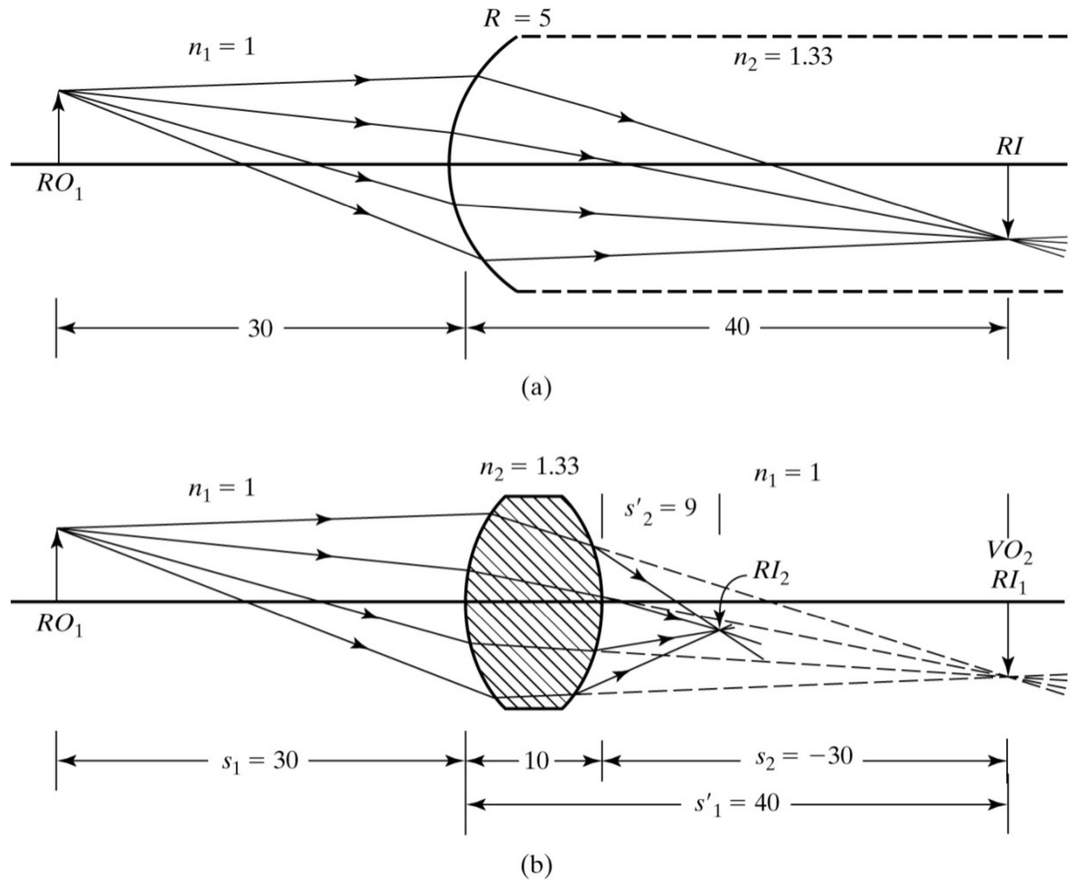
The lateral magnification is, then,

$$m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$$




For a plane refracting surface $s' = -\left(\frac{n_2}{n_1}\right)s$

and $m = -\frac{n_1}{n_2 s}\left(-\left(\frac{n_2}{n_1}\right)s\right) = +1$



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## Example

Find location of the images and lateral magnification in  in a and b.

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R};\ m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$$

$$a)\ \frac{1}{30} + \frac{1.33}{s'_1} = \frac{1.33-1}{5} \rightarrow s'_1 = 40cm$$

the image is real

$$m = -\frac{(1)(+40)}{(1.33)(+30)} = -1$$

the image is inverted same size as object

$b)\ s_2 = -(s'_1 - t) = -30$ the virtual object distance from the second surface.

$\frac{1.33}{-30} + \frac{1}{s'_2} = \frac{1-1.33}{-5} \rightarrow s'_2 = +9cm$ the final real image distance from the second surface.

$m = \frac{(-1.33)(+9)}{(1)(-30)} = +\frac{2}{5}$ final image 2/5 of the virtual object and same orientation.

Total $m = m_1 m_2 = (-1)(+2/5) = -2/5$ The final image is 2/5 of the original object and inverted.

## Thin lenses

Following the technique of example 2 for image of two refracting spherical surfaces while neglecting the thickness of the combined optical system we write:

1st surface: $\frac{n_1}{s_1}+\frac{n_2}{s'_1}=\frac{n_2-n_1}{R_1}$

2nd surface: $\frac{n_2}{s_2}+\frac{n_1}{s'_2}=\frac{n_1-n_2}{R_2}$

Assumption: both sides of the lens have the same refracive index.

2nd object distance: $s_2=t-s'_1$ this produces a correct sign for the distance.

where $t$ is the thickness of the lens.

Neglecting $t$ for the thin lenses: $s_2=-s'_1$

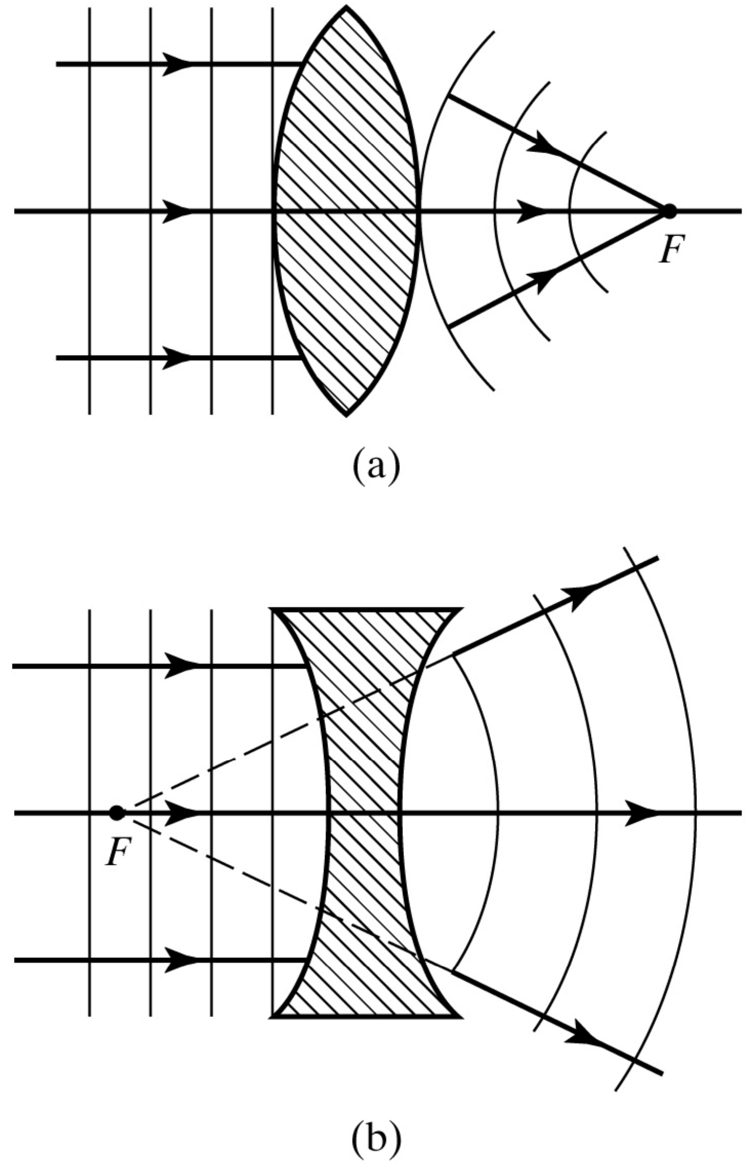
Combining: $-\frac{n_2}{s'_1}+\frac{n_1}{s'_2}=\frac{n_1-n_2}{R_2}\rightarrow \frac{n_1}{s_1}-\frac{n_2-n_1}{R_1}+\frac{n_1}{s'_2}=\frac{n_1-n_2}{R_2}$

$$\frac{n_1}{s_1}+\frac{n_1}{s'_2}=\frac{n_1-n_2}{R_2}+\frac{n_2-n_1}{R_1}\rightarrow\frac{1}{s_1}+\frac{1}{s'_2}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$$

The lensmaker's equation: $\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$

## Wavefront analysis of thin lenses

Plane waves with flat wavefronts arriving at a thin lens are curved to accommodate the Fermat's principle or to stay isochronous.

