Chapter 7
Interference of Light

Lecture Notes for Modern Optics based on
Pedrotti & Pedrotti & Pedrotti
Instructor: Nayer Eradat
Spring 2009

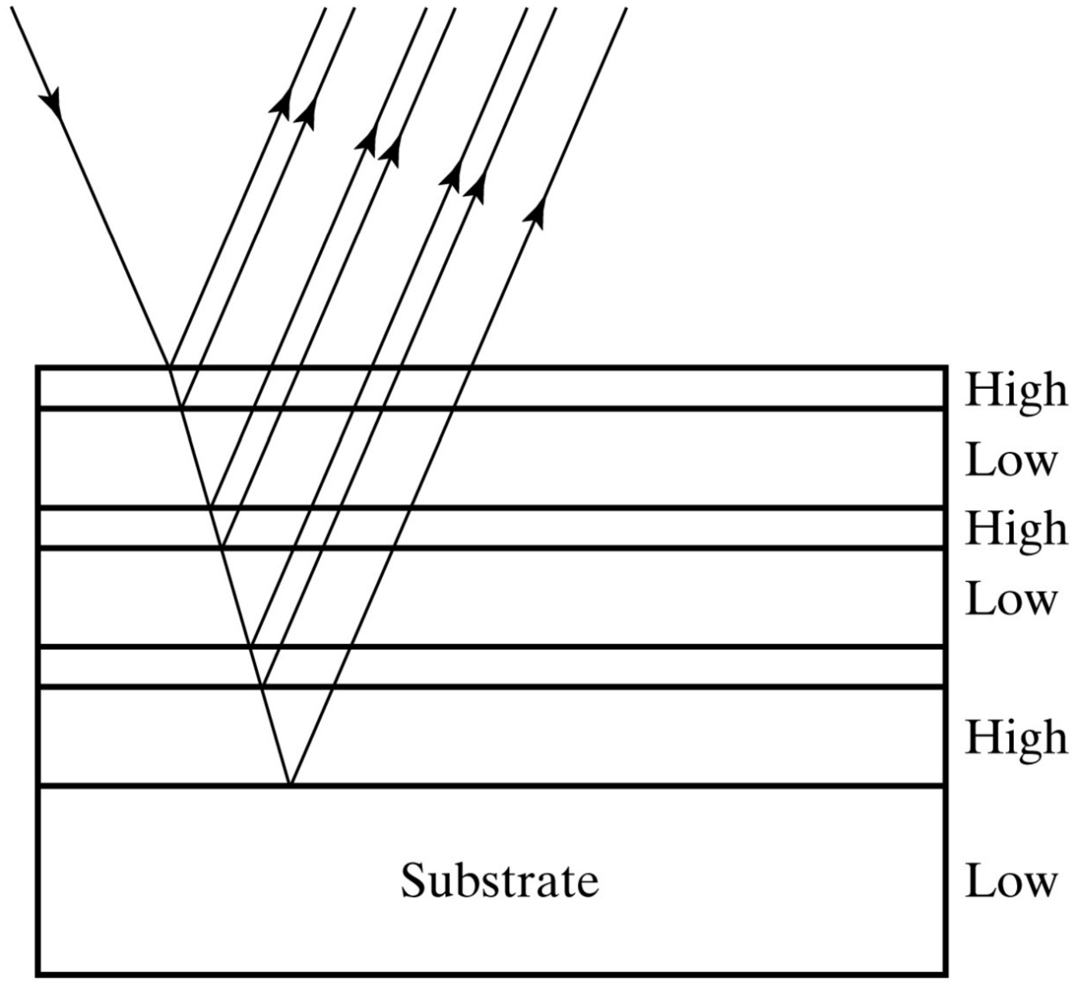
Interference of light

Interference phenomenon depends on superposition of two or more individual waves under strict conditions.

Constructive interference leads to enhancement of the resulting amplitude with respect to that of the constituents.

Destructive interference leads to diminution of the resulting amplitude with respect to that of the constituents.

Interference produces an alternating spatial pattern of **fringes.**



© 2007 Pearson Prentice Hall, Inc.

Two-beam interference: vector treatment

Interference of the plane waves of same frequency arriving at point P:

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{01}\cos\left(ks_1 - \omega t + \phi_1\right)$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{02}\cos\left(ks_2 - \omega t + \phi_2\right)$$

The combined disturbance at point P is: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$

$\vec{\mathbf{E}}s$ are rapidly verying functions in time with frequencies of $\sim 10^{15} Hz$ and average to zero very quickly.

Irradiance or radiant power density is a measure of time-average of the square of the field (detector reading)

$I = E_e\left(W/cm^2\right) \propto \left\langle \vec{E}^2 \right\rangle_t$ in practice t$\left(\text{averaging time } \dfrac{1}{30} s \text{ for the eye}\right)$>>T(period 10^{-15}s)

$$I = \varepsilon_0 c\left\langle \vec{\mathbf{E}}\bullet\vec{\mathbf{E}} \right\rangle = \varepsilon_0 c\left\langle \left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right)\bullet\left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right) \right\rangle$$

$$I = \varepsilon_0 c\left\langle \left(\vec{\mathbf{E}}_1\bullet\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\bullet\vec{\mathbf{E}}_2 + 2\vec{\mathbf{E}}_1\bullet\vec{\mathbf{E}}_2\right) \right\rangle$$

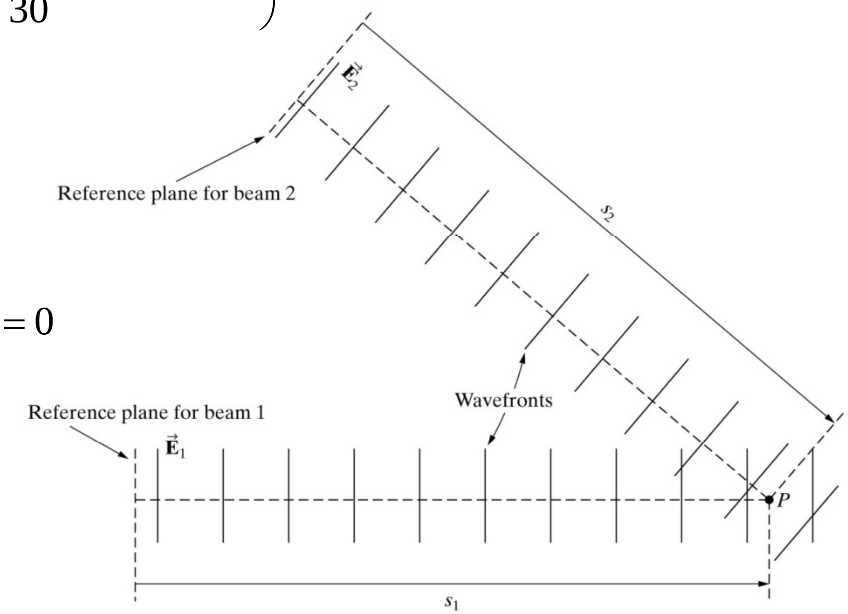
$$I = I_1 + I_2 + I_{12}$$

1) if $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ are orthogonal no interference happens. $\cos\theta = 0$

2) if $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ are parallel interference has maximum effect $\cos\theta = 1$

3) if $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ have angle δ the interference term is:

$$\mathrm{I}_{12} = \varepsilon_0 c\vec{\mathbf{E}}_{01}\bullet\vec{\mathbf{E}}_{02}\left\langle \cos\delta \right\rangle$$



© 2007 Pearson Prentice Hall, Inc.

where $\delta = k\left(s_2 - s_1\right) + \phi_2 - \phi_1$ is the phase difference between the waves.

