Chapter 6
Properties of Lasers

Lecture Notes for Modern Optics based on
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Energy quantization in light and matter

Quantization of EM fields: energy of EM radiation is quantized in units of $h\nu$

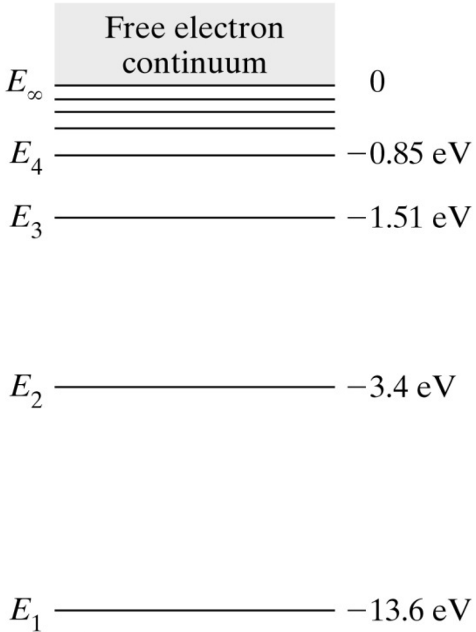
$$E_{photon} = h\nu,\ where\ h = 6.63\times10^{-34}\ J.s$$

Total energy stored in EM field of frequency ν is: $\boxed{E_n^{EM} = \underbrace{h\nu/2}_{?} + nh\nu}$ where $n = 0,1,2,3,\ldots$ is number of photons in the EM radiation. $h\nu/2$ is the energy associated with electromagnetic vacuum.

In absence of photons there is still some energy stored in vacuum.

Energy quantization in matter: Atoms are composed of charged particles thus they interact with EM fields.

Atoms have quantized energy levels. Energy levels of hydrogen atom: $E_n = -\dfrac{13.6eV}{n^2}$ where $n = 1,2,4,\ldots$



A strong interaction between the EM field and atom can occur if some constituents of the matter have allowed energy levels that is in resonance with the energy of photons in EM radiation or:

$$\underbrace{E_n - E_m}_{\text{Energy of an atomic or molecular transition}} = \underbrace{E_{n+1}^{EM} - E_n^{EM}}_{\text{Energy of a photon}} = h\nu_0$$

Ground state of an atom is the most stable state of it which corresponds to the lowest energy level of the atom.

Excited states of an atom: are states above the ground state and less stable and correspond to higher energy levels.

Energy of free electrons is not quantized so they can interact with photons of any energy.

Boud electrons absorb energies in packets that matches the quantized energy levels available to them.

Lineshape function

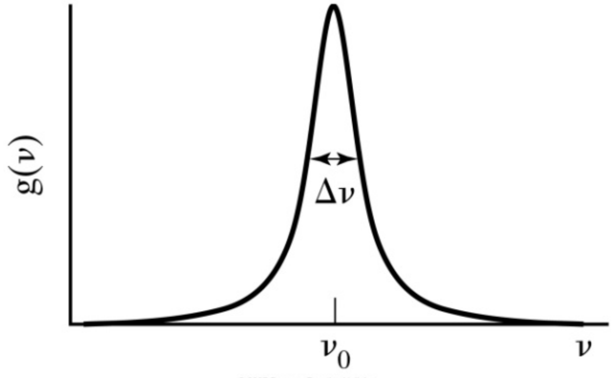
The energy levels of atoms are not discrete lines.

In reality energy levels have a finite width ΔE that is a result of interaction of atom with its environment.

A typical value of width of an energy level: $\Delta E \sim 10^{-7}\, eV$

This allows photons with energies within a small range of the energy difference between the levels to interact with the atom (a more relax criteria than the exact match). A photon should have a energy in the range:

$$h\nu = E_n - E_m - \frac{(\Delta E_n + \Delta E_m)}{2} \quad \text{to} \quad h\nu = E_n - E_m + \frac{(\Delta E_n + \Delta E_m)}{2}$$



Only fields of frequencies $\nu = \frac{E_n - E_m}{h} \pm \frac{(\Delta E_n + \Delta E_m)}{2h} \rightarrow \boxed{\nu = \nu_0 \pm \frac{\Delta\nu}{2}}$

are likely to have significant interaction with the atom.

The center frequency of interaction or line center $\boxed{\nu_0 = \frac{E_n - E_m}{h}}$ and frequency linewidth $\boxed{\Delta\nu = \frac{(\Delta E_n + \Delta E_m)}{h}}$

Range of linewidths for different material: $10^6 - 10^9\, Hz$

Probability of interaction of an atom with energy levels of E_m, E_n with a photon of frequency ν is proportional to the lineshape function $g(\nu)$ which usually is a symmetric function with width of $\Delta\nu$ and peak at $\nu = \nu_0$.

By convention lineshape function $g(\nu)$ is normalized so that probability stays below 1: $\int_{All\ \nu} g(\nu)\, d\nu = 1$

Thermal Equilibrium and Boltzmann distribution of atoms

For a system in equilibrium with its sourroundings there is no net energy exchange.

Thus the system and its surroundings is characterized by one temperature T.

Boltzman Distribution for an assambly of atoms at thermal equilibrium:

$$P_i = P_1 e^{-(E_i - E_1)/K_B T}$$

P_1 : liklihood of the atom being in ground state

P_i : liklihood of the atom being in the ith excited state

$\mathrm{k_B} = 1.38\times 10^{-23} J/K = 8.62\times 10^{-5} eV/K$ is Boltzman constant.

T: the temperature in Kelvin

At thermal equilibrium atoms are more likely to be in lower energy states than higher.

Blackbody radiation

A blackbody is a perfect absopber and also a perfect emitter. They are also called Plankian sources.

In 1900 Max Plank suggested quantization of the radiation emitted from a blackbody and absorption by a blackbody in order to explain experimental data. Plank's suggestion for spectral excitance, M_λ of a balckbody was: $\boxed{M_\lambda = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{e^{hc/\lambda k_B T} - 1}\right)}$

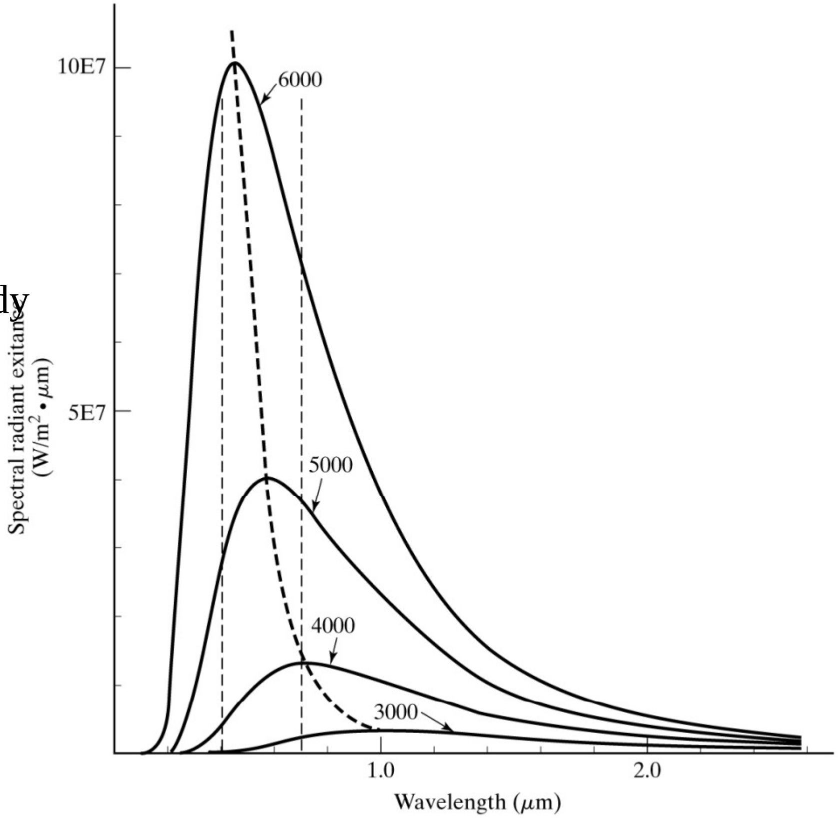
where M_λ is the power per unit area per wavelength interval at temperature T, emitted by a source.