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## Graphical ray tracing of the thin lenses

Magnification of the thin lenses:

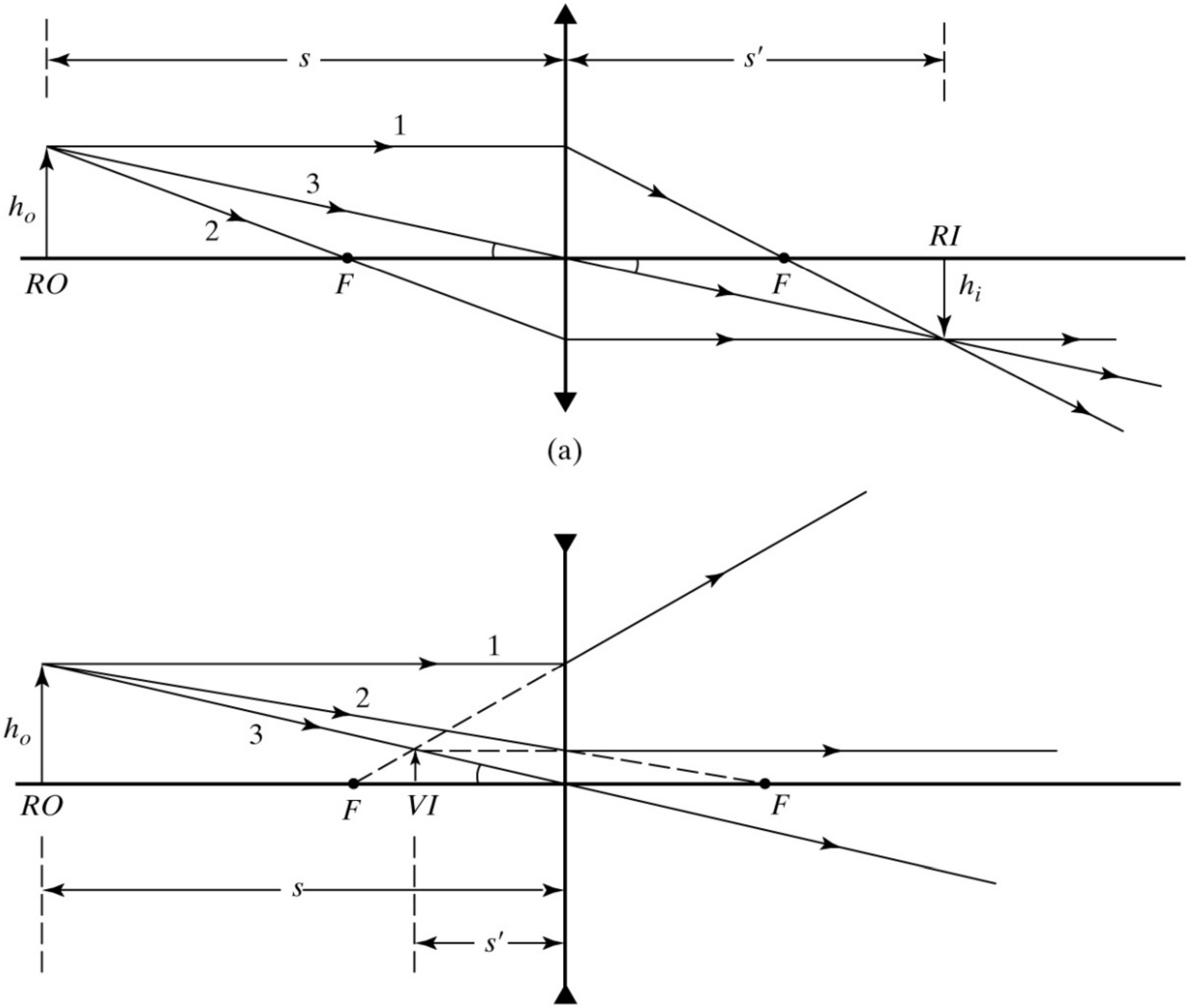
$$|m| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s'}{s}\right|$$

With proper sign conventions:

$$m = -\frac{s'}{s}$$

$m > 0$ for images with the same orientation as objects.

$m < 0$ for inverted images.




(a)

(b)

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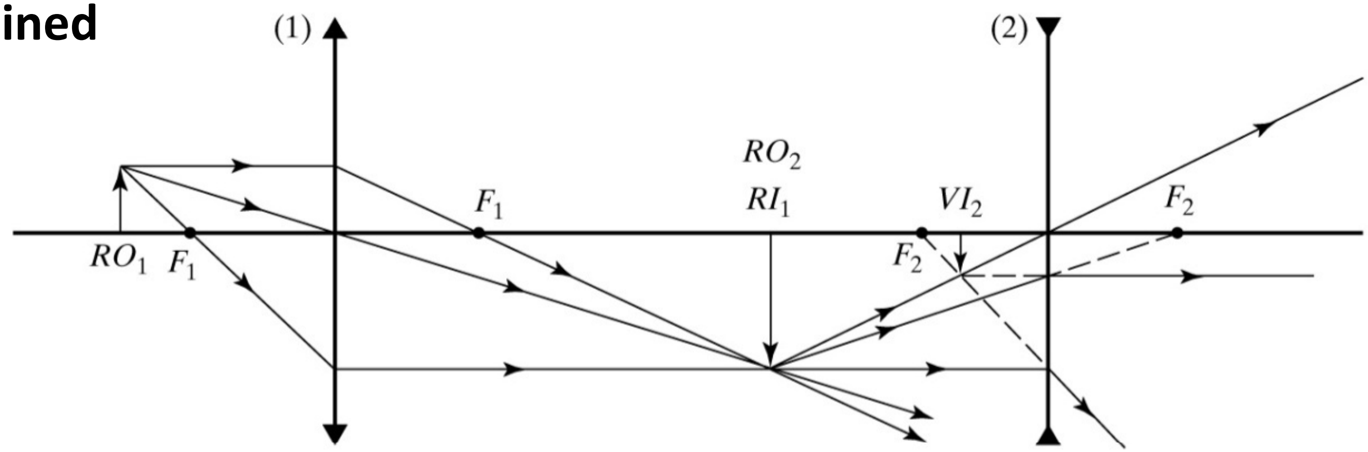
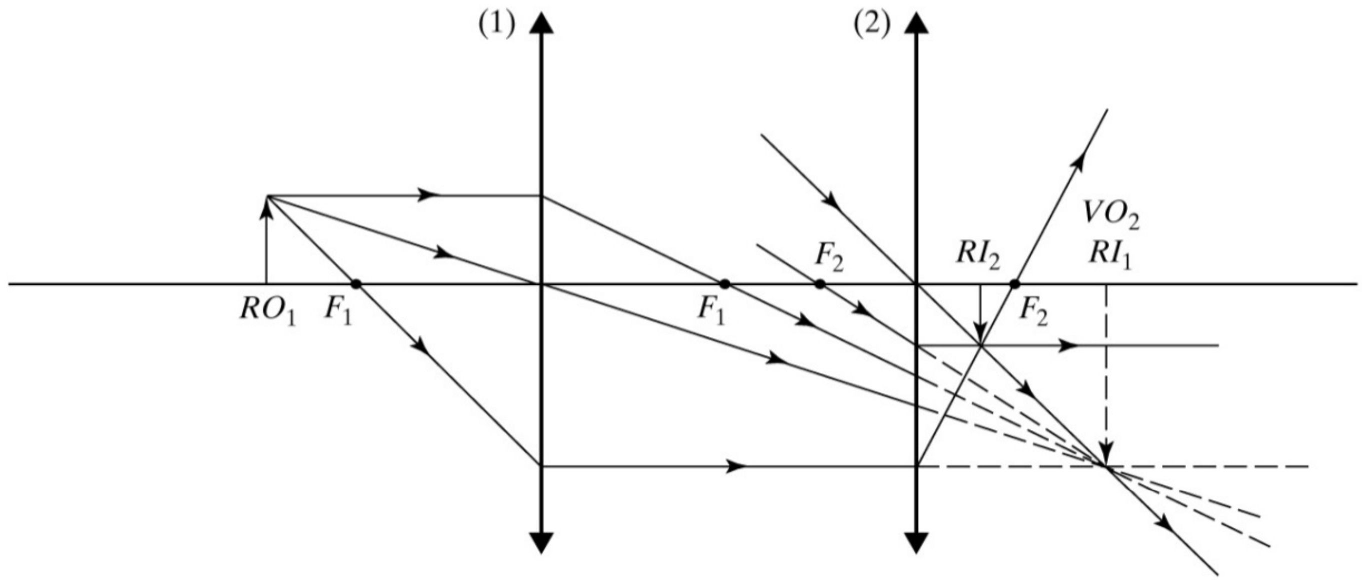
# Ray tracing in combined systems