For a purely monochromatic wave δ is independent of time (a constant phase relationship between two waves).

Two-beam interference

Interference of the plane waves of same frequency arriving at point P: $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\left\langle \cos\delta \right\rangle}$

$I_1 = \frac{1}{2}\varepsilon_0 c E_{01}^2;\ I_1 = \frac{1}{2}\varepsilon_0 c E_{02}^2;\ \ \mathrm{I}_{12} = \varepsilon_0 c \vec{\mathbf{E}}_{01} \bullet \vec{\mathbf{E}}_{02} \left\langle \cos\delta \right\rangle = 2\sqrt{I_1 I_2}\left\langle \cos\delta \right\rangle$ where $\delta = k\left(s_2 - s_1\right) + \phi_2 - \phi_1$

For **mutually incoherent fields** and low quality laser sources the phases are random functions of time that very on time scales much larger than the E field oscillations but shorter than the detector averaging time.

The interference term in this case will be:

$2\sqrt{I_1 I_2}\left\langle \cos\left(k\left(s_2 - s_1\right) + \phi_2\left(t\right) - \phi_1\left(t\right)\right)\right\rangle = 0$ for all incoherent sources and low quality lasers and

$\boxed{I = I_1 + I_2}$ Light from independent sources even if they are same kind of lasers do not interfere.

For **mutually coherent beams** $\left(\text{light from same lase split and recombined}\right)$ the phase difference at detector $\phi_2\left(t\right) - \phi_1\left(t\right) = 0$ if the light paths are equal and $\delta =$ constant and the interference term is:

$$2\sqrt{I_1 I_2}\left\langle \cos\left(k\left(s_2 - s_1\right) + \phi_2\left(t\right) - \phi_1\left(t\right)\right)\right\rangle = 2\sqrt{I_1 I_2}\left\langle \cos\left(k\left(s_2 - s_1\right)\right)\right\rangle$$

Even if travel distance difference is nonzero $\phi_2\left(t\right) - \phi_1\left(t + \delta t\right) \cong 0$ so long as $\delta t << t_c$

The coherence time of a source is inversely proportional to the range of the frequency components that make up the electric field $\boxed{t_c = \tau_0 = \frac{1}{\Delta\nu}}$.

Coherence length is the distance that electric field travels in coherence time: $\boxed{L_c = l_t = c\tau_0 = ct_c}$

For mutually coherent sources we assume the length traveled from the sources is much shorter than the coherence length, then the irradiance of the combined fields is: $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$.

Two-beam interference

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

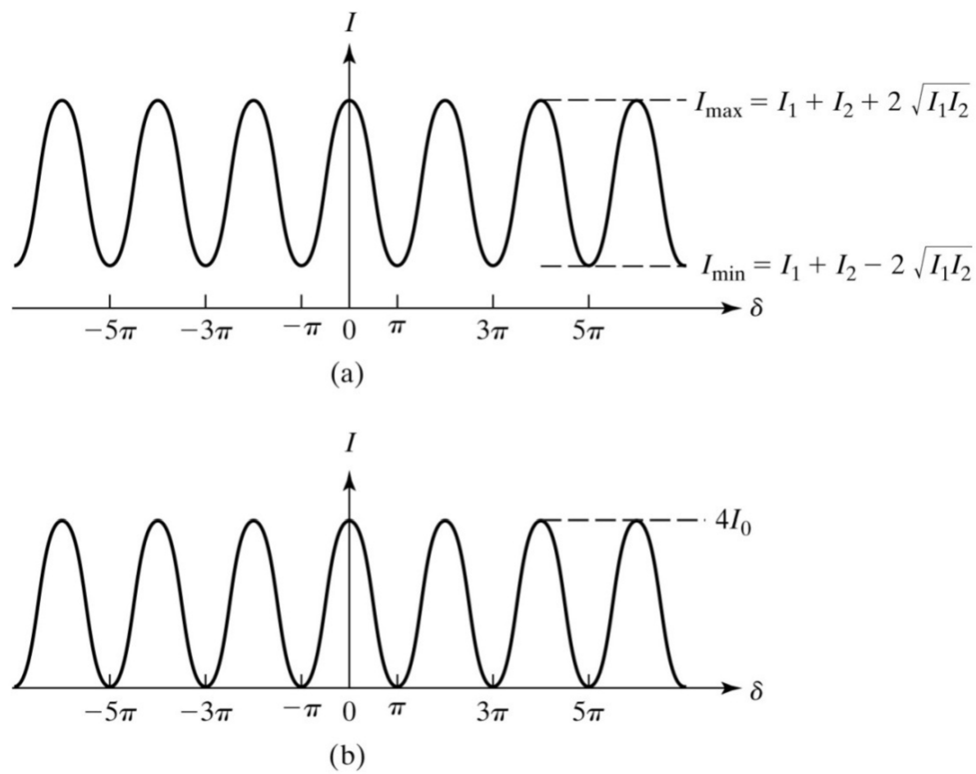
δ is the total phase difference between the beam from the point of separation. It can be due to

a) path length difference

b) phase shift at reflecting beam splitters

c) differing indecies of refraction in the separate paths

depanding on the <u>sign of the cosδ</u> the interference will be <u>constructive</u> (+) <u>or destructive</u> (–). A pattern of alternating maxima and minima (<u>fringes</u>) will form at the plane of observation due to varying δ at different locations of the observation screen.



© 2007 Pearson Prentice Hall, Inc.

$$\begin{cases} I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(E_{01} + E_{02}\right)^2 \\ I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(E_{01} - E_{02}\right)^2 \end{cases}$$

If $I_1 = I_2 = I_0 \;\rightarrow\; I_{max} = 4I_0$ and $I_{min} = 0$

Fring visibility is a measure of fringe contrast

and is defined as: $\boxed{\textit{Fringe visibility} = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}}$

To ensure maximum visibility in experimental settings, we make sure the amplitudes are same in both arms when they arrive at the screen.

Conservation of energy

Rewriting the irradiance for the interference pattern of two equal amplitude components, we get:

$$I = I_0 + I_0 + 2\sqrt{I_0^2}\cos\delta = 2I_0\left(1+\cos\delta\right)$$

Using $1+\cos\delta = 2\cos^2\frac{\delta}{2}$ we get

$$I = 4I_0\cos^2\frac{\delta}{2}$$

The energy is not conserved at each point of the interference pattern $I \neq 2I_0$

but it is conserved over one spatial period $I_{av} = 2I_0$.