Wien's displacement law: M_λ peak is inversly proportional to the temperature $\boxed{\lambda_{\max} T = hc/5k_B = 2.898\times10^3 (\mu m.K)}$

This can be derived from equating the T-derivative of M_λ to zero.

Stephan-Boltzman law: total radiant excitance of a blackbody is proportional to 4th power of its temperature in Kelvin.

$$M = \int_0^\infty M_\lambda d\lambda = \sigma T^4 \text{ where } \sigma = 5.67\times10^{-8} \text{ W}/\left(\text{m}^2.K^2\right).$$



Real sources emit less than blakbody.

Emissivity of a surface is defined as:

$\varepsilon(\mathrm{T}) = M / M_{bb}$ and $\varepsilon_{bb}(T) = 1$

$\varepsilon_{nbb}(\mathrm{T}) < 1$

lasers

Lasers introduced in 1960s perhaps the most important optical device.

$$\text{LASER: } \underbrace{\text{Light}}_{\text{EM waves}} \ \underbrace{\text{Amplification}}_{\text{Resonator}} \text{ by the } \underbrace{\text{Stimulated Emission}}_{\substack{\text{Laser cavity, population inversion,} \\ \text{laser material, pump}}} \text{ of } \underbrace{\text{Radiation}}_{\substack{\text{Property of the} \\ \text{material}}} \text{ is an}$$

$$\left\{ \begin{array}{l} \text{intense,} \\ \text{highly directional,} \\ \text{nearly monochromatic,} \\ \text{coherent,} \end{array} \right\} \text{electromagnetic energy source.}$$

Theoretical prediction of stimulated emission by Albert Einstein in 1916.

Experimental demonstration by C.H. Townes in microwave regime in 1954 (MASER).

A. Schawlow and C.H. Townes adapted the principles of masers in the visible 1958 (LASER)

The first solid state laser (Ruby 694.3 nm) was built by T.H. Meiman (1960)

The first gas laser (HeNe: 1.15 μm and 632.8nm) was built by A. Javan (1960)

For the most of 1960s the lasers were described as "A solution in search of a problem".

Today the applications are being discovered on almost weekly basis.

Laser light is a manifestation of a particular interaction between charged particles and electromagnetic waves that requires understanding of quantization of energy of EM waves and energy of atoms.

Einstein’s theory of light-matter interaction

In 1916 Eninstein showed that existance of thermal equilibrium between light and matter could be explained by three basic interactions.

1) Stimulated absorption
2) Stimulated emission
3) Spontaneous emission.

We introduce the following parameters:

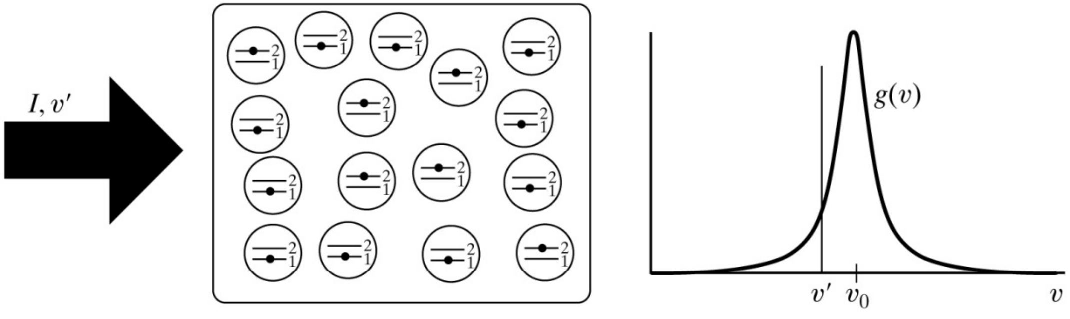
$N_1,\ N_2$: population density of the lower and upper states

I: irradiance of the incident radiation (numberof photons)

$g(\nu')$: lineshape value at frequency of the incident light

B_{12}: the Einstein B coefficient for stimulated absorption

B_{21}: the Einstein B coefficient for stimulated emission



(a) (b)

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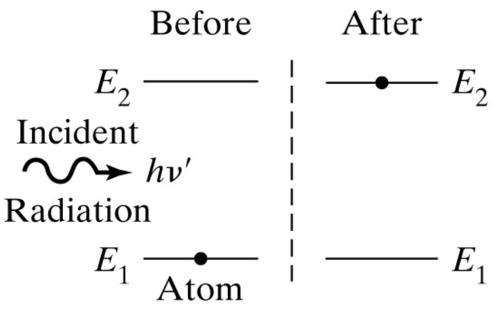
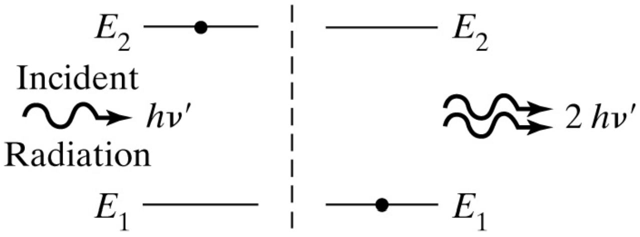
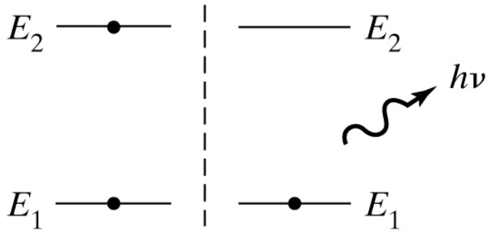


(a) Stimulated absorption



(b) Stimulated emission



(c) Spontaneous emission

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Einstein rate equations

1) Stimulated absorption: a process by which EM waves transfer energy to matter and excite it from a lower state to a higher state of energy $\boxed{h\nu' \approx E_2 - E_1 = h\nu_0}$

The rate of occurence of stimulated absorption per unit volume is: $\boxed{R_{St.Abs} = B_{12} g(\nu')(I/c) N_1}$

$I: Irradiance$

2) Stimulated emission: when a photon of energy $h\nu' \approx E_2 - E_1 = h\nu_0$ encounters an atom initially in an excited state E_2, it can stimulate the atom to drop to a lower state E_1.

In the process the atom releases a photon with same energy, direction, phase, polarization as that of the initial photon. Thus light can be amplified and the result is a highly monochromatic, directional and coherent radiation.

The rate of occurence of stimulated emission per unit volume is: $\boxed{R_{St.Em} = B_{21} g(\nu')(I/c) N_2}$

3) Spontaneous emission: an atom in excited state E_2 "spontaneously" gives up its energy in the form of a photon in a random direction $h\nu' \approx E_2 - E_1 = h\nu_0$ and falls to a lower state E_1.

The spectrum of the spontaneous emission is same as that of the lineshape $g(\nu')$. Spontaneous emission occures because aoms in excited state are not stable and have limitted lifetime.

The rate of occurence of spontaneous emission per unit volume is: $\boxed{R_{Sp.Em} = A_{21} N_2}$

Relations between Einstein A and B coefficients

$$R_{St.Abs} = B_{12} g(\nu')(I/c) N_1$$

$$R_{St.Em} = B_{21} g(\nu')(I/c) N_2$$

$$R_{Sp.Em} = A_{21} N_2$$

Einstein showed that a thermal equilibrium between a radiation field and an assembly of atoms will exist if

$\boxed{\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h\nu^3}{c^3}}$ and $\boxed{B_{12} = B_{21}}$ (for non-degenerate states)

Since rate coefficients for stimulated emission and absorption are equal, the ratio of these processes is proportional to the ratio of population of each state:

$$\frac{R_{St.Em}}{R_{St.Abs}} = \frac{B_{21} g(\nu')(I/c) N_2}{B_{12} g(\nu')(I/c) N_1} = \frac{N_2}{N_1}$$

In an assambly of atoms in thermal equilibrium, probability of finding an atoms in a particular energy state, i at temperature T is given by Maxwell-Boltzman distribution: $P_i = P_1 e^{-(E_i - E_1)/K_B T}$

Where $K_B = 1.38 \times 10^{-23} J/K = 8.62 \times 10^{-5} eV/K$ is the Boltzman constant, T is in Kelvin

$KT = 25meV \ at \ 300K$

$$\frac{R_{St.Em}}{R_{St.Abs}} = \frac{N_2}{N_1} = \frac{P_2}{P_1} = \frac{P_1 e^{-(E_2 - E_1)/K_B T}}{P_1 e^{-(E_1 - E_1)/K_B T}} = e^{-(E_2 - E_1)/K_B T} < 1$$

So stimulated absorption will occur more often than the stimulated emission.