(a)




(b)

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**TABLE 2-1** SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

| | Spherical surface | Plane surface |
|---|---|---|
| Reflection | $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, f = -\frac{R}{2}$ | $s' = -s$ |
| | $m = -\frac{s'}{s}$ | $m = +1$ |
| | Concave: $f > 0, R < 0$ | |
| | Convex : $f < 0, R > 0$ | |
| Refraction Single surface | $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$ | $s' = -\frac{n_2}{n_1}s$ |
| | $m = -\frac{n_1 s'}{n_2 s}$ | $m = +1$ |
| | Concave: $R < 0$ | |
| | Convex : $R > 0$ | |
| Refraction Thin lens | $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ | |
| | $\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ | |
| | $m = -\frac{s'}{s}$ | |
| | Concave: $f < 0$ | |
| | Convex : $f > 0$ | |


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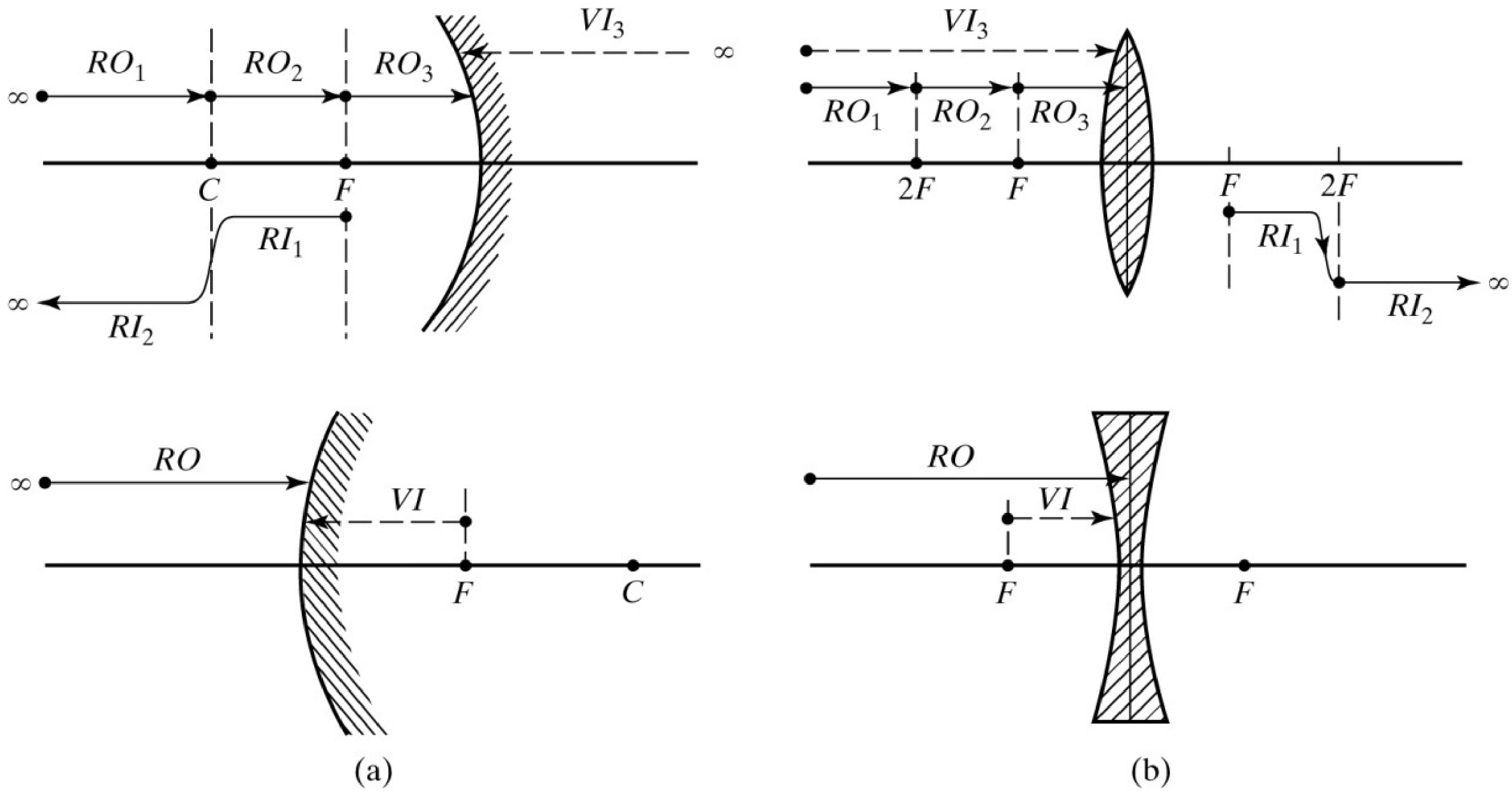
## Summary of image formation by spherical mirrors and thin lenses

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## Vergence and refractive power

Vergence or reciprocal of the image/object distance describes the curvature of the wavefront.