So inteference causes redistribution of the energy of sources over the fringe pattern.

The frine analysis performed here was for plane harminc waves but the results are general and hold for spherical and cylindrical waves. The only difference is that these waves do not have constant amplitudes and irradiances as they propagate.

Young's (1802) double slit experiment

To demonstrate the interference pattern and fringes Young generated two coherent point sources by splitting the wavefront of a wave generated by a point source to two in-phase point sources. Today we uese a laser as the first source. We can assume that the amplitudes of the splitted waves are equal. Our goal is to develope an expression for the irradiance at an arbitrary point, P on a screen at a distance L from the sources. a is the separation of the sources. The irradiance at P is:

$I = 4I_0 \cos^2(\delta/2)$ where δ is the phase difference of the waves arriving at P

In this geometry phase difference is caused by the optical path difference

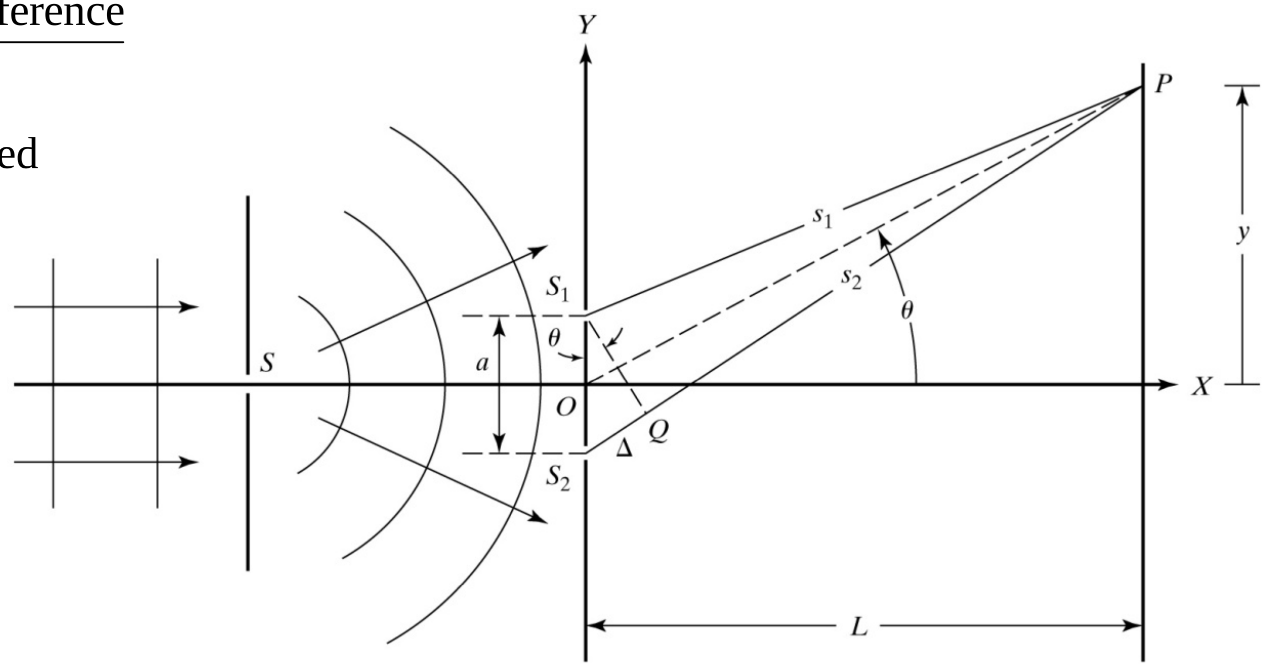
$$\Delta = s_2 - s_1 \cong a\sin\theta$$

The phase difference associated with Δ is

$$\delta = k(s_2 - s_1) = (2\pi/\lambda)\Delta$$

$$I = 4I_0 \cos^2\left(\frac{\pi\Delta}{\lambda}\right)$$

$$\boxed{I = 4I_0 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}$$



© 2007 Pearson Prentice Hall, Inc.

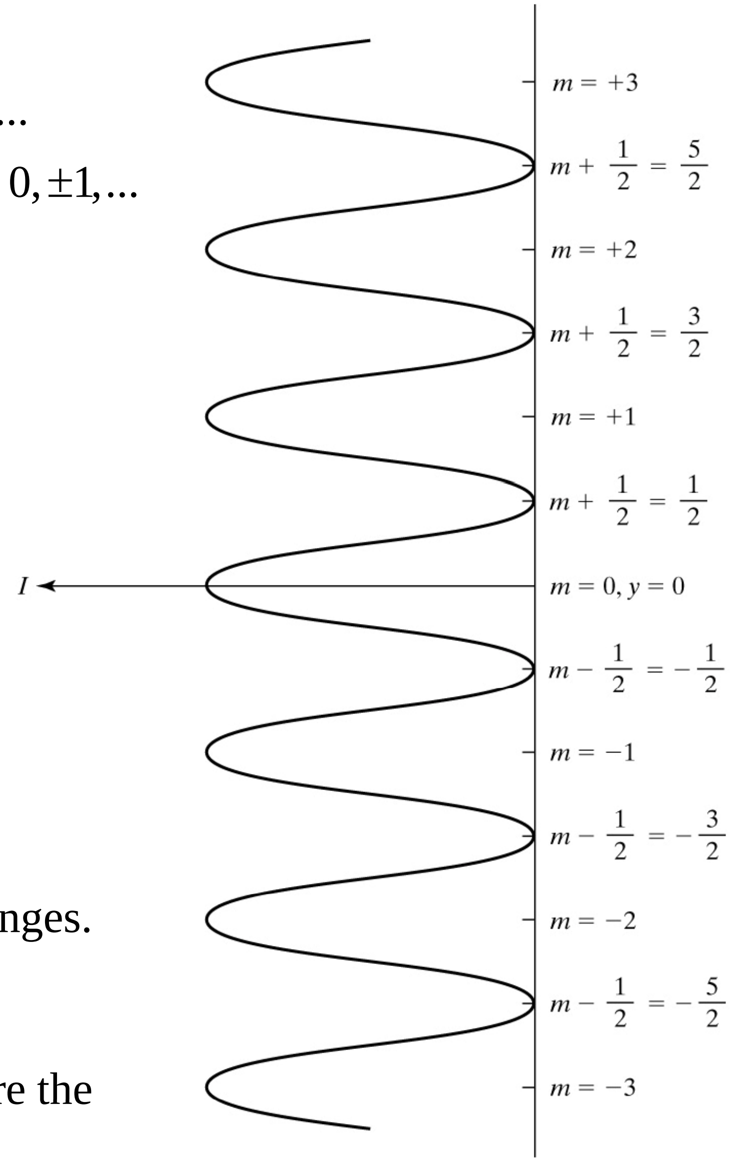
Young's (1802) double slit experiment

$$\boxed{I = 4I_0\cos^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}$$

$$\begin{cases}\text{constructive interference} \rightarrow a\sin\theta = \Delta = m\lambda,\ m = 0, \pm 1, \ldots \\ \text{destructive interference} \rightarrow a\sin\theta = \Delta = (m+1/2)\lambda, m = 0, \pm 1, \ldots\end{cases}$$

For $y << L \rightarrow \sin\theta = \tan\theta \cong \frac{y}{L}$ then $\boxed{I \cong 4I_0\cos^2\left(\frac{\pi a y}{L\lambda}\right)}$

$$\begin{cases}\text{bright fringe } \frac{\pi a y}{L\lambda} = m\pi \rightarrow y_{m,\max} = \frac{m\lambda L}{a}, m = 0, \pm 1, \ldots \\ \text{dark fringe } \frac{\pi a y}{L\lambda} = \left(m+\frac{1}{2}\right)\pi \rightarrow y_{m,\min} = \frac{(m+1/2)\lambda L}{a}, \\ m = 0, \pm 1, \ldots\end{cases}$$



© 2007 Pearson Prentice Hall, Inc.