At thermal equilibrium an assembly of atoms is a net absorber.

Pumping, population inversion

To achieve the amplification of the EM radiation we need to take the atoms out of thermal equilibrium.

Pumping: is a way of exciting the atoms from $\mathrm{E}_1 \rightarrow \mathrm{E}_2$ can be done optically or electrically.

Population inversion: increasing population of the atoms in excited state to the extend at which $N_2 > N_1$

Under population inversion condition: $R_{St.Em} > R_{St.Abs}$

Spontaneous emission will always exist and acts as derementaI factor (noise generating) in laser systems.

Spontaneous emission broadens the laser linewidth so inhibition of the SE is a big research topic today.

$\frac{A_{21}}{B_{21}} \propto \nu^3$ that is why making blue lasers is more challenging than the red ones.

Essential elements of a laser

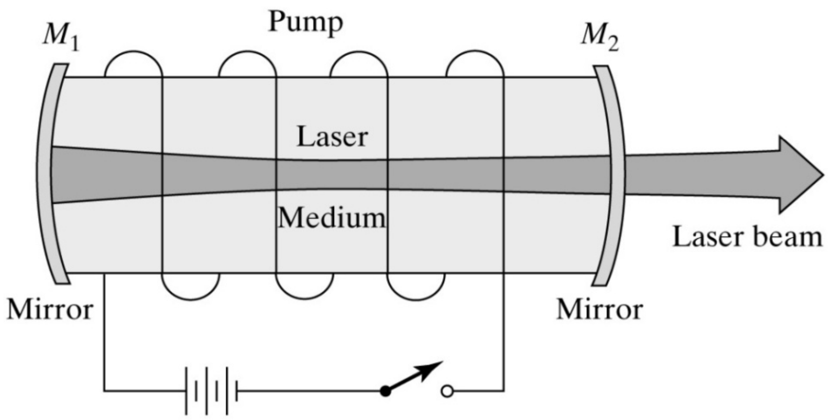
Pump: is a source of energy that excites the atoms from $E_1 \rightarrow E_2$ can be done optically or electrically.

Laser medium: absorbs the pump and emits it as stimulated and spontaneous emission.

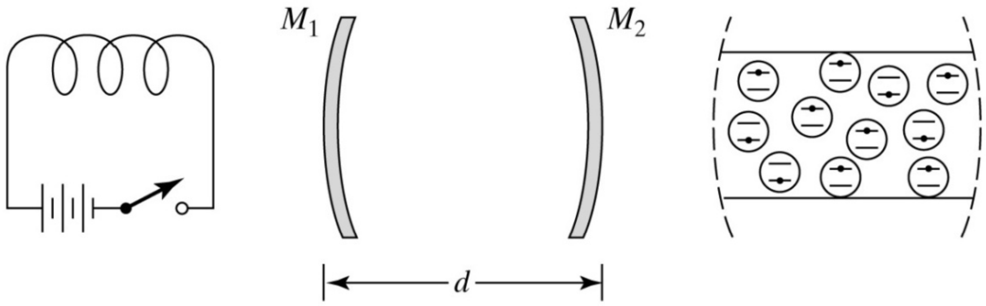
Laser cavity: maximizes the pump absorption by the laser medium.

Resonator: provides feedback to the laser medium and promotes the stimulated emission. Also selects the frequency of the laser (will talk about it more).

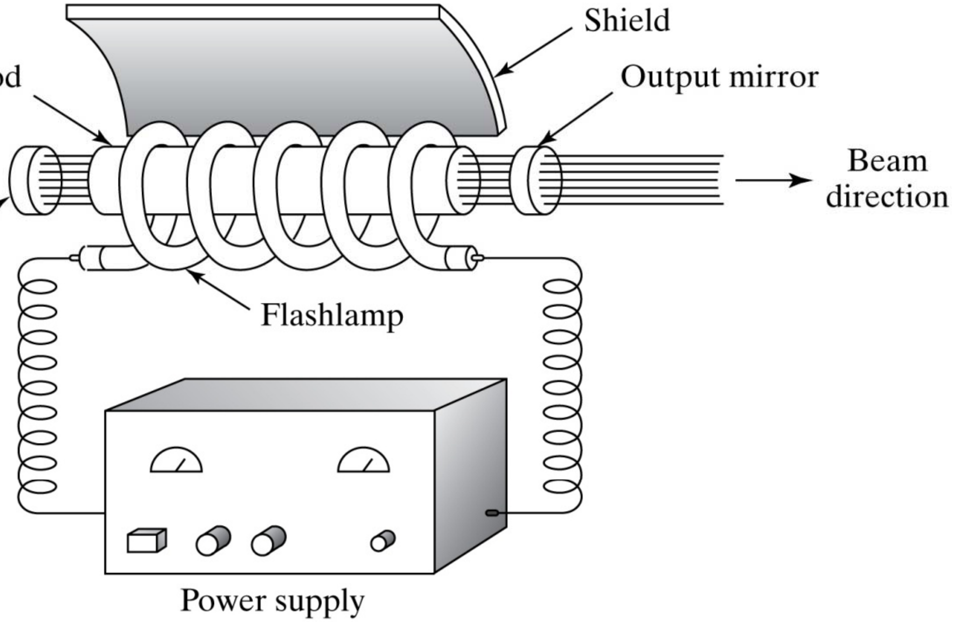
Laser driver: powers the pump.