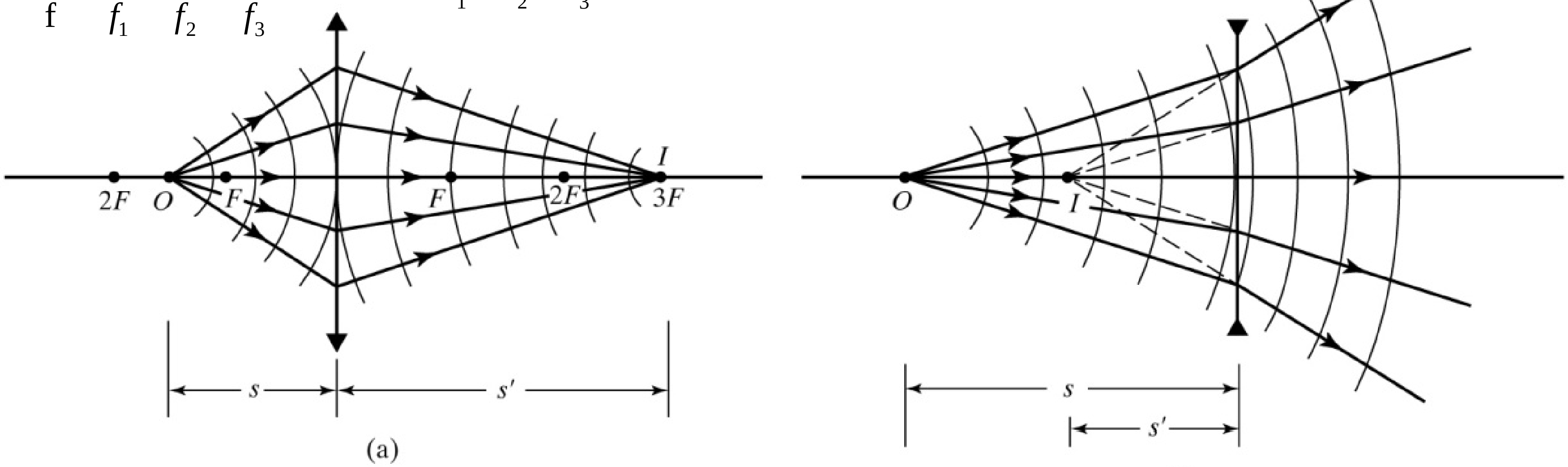
Vergence is measured in unitst of $\frac{1}{\text{m}}$ or Diopter. $\text{V}=\frac{1}{\text{s}}$ and $\text{V'}=\frac{1}{\text{s'}}$

Refractive power of an optical system is $P=\frac{1}{f}$

So the lens equation becomes simpler: $\frac{1}{\text{s(m)}}+\frac{1}{s'(m)}=\frac{1}{f(m)} \rightarrow V(D)+V'(D)=P(D)$

We can use this approach to show that for combination of lenses:

$$\frac{1}{\text{f}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+.... \rightarrow P=P_1+P_2+P_3+....$$




(a)

(b)

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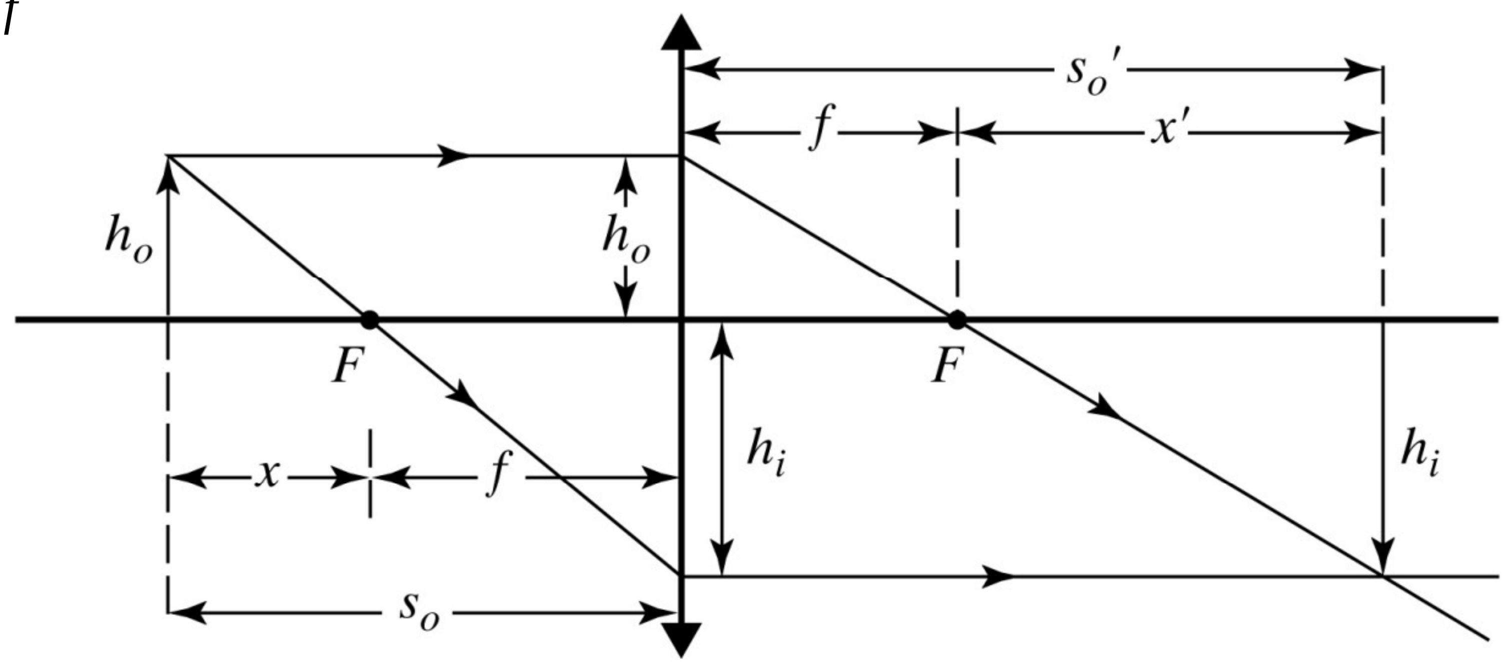
## Newtonian equation for the thin lens

The object and image distances are measured from the focal points like the picture.

The equation is simpler and is used in certain applications

$$m = \frac{h_i}{h_o} = \frac{f}{x} = \frac{x'}{f} \rightarrow xx' = f^2$$

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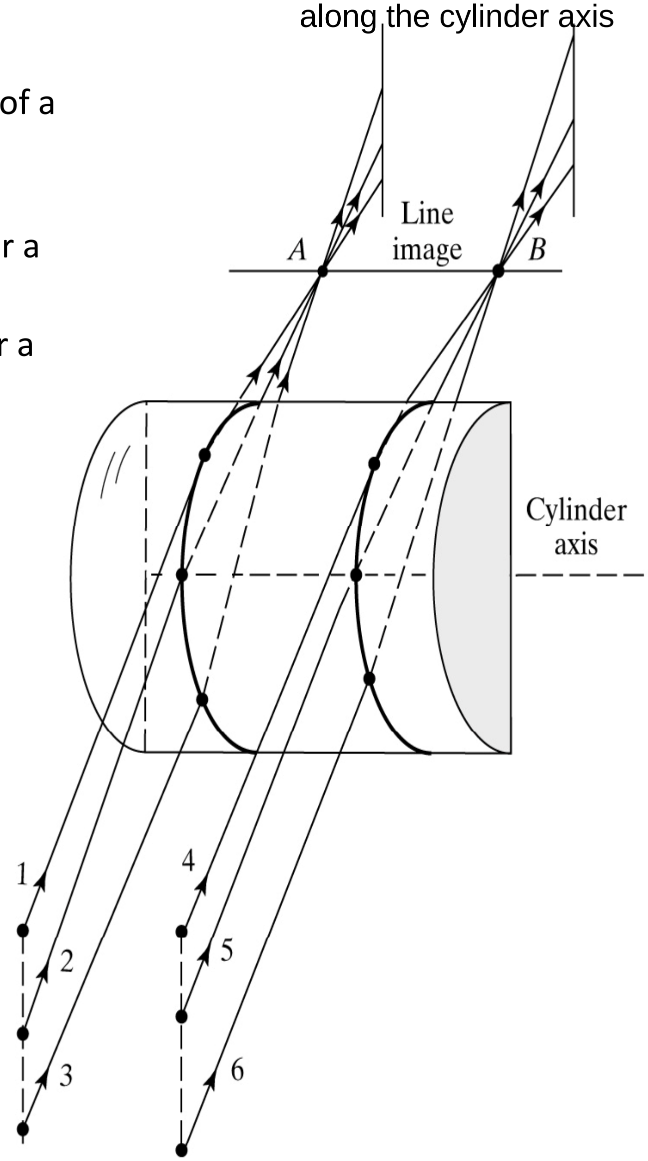
# Cylindrical lenses

Asymmetrical lenses formed by a section of a cylinder that are useful for line focusing, correcting astigmatism in human eye.