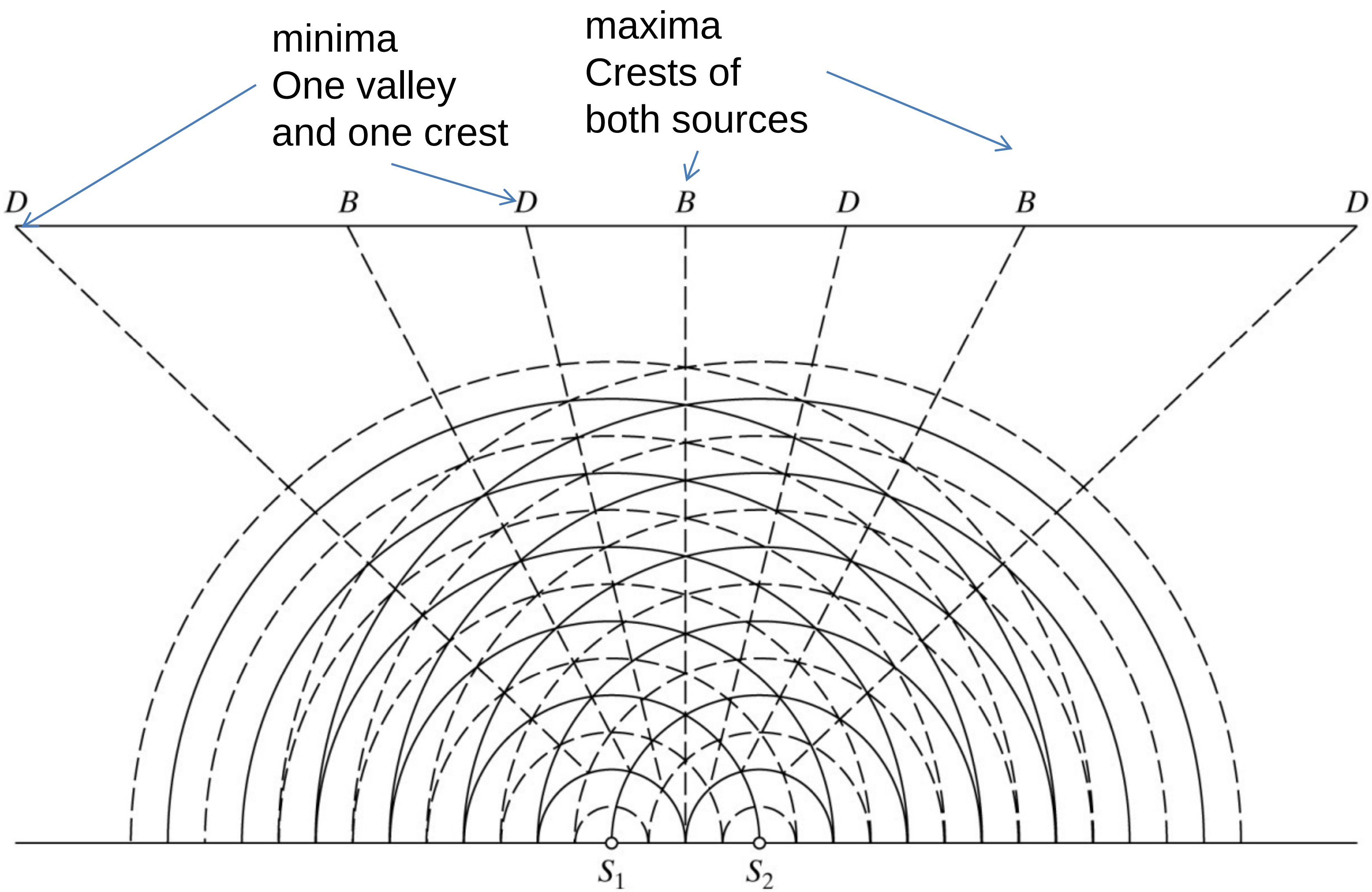
Fringe separtion (peak to peak):

$\boxed{\Delta y_{\text{maxima}} = \Delta y_{\text{minima}} = y_{m+1} - y_m = \frac{\lambda L}{a}}$ is constant for all fringes.

The minima are situated midway between the maxima.

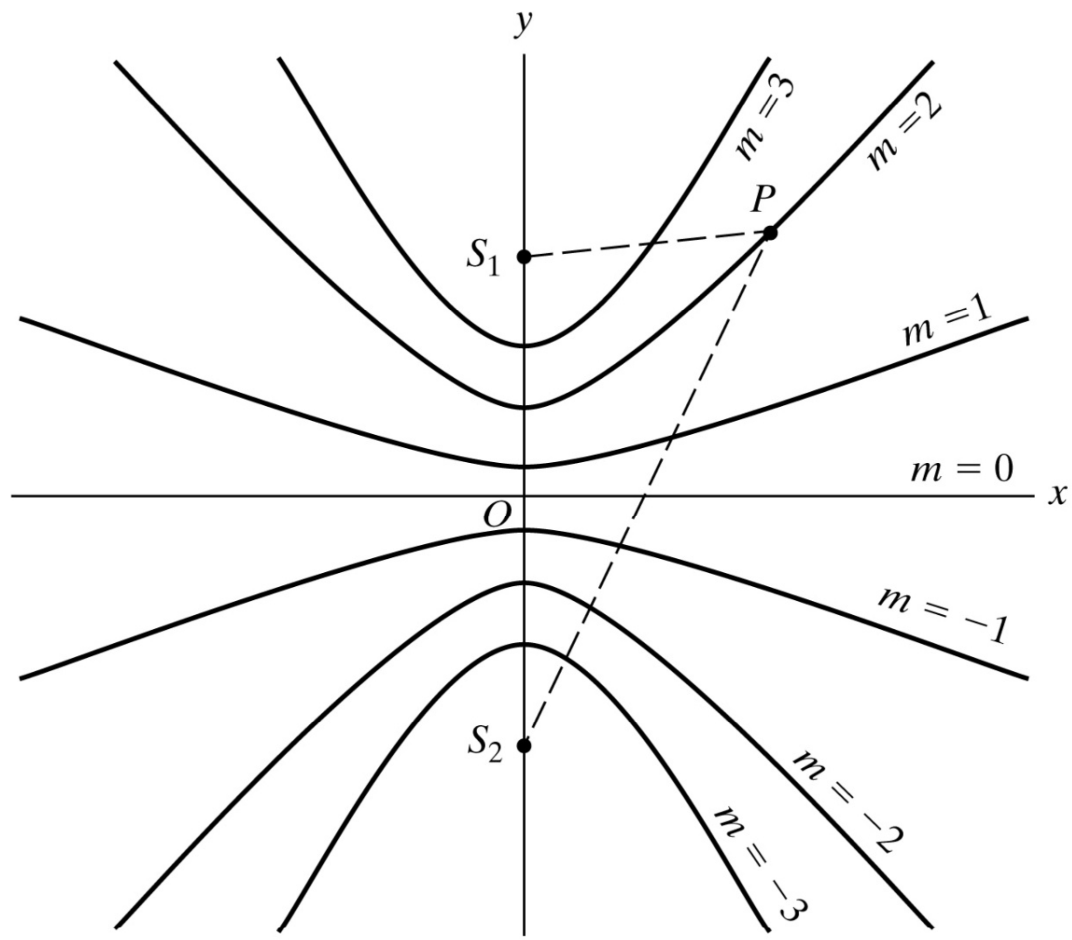
We can design experiments with this technique to measure the wavelength or width of the very tiny slits.