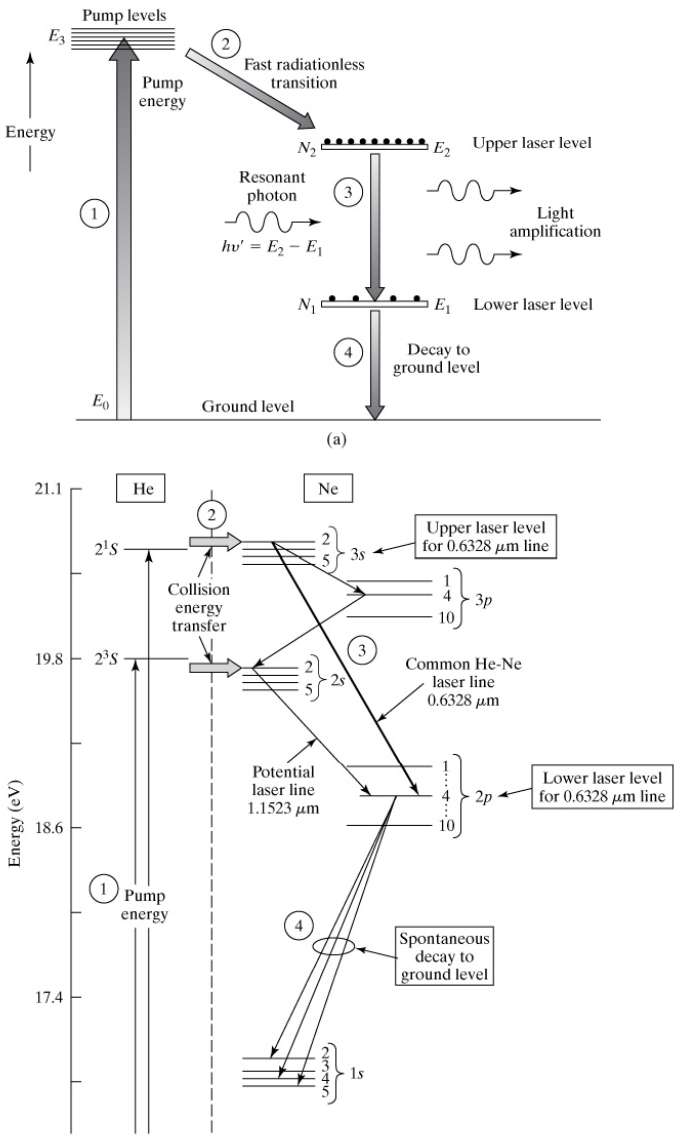


(a) Laser



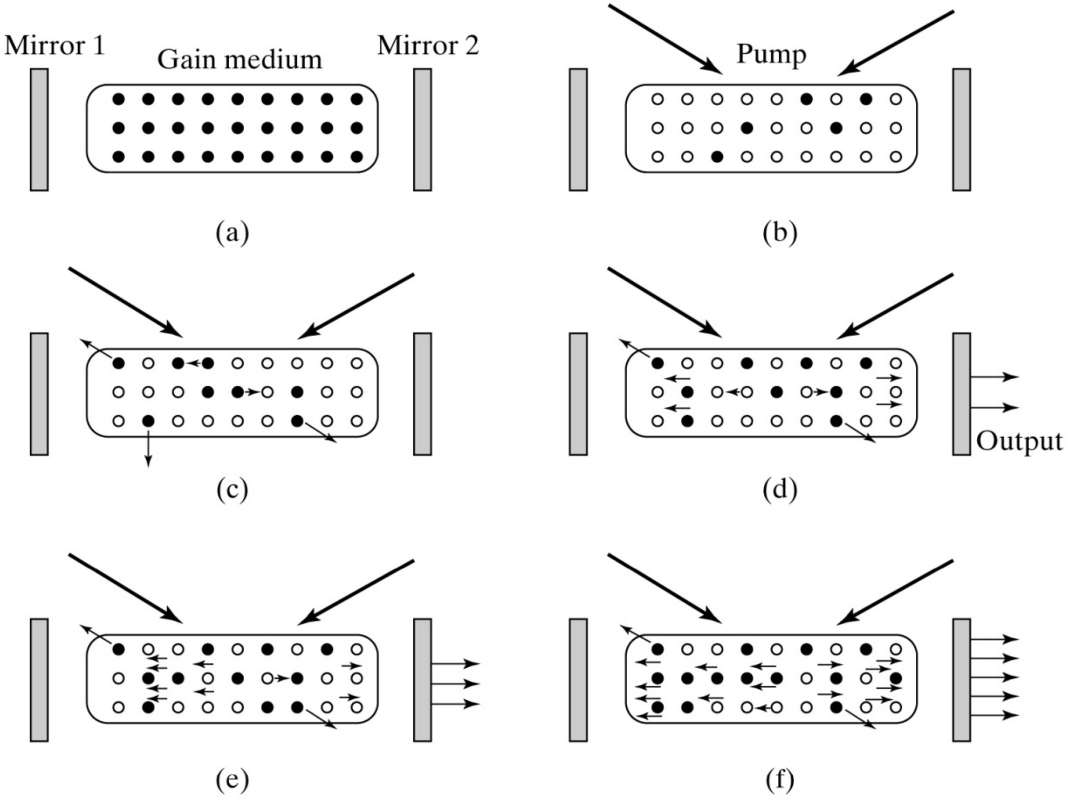
(b) Pump (c) Resonator (d) Laser medium

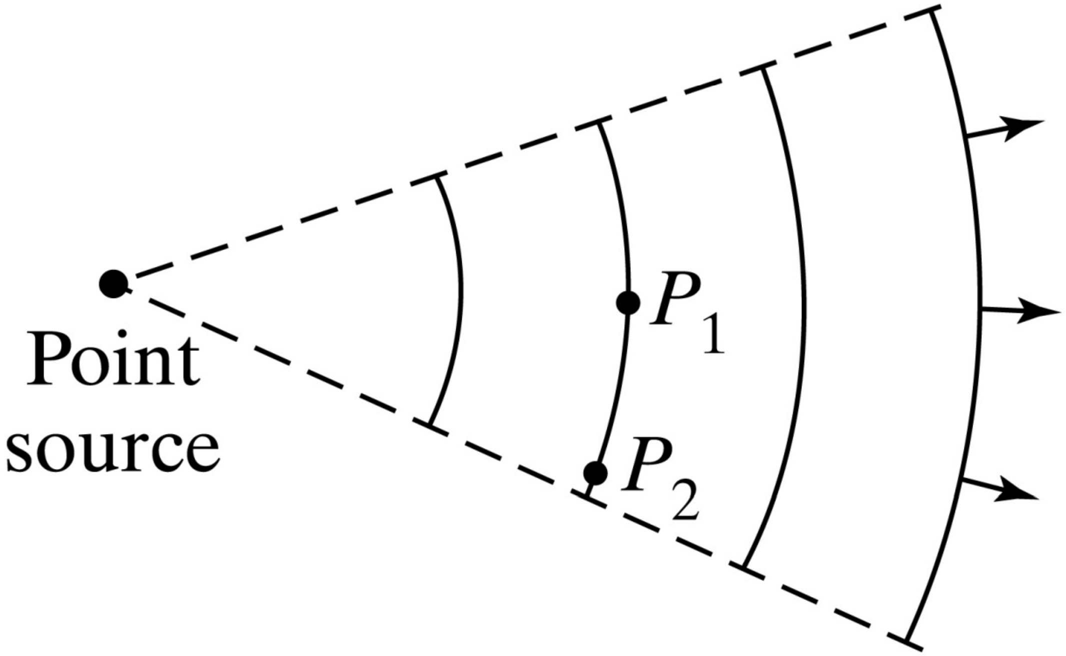

Shield
Ruby rod
Output mirror
Beam direction
End mirror
Flashlamp
Power supply


Pump levels
E_3
Energy
Pump energy
①
②
Fast radiationless transition
N_2
E_2
Upper laser level
Resonant photon
③
$h\nu' = E_2 - E_1$
Light amplification
N_1
E_1
Lower laser level
④
Decay to ground level
E_0
Ground level
21.1
19.8
18.6
17.4
Energy (eV)
He
Ne
2^1S
2^3S
②
Collision energy transfer
2
5
3s
Upper laser level for 0.6328 μm line
1
4
10
3p
③
2
5
2s
Common He-Ne laser line 0.6328 μm
Potential laser line 1.1523 μm
1
4
10
2p
Lower laser level for 0.6328 μm line
① Pump energy
④
Spontaneous decay to ground level
2
3
4
5
1s

(a)

(b)


Mirror 1
Gain medium
Mirror 2
Pump
Output
(a)
(b)
(c)
(d)
(e)
(f)