Spherical lenses produce a point image for a point object (**stigmatic**).

Cylindrical lenses produce a line image for a point object (**Astigmatic**).

Cylindrical lens with
No focusing power
along the cylinder axis

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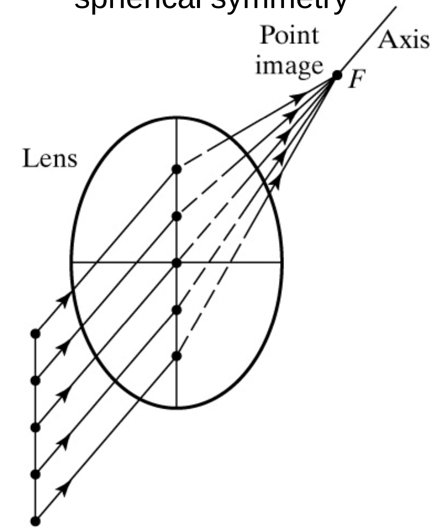
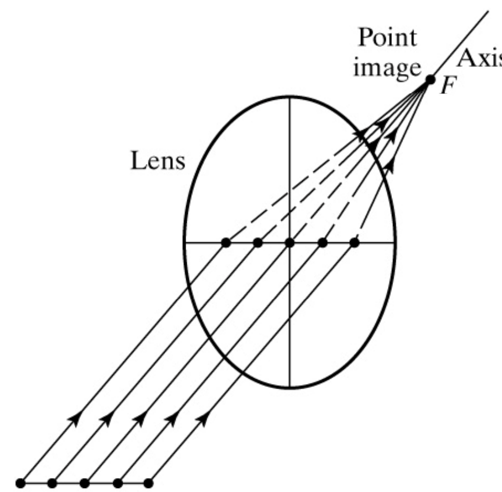
Spherical lens with
spherical symmetry




Vertical fan of rays

(a)




Horizontal fan of rays

(b)

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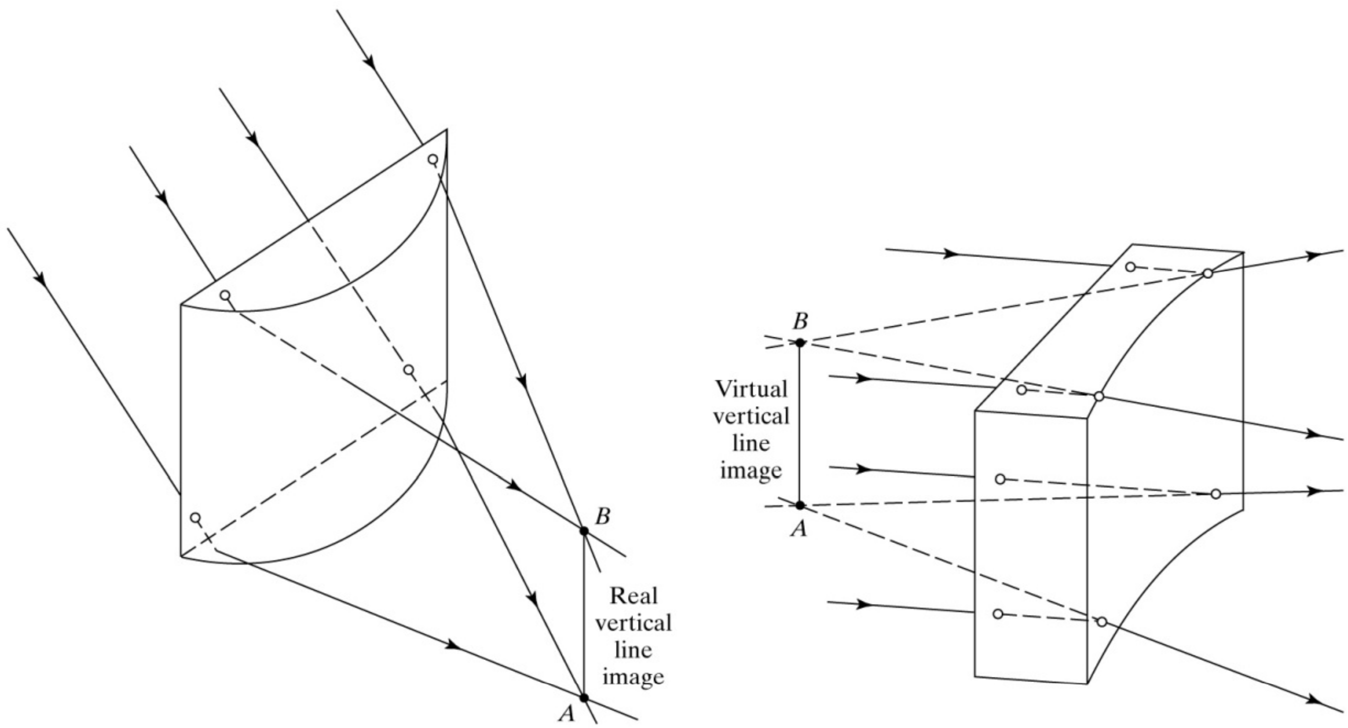
**Types of cylindrical lens**
**Generation of real and virtual line images by cylindrical lenses**




(a) (b)

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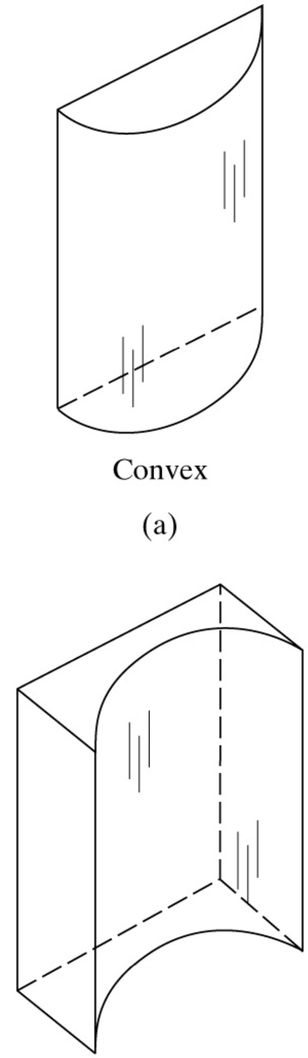

Convex

(a)

Concave

(b)

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# Astigmatic imaging property of the cylindrical lenses (point to line)

We use the same sign convention as the spherical lenses.