Formation of the fringes with analysis of the crests and valleys of the spherical waves from two sources



© 2007 Pearson Prentice Hall, Inc.

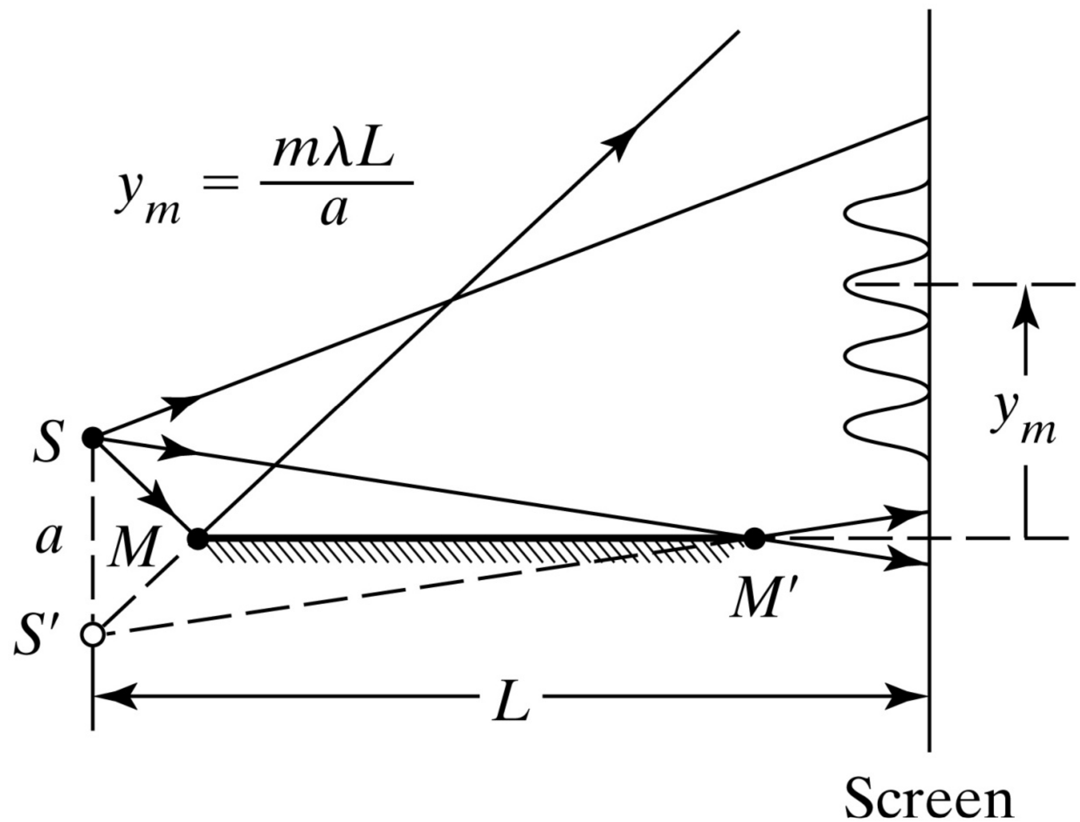
The bright fringe surfaces for two coherent point source generated by rotating the pattern around the x axis



© 2007 Pearson Prentice Hall, Inc.

Double-slit experiments with virtual sources I
Lloyd's mirror

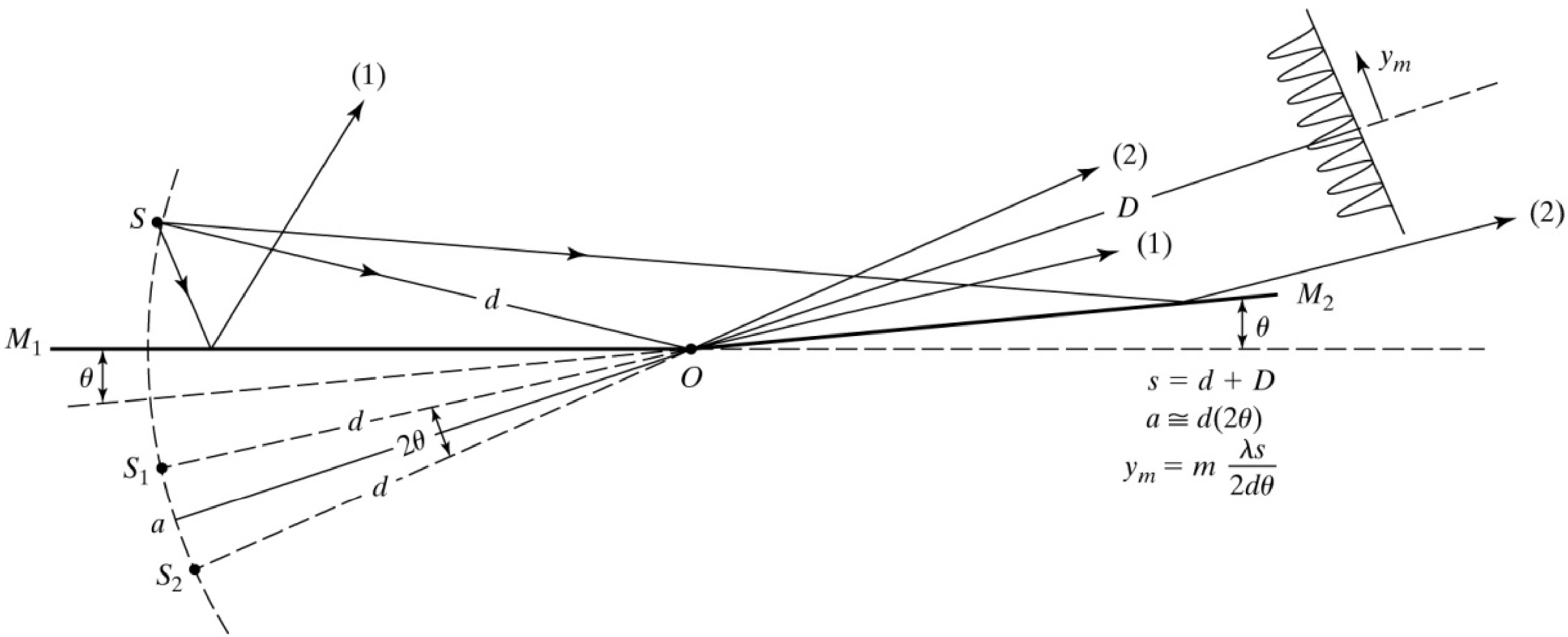
The interference pattern is generated by superposition of the light from the actual source S and the virtual source S' that is image of the S on the mirror MM'