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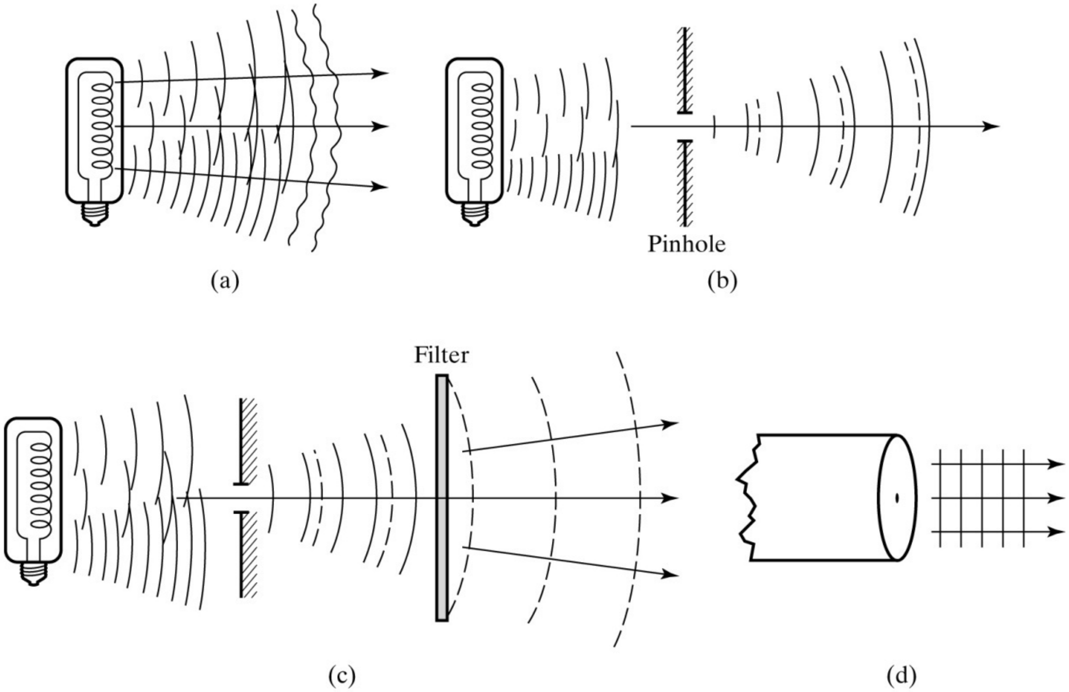
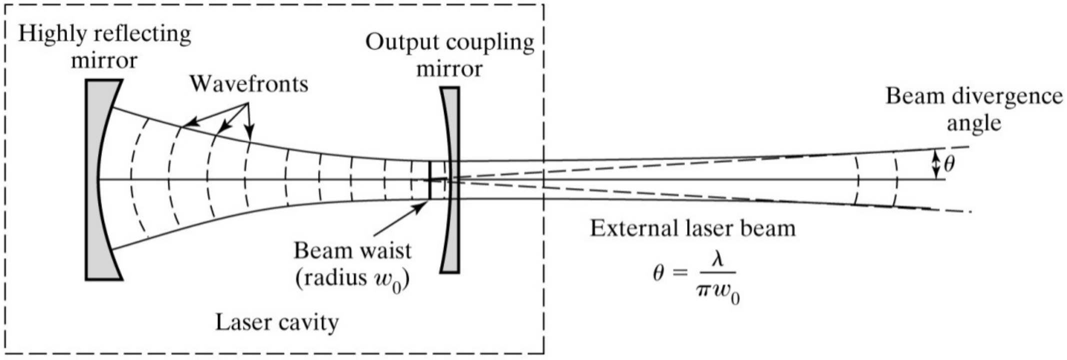
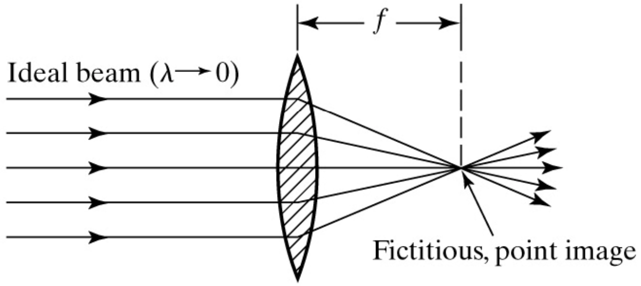
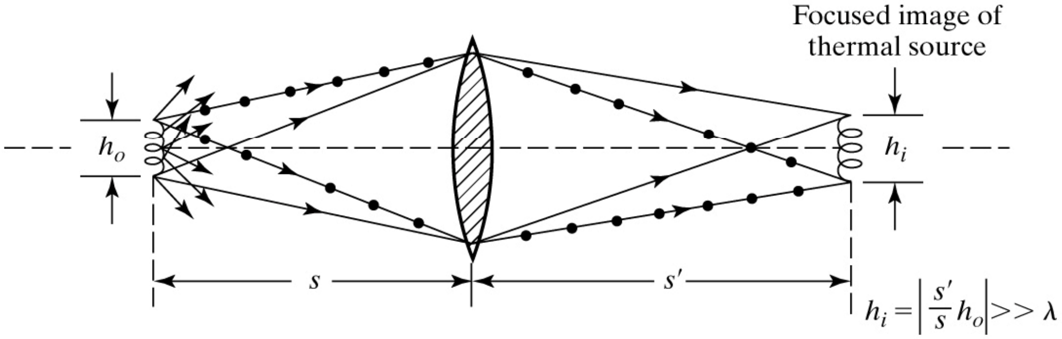

Pinhole
(a)
(b)
Filter
(c)
(d)

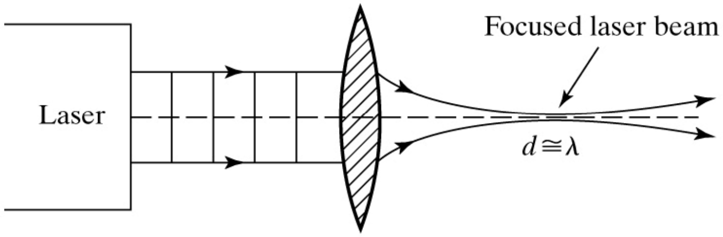

Highly reflecting mirror
Wavefronts
Output coupling mirror
Beam divergence angle
θ
Beam waist (radius w_0)
Laser cavity
External laser beam
$\theta = \frac{\lambda}{\pi w_0}$



(a) Ideal source



(b) Ordinary source



(c) Laser source

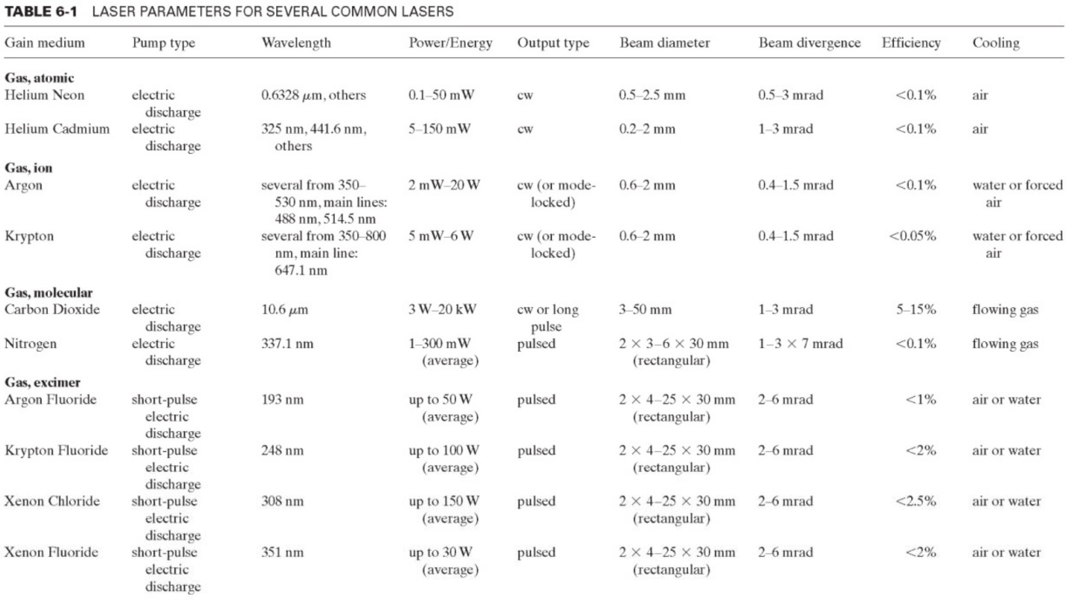


TABLE 6-1 LASER PARAMETERS FOR SEVERAL COMMON LASERS

Gain medium	Pump type	Wavelength	Power/Energy	Output type	Beam diameter	Beam divergence	Efficiency	Cooling
Gas, atomic								
Helium Neon	electric discharge	0.6328 μm, others	0.1–50 mW	cw	0.5–2.5 mm	0.5–3 mrad	<0.1%	air
Helium Cadmium	electric discharge	325 nm, 441.6 nm, others	5–150 mW	cw	0.2–2 mm	1–3 mrad	<0.1%	air
Gas, ion								
Argon	electric discharge	several from 350–530 nm, main lines: 488 nm, 514.5 nm	2 mW–20 W	cw (or mode-locked)	0.6–2 mm	0.4–1.5 mrad	<0.1%	water or forced air
Krypton	electric discharge	several from 350–800 nm, main line: 647.1 nm	5 mW–6 W	cw (or mode-locked)	0.6–2 mm	0.4–1.5 mrad	<0.05%	water or forced air
Gas, molecular								
Carbon Dioxide	electric discharge	10.6 μm	3 W–20 kW	cw or long pulse	3–50 mm	1–3 mrad	5–15%	flowing gas
Nitrogen	electric discharge	337.1 nm	1–300 mW (average)	pulsed	2 × 3–6 × 30 mm (rectangular)	1–3 × 7 mrad	<0.1%	flowing gas
Gas, excimer								
Argon Fluoride	short-pulse electric discharge	193 nm	up to 50 W (average)	pulsed	2 × 4–25 × 30 mm (rectangular)	2–6 mrad	<1%	air or water
Krypton Fluoride	short-pulse electric discharge	248 nm	up to 100 W (average)	pulsed	2 × 4–25 × 30 mm (rectangular)	2–6 mrad	<2%	air or water
Xenon Chloride	short-pulse electric discharge	308 nm	up to 150 W (average)	pulsed	2 × 4–25 × 30 mm (rectangular)	2–6 mrad	<2.5%	air or water
Xenon Fluoride	short-pulse electric discharge	351 nm	up to 30 W (average)	pulsed	2 × 4–25 × 30 mm (rectangular)	2–6 mrad	<2%	air or water