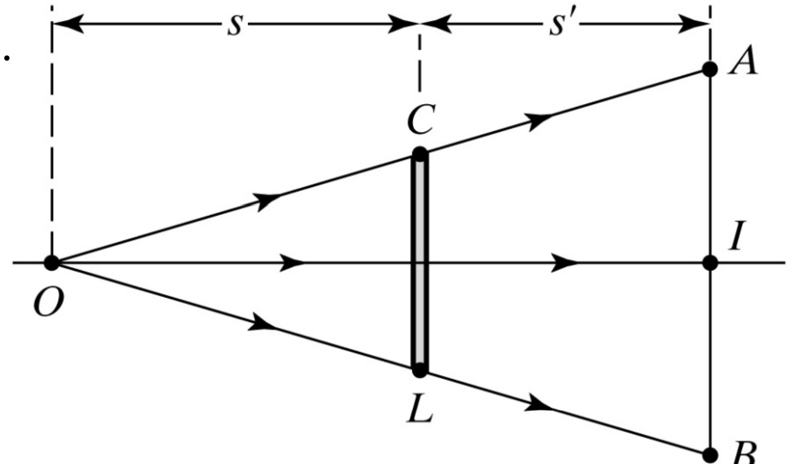
$AB$ image length is always positive

CL length of lens along the axis of cylinder

From the similar triangles:

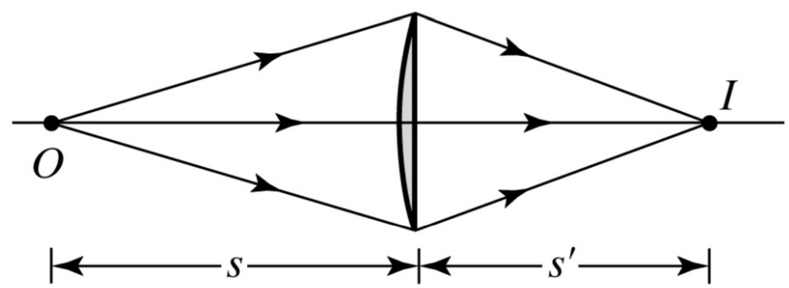
$$\frac{AB}{CL} = \frac{s+s'}{s}$$

$$\boxed{AB = \left(\frac{s+s'}{s}\right)CL}$$




Top view of the non-focusing axis

(a)




(b)

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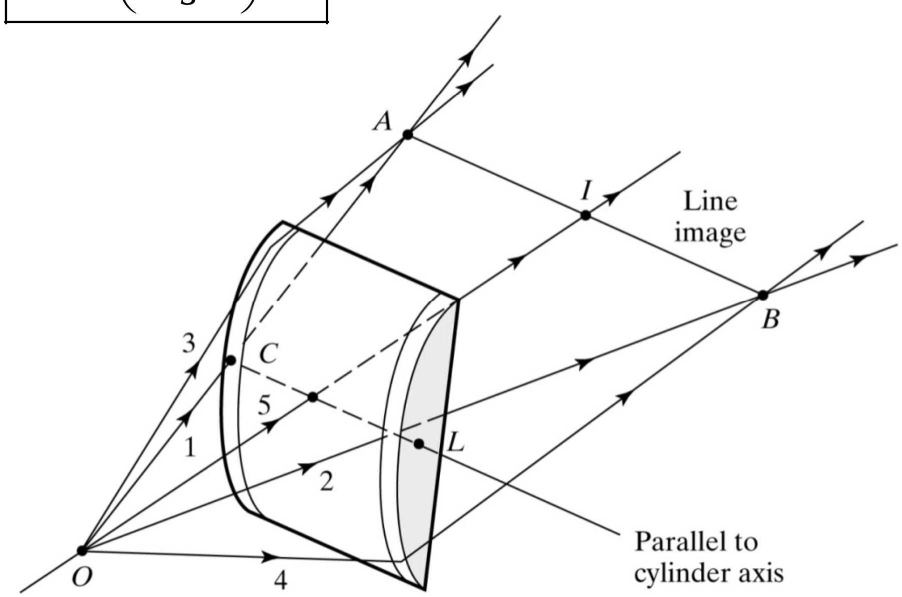
Top view of the focusing axis

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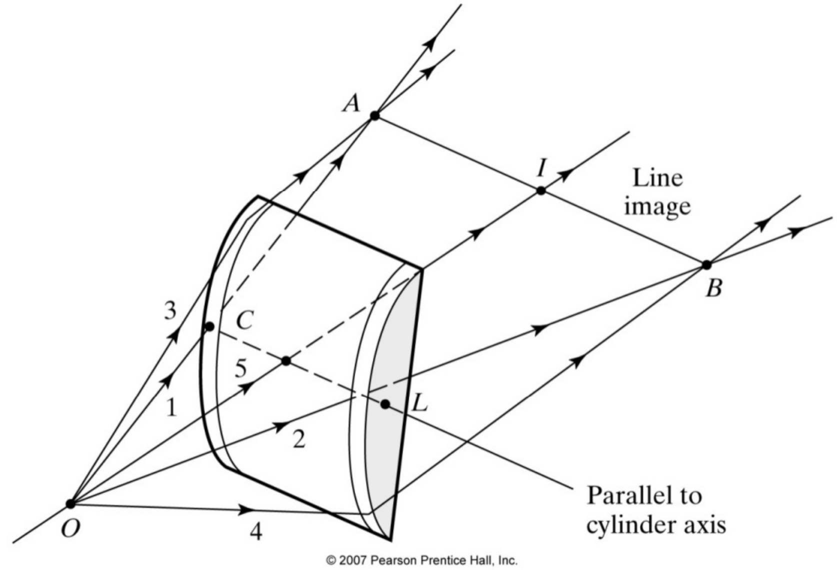
**Example**

A thin plano-cylindrical lens in air with R=10cm and n=1.50 and CL=5cm. Light from point object at s=25cm incident on the convex cylindrical surface from left. Find the **position and length of the line image**.



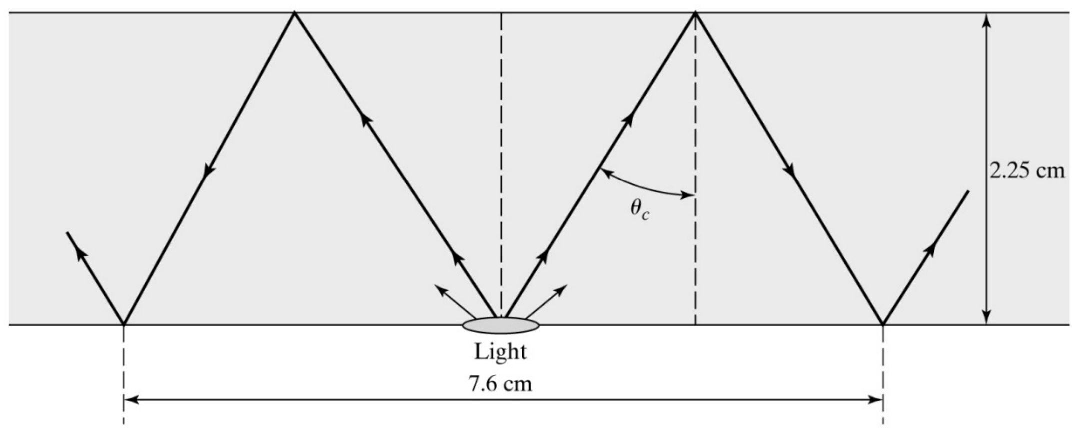

First we find the image distance from the formula for the spherical refracting surface in 2 dimension

for the convex surface: $\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R} \rightarrow \frac{1}{25}+\frac{1.5}{s'}=\frac{1.50\text{-}1.00}{10} \rightarrow s'=150cm$ that is a real image.

for the flat surface: $\frac{n_1}{s}+\frac{n_2}{s'}=0 \rightarrow \frac{1.5}{\text{-}150}+\frac{1.0}{s'}=0$ (thickness is ignored) $\rightarrow \text{s}'=100cm$

Now finding the image length: $AB=\left(\frac{25+100}{25}\right)5=25cm$

The line image is parallel to the ctlinder axis elongated to 25 cm at 1m from the lens.