© 2007 Pearson Prentice Hall, Inc.

Double-slit experiments with virtual sources II
Fresnel's mirror

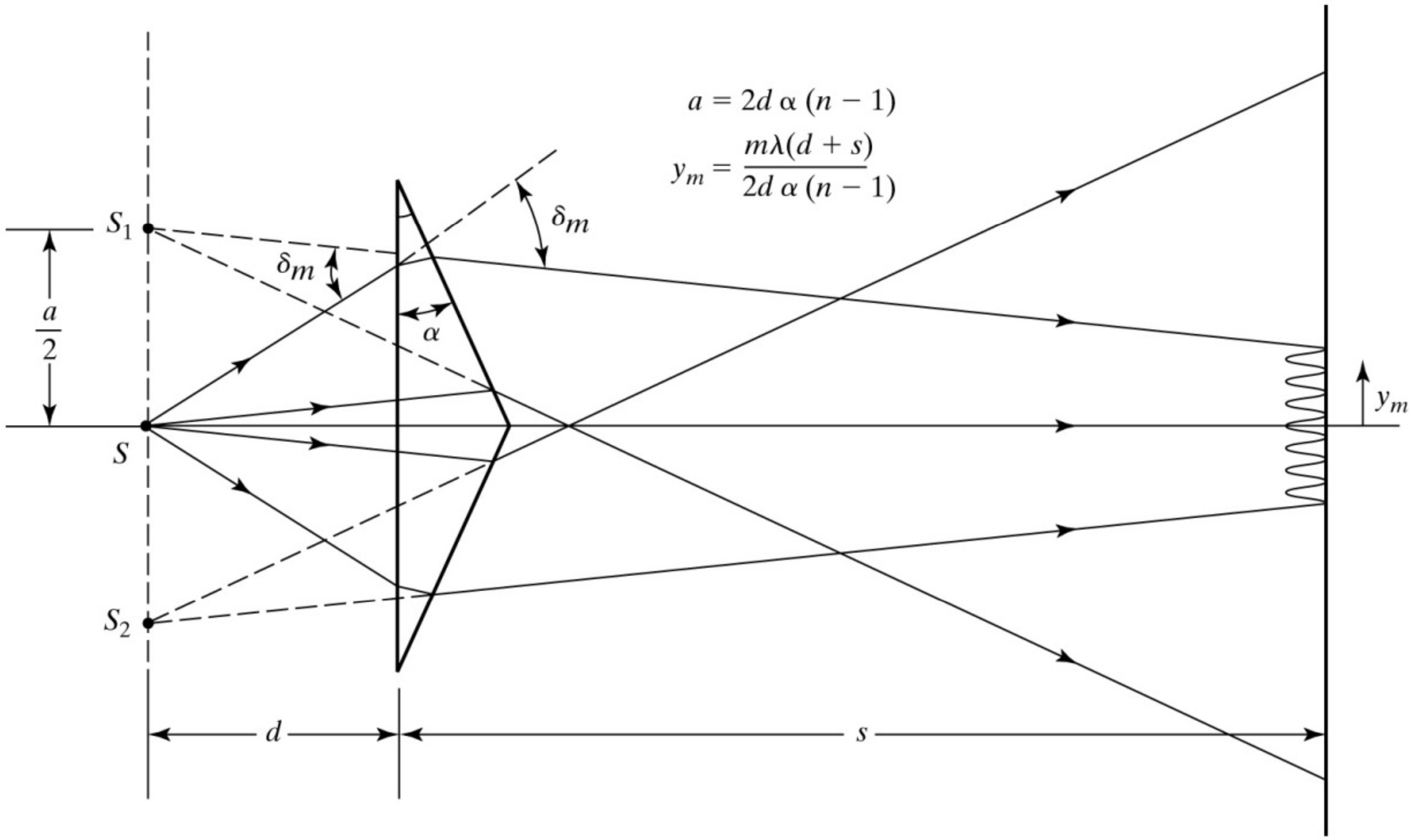
The interference pattern is generated by superposition of the light from the two virtual source S_1 and S_2 that are images of the S on the mirror M_1 and M_2



© 2007 Pearson Prentice Hall, Inc.

Double-slit experiments with virtual sources II
Fresnel's biprisms

The interference pattern is generated by superposition of the light from the two virtual source S_1 and S_2 that are formed by refraction in the two halves of the biprism.



© 2007 Pearson Prentice Hall, Inc.

Interference in dielectric films

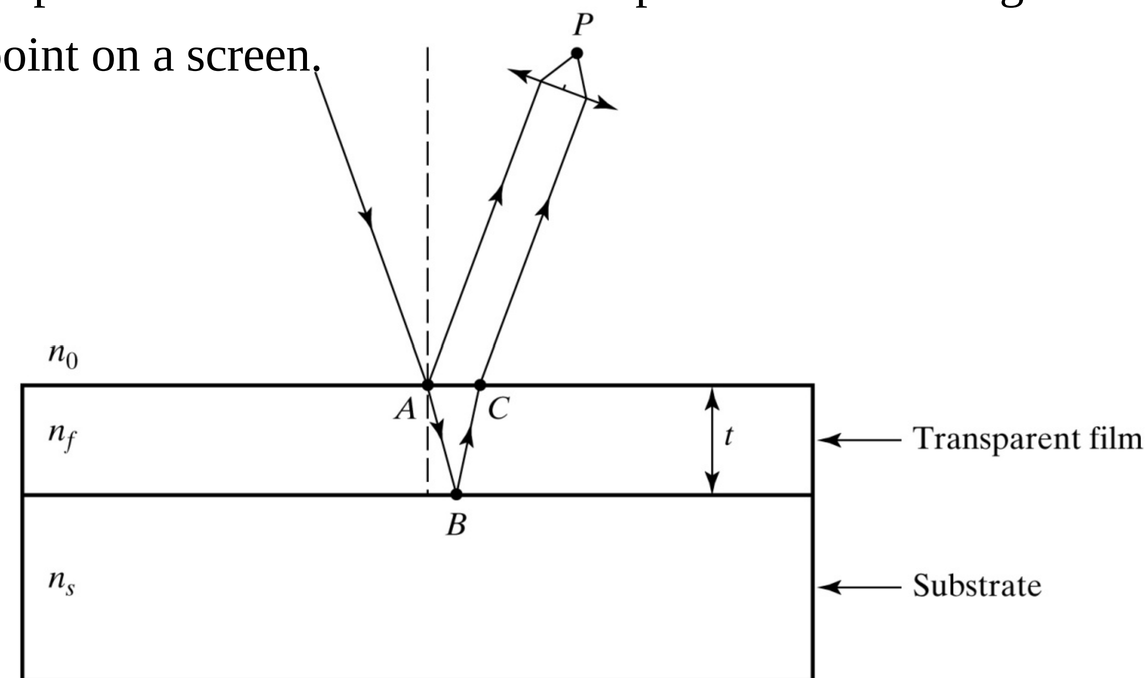
Dielectric here means index of refraction is real and positive. No considerable absorption hapens at the wavelength of interest.