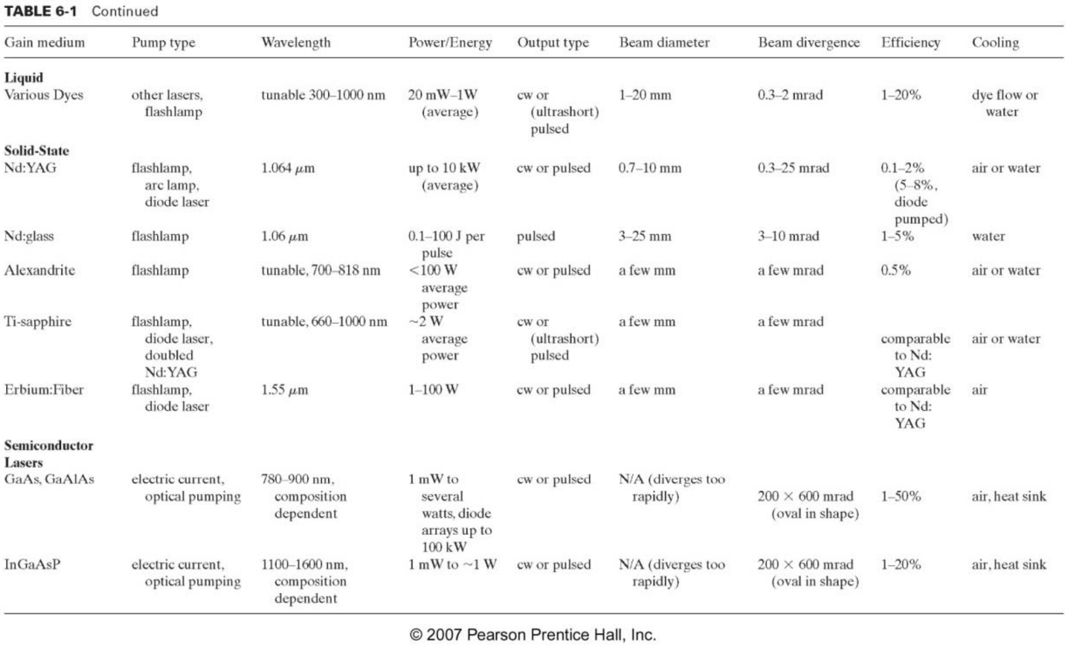
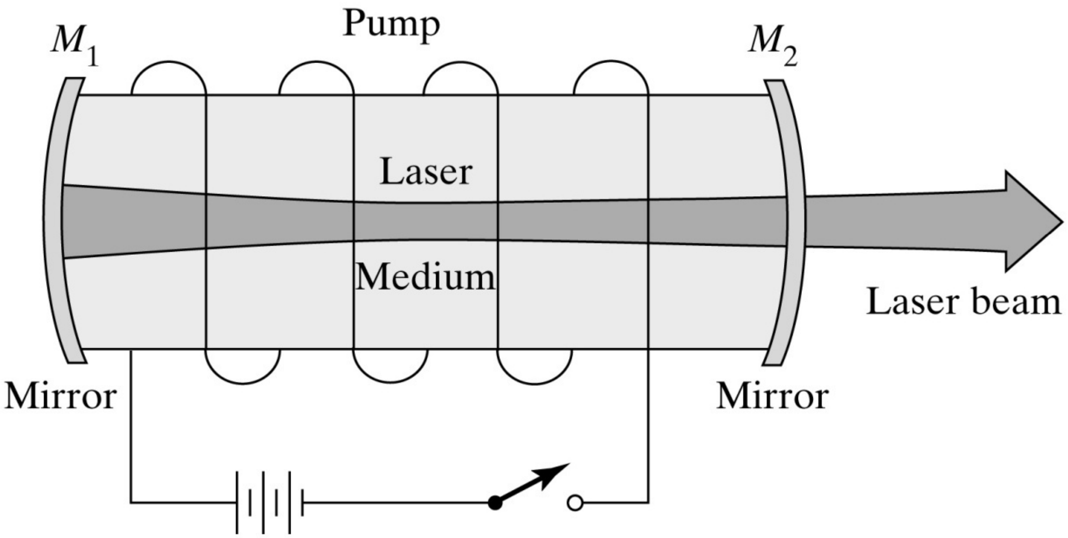
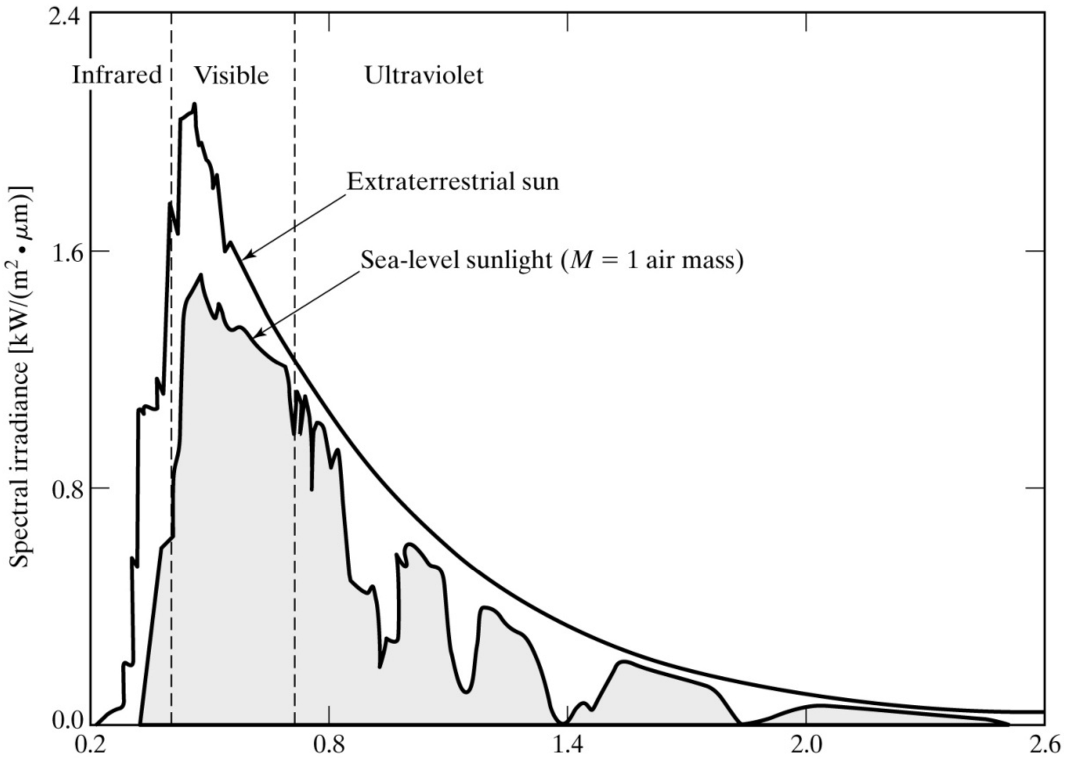