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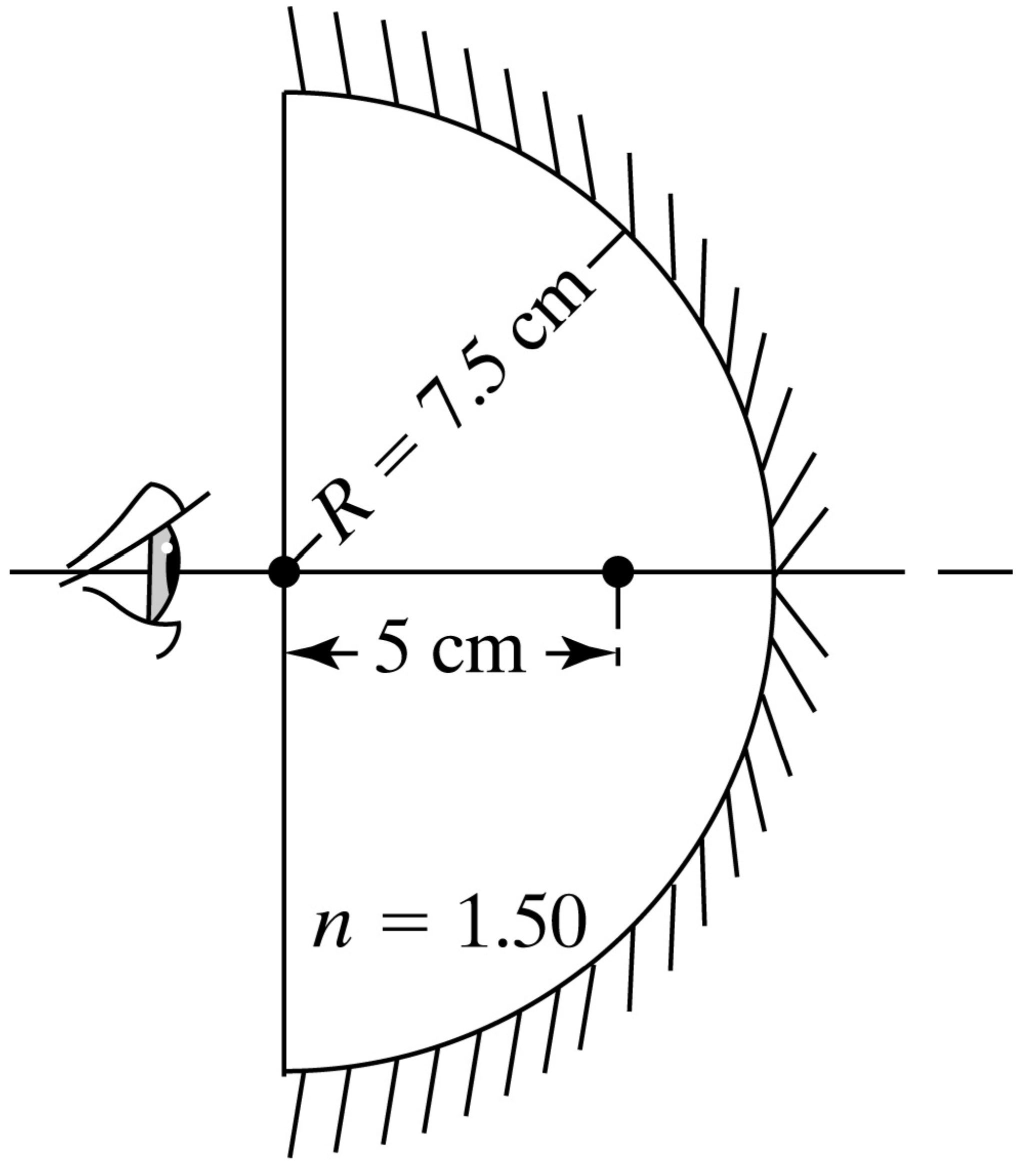


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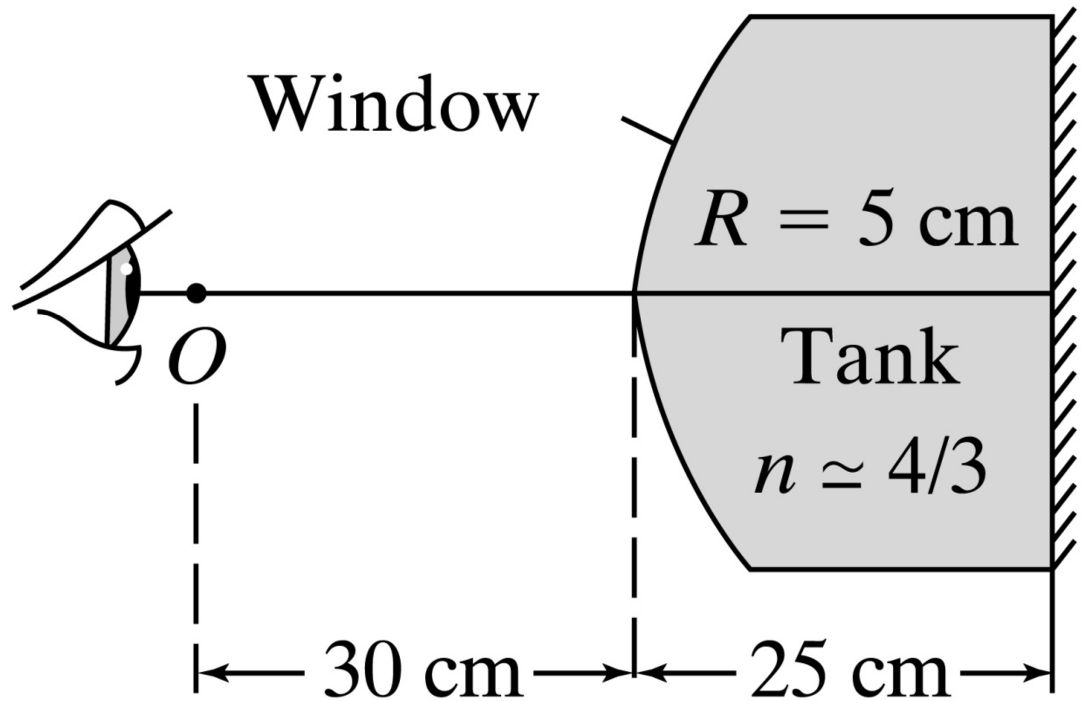