To create two coherent sources from one we can use two techniques:

1) Amplitude division. For example by partial reflection at two different interfaces.

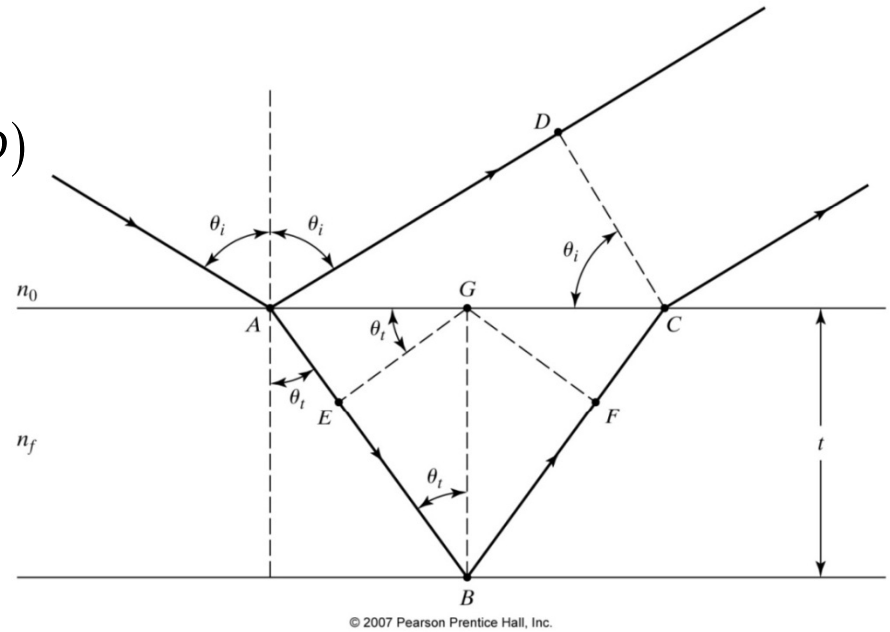
2) Wavefrot division. Fro example placing two slits in front of the waverfront.

A transparent film bounded by parallel planes divides a beam of light into two parts. (amplitude division) some of the light is reflected from the first interface and some from the second interface. Two reflected portions from first and second planes can be brought together by a lens to interfer at a point on a screen.



© 2007 Pearson Prentice Hall, Inc.

Thin film interference



The path difference is: $\Delta = n_f\left(AB + BC\right) - n_0\left(AD\right)$

$$\Delta = \underbrace{\left[n_f\left(AE + FC\right) - n_0\left(AD\right)\right]}_{=0 \text{ it can be shown}} + n_f\left(EB + BF\right)$$

Snell's law: $n_0 \sin\theta_i = n_f \sin\theta_t$

$$AE = AG\sin\theta_t = \left(AC/2\right)\sin\theta_t$$

$$\left.\begin{array}{l} 2AE = AC\sin\theta_t \\ AD = AC\sin\theta_i \end{array}\right\} 2AE = AD\frac{\sin\theta_t}{\sin\theta_i} = AD\frac{n_0}{n_f}$$

$$n_0 AD = 2AEn_f = n_f\left(AE + FC\right) \rightarrow \left[n_f\left(AE + FC\right) - n_0\left(AD\right)\right] = 0$$

$\Delta = n_f\left(EB + BF\right) = 2n_f EB \rightarrow \boxed{\Delta = 2n_f t\cos\theta_t}$ this can be related to the angle of inceidence through the Snell's law. For normal incidence $\Delta = 2n_f t$.

The phase difference corresponding to the path difference Δ is: $\delta = k\Delta = 2\pi/\lambda$.

The net phase difference has to take into account the possible phase change by the reflection.

Δ_{P} optical path difference, Δ_{r} equivalent path difference arrising from the phase change.

Constructive interference: $\Delta_P + \Delta_r = m\lambda$ where $m = 0,1,2,...$

Destructive interference: $\Delta_P + \Delta_r = \left(m + 1/2\right)\lambda$ where $m = 0,1,2,...$

What is the value of Δ_{r} for dirrerent types of interface?

Reflected wave amplitude

Amplitude of the reflected electromagnetic wave at an interface

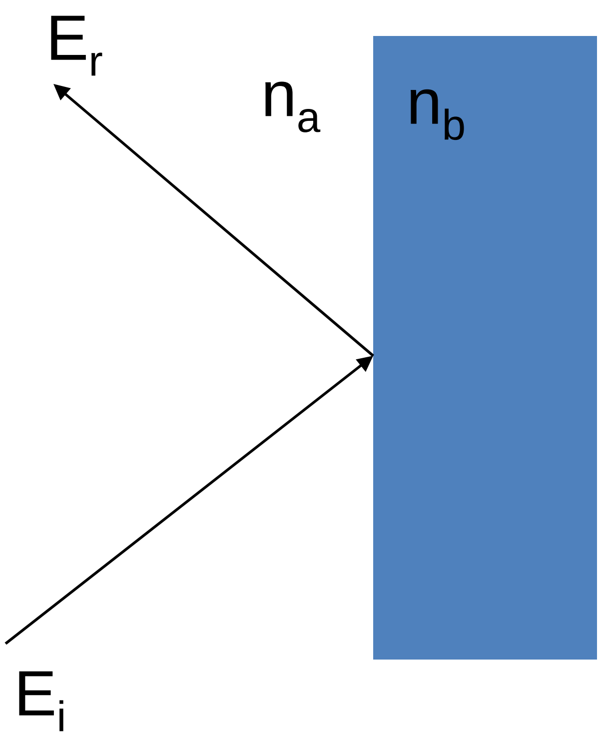
$$\boxed{\left|E_r\right| = \frac{n_a - n_b}{n_a + n_b}\left|E_i\right|}$$

Three cases are recognized:

1) If $\underline{n_a < n_b}$ $\left(\underline{\text{external}}\ \text{reflection}\right)$ then E_r and E_i have different signs so <u>the reflected wave has half-cycle phase difernce</u> with respect to the incident wave.