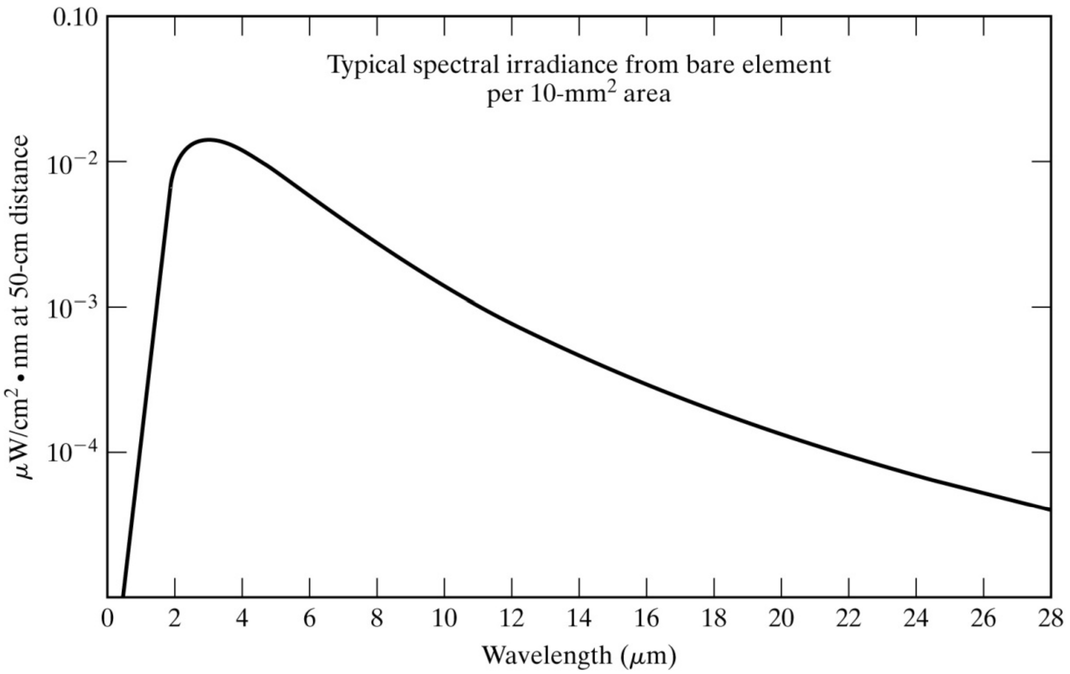
TABLE 6-1 Continued

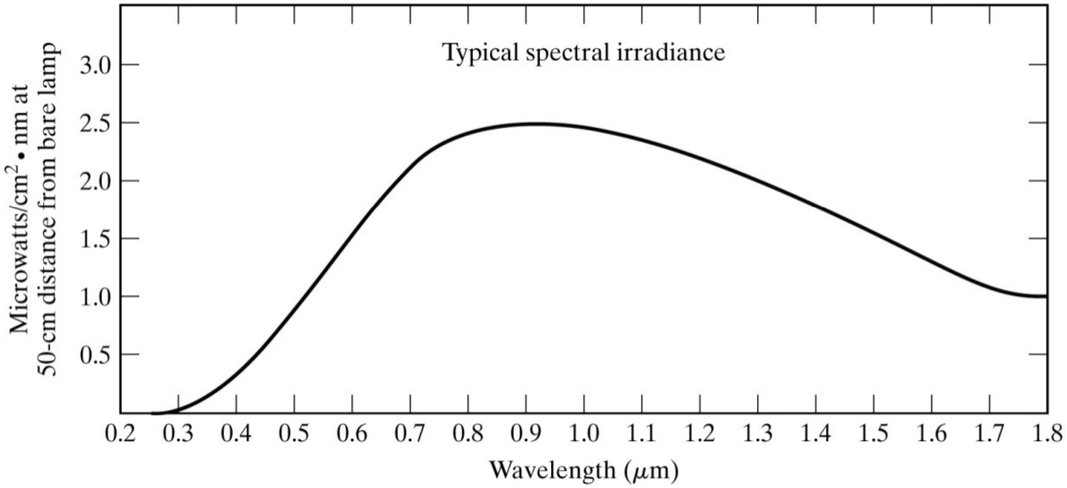
Gain medium	Pump type	Wavelength	Power/Energy	Output type	Beam diameter	Beam divergence	Efficiency	Cooling
Liquid								
Various Dyes	other lasers, flashlamp	tunable 300–1000 nm	20 mW–1W (average)	cw or (ultrashort) pulsed	1–20 mm	0.3–2 mrad	1–20%	dye flow or water
Solid-State								
Nd:YAG	flashlamp, arc lamp, diode laser	1.064 μm	up to 10 kW (average)	cw or pulsed	0.7–10 mm	0.3–25 mrad	0.1–2% (5–8%, diode pumped)	air or water
Nd:glass	flashlamp	1.06 μm	0.1–100 J per pulse	pulsed	3–25 mm	3–10 mrad	1–5%	water
Alexandrite	flashlamp	tunable, 700–818 nm	<100 W average power	cw or pulsed	a few mm	a few mrad	0.5%	air or water
Ti-sapphire	flashlamp, diode laser, doubled Nd:YAG	tunable, 660–1000 nm	~2 W average power	cw or (ultrashort) pulsed	a few mm	a few mrad	comparable to Nd: YAG	air or water
Erbium:Fiber	flashlamp, diode laser	1.55 μm	1–100 W	cw or pulsed	a few mm	a few mrad	comparable to Nd: YAG	air
Semiconductor Lasers								
GaAs, GaAlAs	electric current, optical pumping	780–900 nm, composition dependent	1 mW to several watts, diode arrays up to 100 kW	cw or pulsed	N/A (diverges too rapidly)	200 × 600 mrad (oval in shape)	1–50%	air, heat sink
InGaAsP	electric current, optical pumping	1100–1600 nm, composition dependent	1 mW to ~1 W	cw or pulsed	N/A (diverges too rapidly)	200 × 600 mrad (oval in shape)	1–20%	air, heat sink



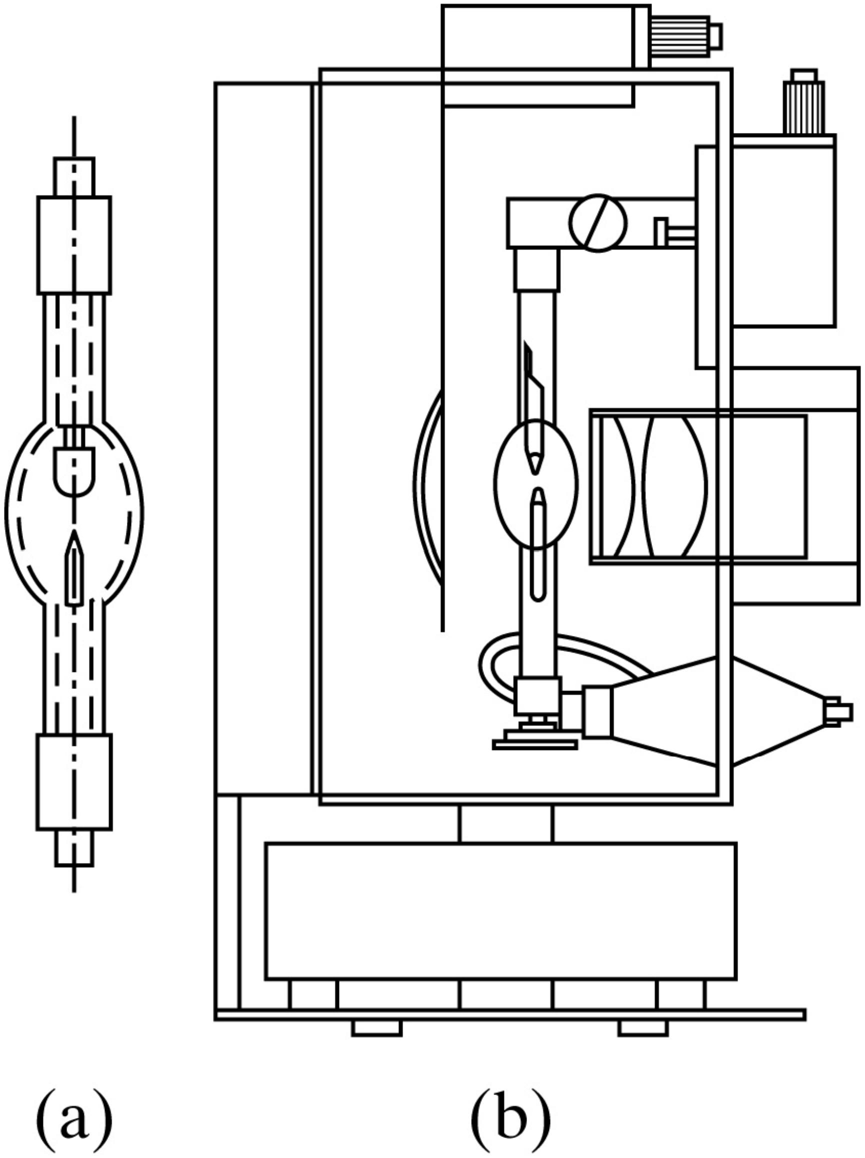
M_1
Pump
M_2
Laser
Medium
Laser beam
Mirror
Mirror