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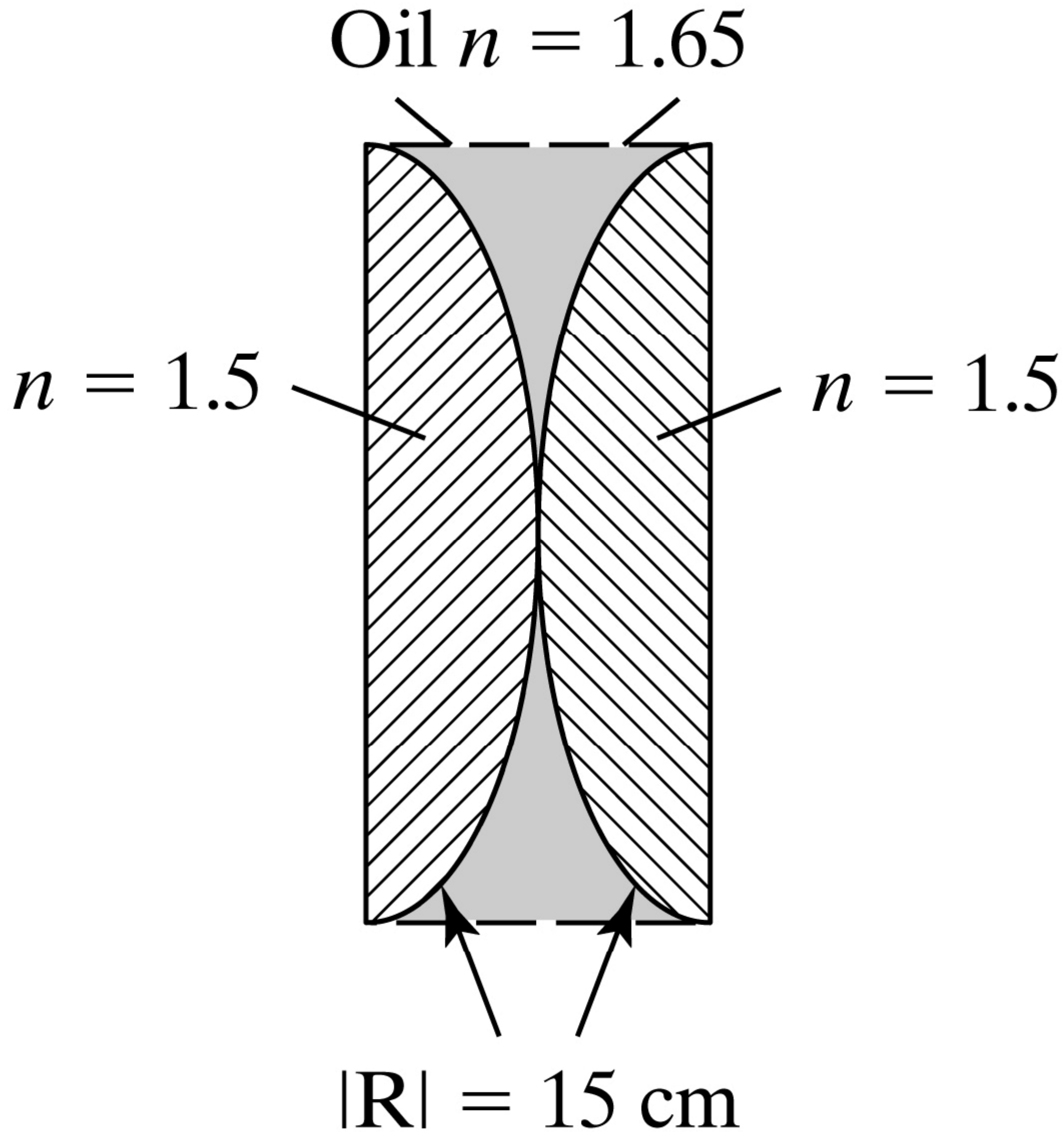

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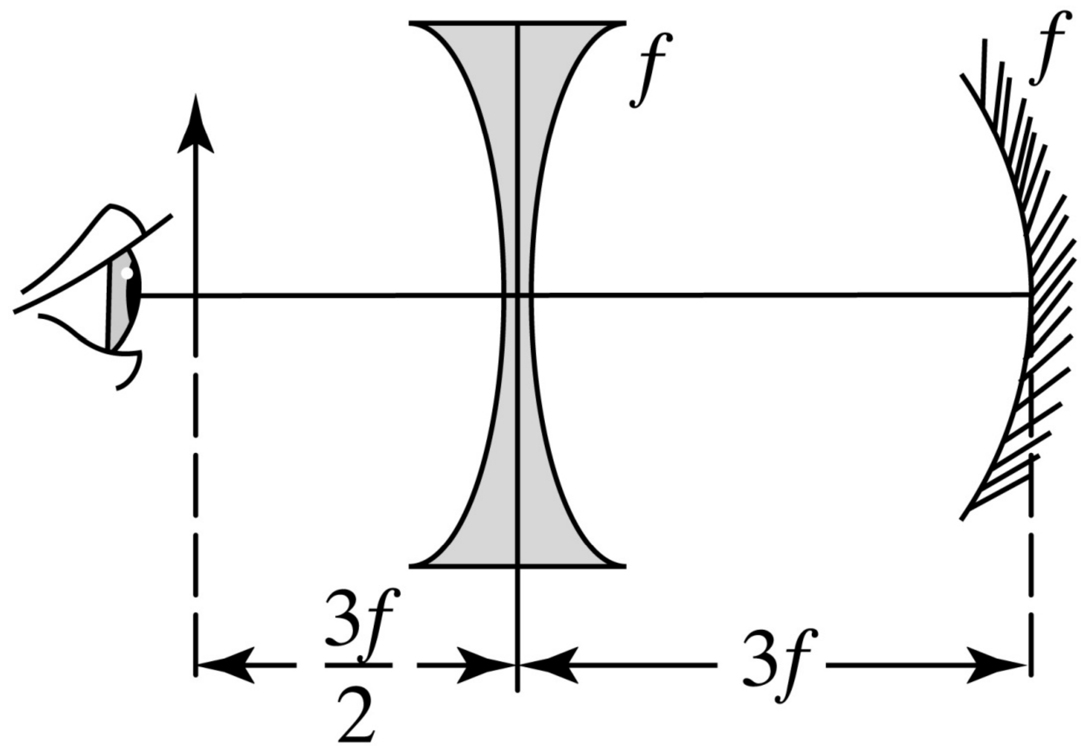


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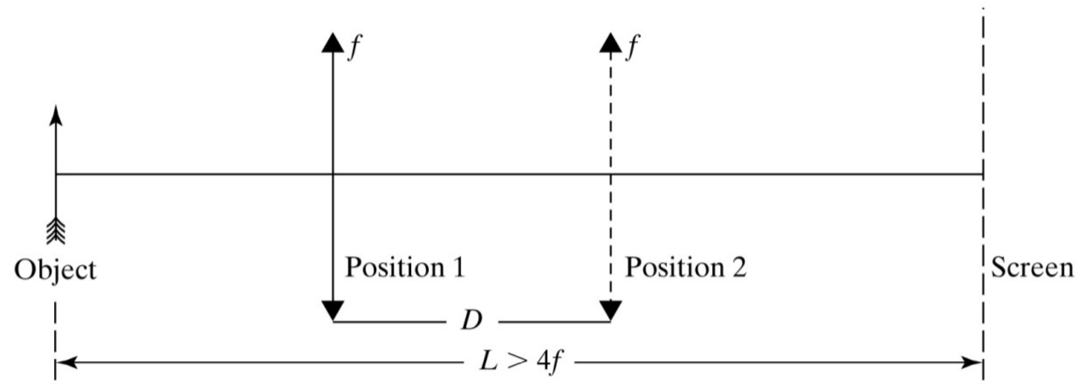


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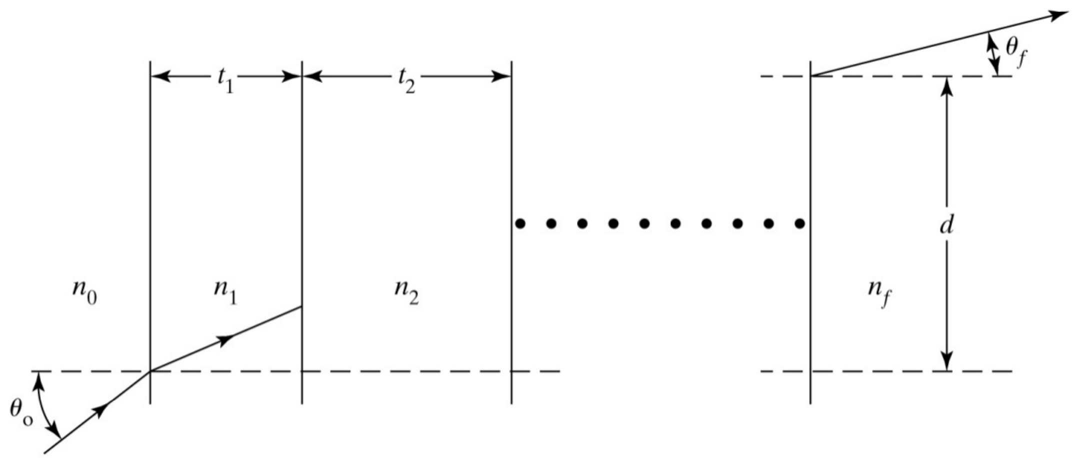


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