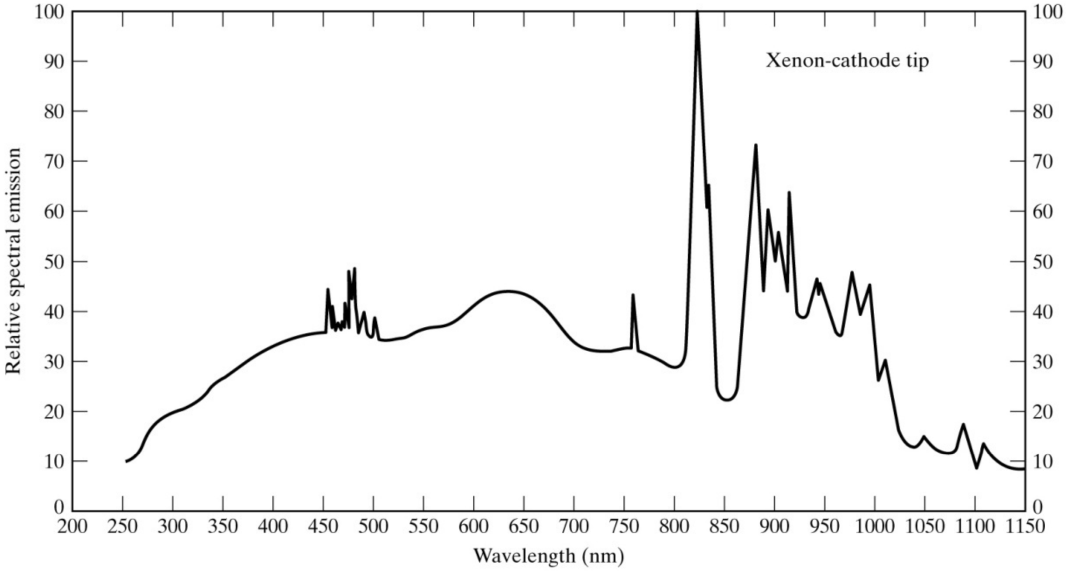

Infrared
Visible
Ultraviolet
Extraterrestrial sun
Sea-level sunlight (M = 1 air mass)
Spectral irradiance [kW/(m^2 • μm)]
Wavelength (μm)
2.4
1.6
0.8
0.0
0.2
0.8
1.4
2.0
2.6

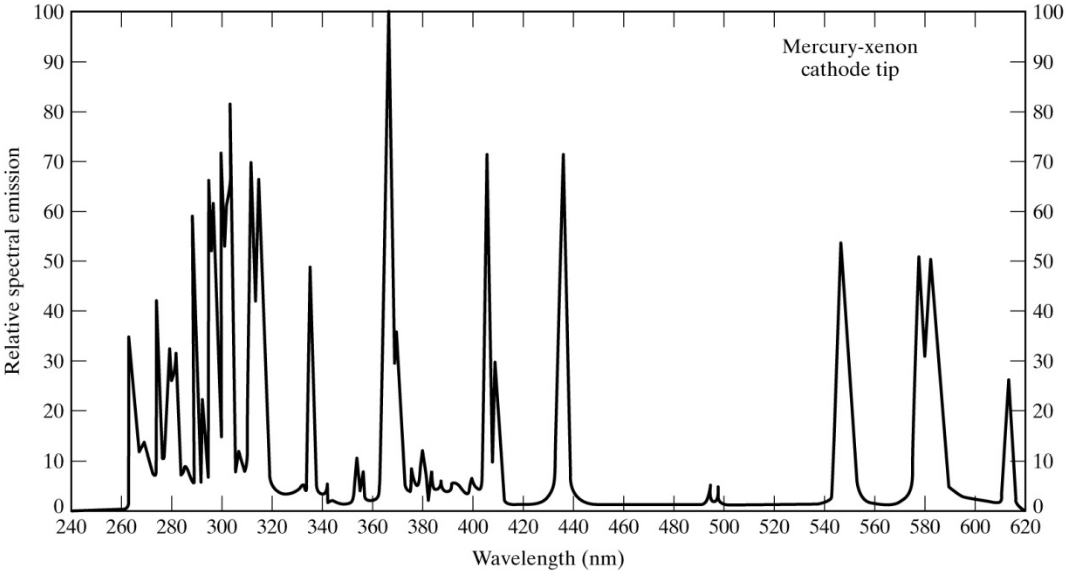

Typical spectral irradiance from bare element
per 10-mm^2 area
μW/cm^2 • nm at 50-cm distance
0.10
10^{-2}
10^{-3}
10^{-4}
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
Wavelength (μm)

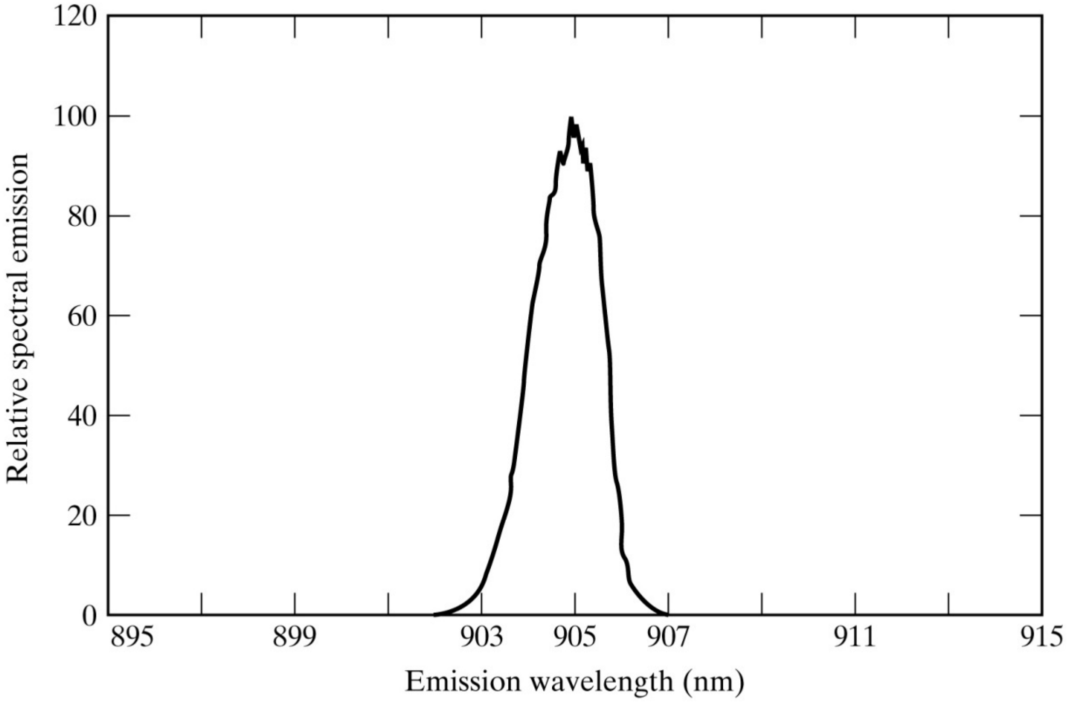


Typical spectral irradiance
Microwatts/cm^2 • nm at 50-cm distance from bare lamp
3.0
2.5
2.0
1.5
1.0
0.5
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Wavelength (μm)


(a)
(b)


Xenon-cathode tip
Relative spectral emission
0
10
20
30
40
50
60
70
80
90
100
200
250
300
350
400
450
500
550
600
650
700
750
800
850
900
950
1000
1050
1100
1150
Wavelength (nm)


Mercury-xenon
cathode tip
Relative spectral emission
Wavelength (nm)
0
10
20
30
40
50
60
70
80
90
100
240
260
280
300
320
340
360
380
400
420
440
460
480
500
520
540
560
580
600
620


120
100
80
60
40
20
0
Relative spectral emission
895
899
903
905
907
911
915
Emission wavelength (nm)