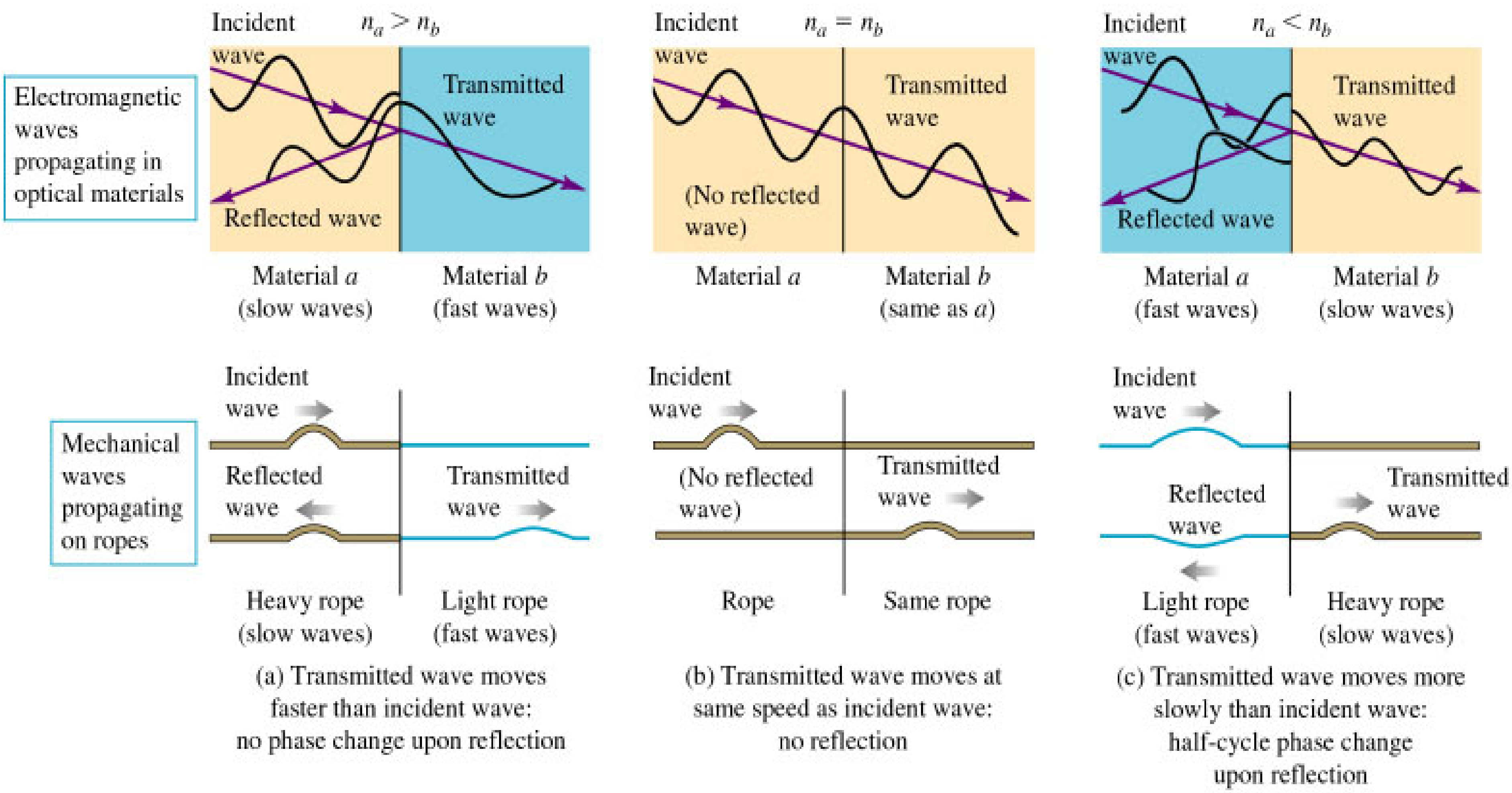
2) If $\underline{n_a > n_b}$ $\left(\underline{\text{internal}}\ \text{reflection}\right)$ then E_r and E_i have the same signs so <u>the reflected wave and incident wave are in-phase</u>.

3) If $\underline{n_a = n_b}$ then $E_r = 0$ <u>no reflection happens</u>.



Amplitude of reflected EM wave from an interface

$$|E_r| = \frac{n_a - n_b}{n_a + n_b}|E_i|$$



Condition for constructive and destructive interference by a thin film

No relative phase shift at interfaces :

Constructive: $\Delta_\mathrm{r} + \Delta_p = 0 + 2n_f t\cos\theta_t = m\lambda \rightarrow 2n_f t\cos\theta_t = m\lambda$

Destructive: $\Delta_\mathrm{r} + \Delta_p = 0 + 2n_f t\cos\theta_t = \left(m + \frac{1}{2}\right)\lambda$

$m = 0,\ 1,\ 2,\ 3, \ldots$

Half - cycle relative phase shift at interface :

Constructive: $\Delta_\mathrm{r} + \Delta_p = \frac{\lambda}{2} + 2n_f t\cos\theta_t = m\lambda \rightarrow \boxed{2n_f t\cos\theta_t = \left(m + \frac{1}{2}\right)\lambda}$

for normal incidence $2n_f t = \left(m + \frac{1}{2}\right)\lambda$

Destructive: $\Delta_\mathrm{r} + \Delta_p = \frac{\lambda}{2} + 2n_f t\cos\theta_t = \left(m + \frac{1}{2}\right)\lambda \rightarrow \boxed{2n_f t\cos\theta_t = m\lambda}$

for normal incidence $2n_f t = m\lambda$

$m = 0,\ 1,\ 2,\ 3, \ldots$

Antireflecting coating

For antireflecting coatings we want the reflected wave to vanish.

if $\Delta_{\mathrm{r}} + \Delta_p = \left(m + \frac{1}{2}\right)\lambda$ then the reflected wave is out of phase and desctructive interference will happen.

Usually $\mathrm{n_s} > n_f > n_a$ so both reflectons are external and no phase shift happens at the interfeces $\Delta_{\mathrm{r}} = 0.$

To minimize the film thickness and loss we take $m = 1$ and for one layer antireflection coating

$\Delta_p = 2t = \frac{\lambda}{2} \rightarrow \boxed{t = \frac{\lambda}{4}}$ this is the so-called quarter wavelength layer.

This works only for one wavelength. In order to make it for a brader spectrum designers use many layers.

An antireflection coating is perfect if amplitudes of the waves coming from both surfaces are equal.

To achieve that we need a special relationship between the indexes of the layers. The reflection coefficient of a surface is: $r = \frac{1 - n_2 / n_1}{1 + n_2 / n_1}$ the r at two interface have to be equal.

$$\left.\begin{array}{l} \text{At air-film interface } r_{a-f} = \dfrac{1 - n_f / n_0}{1 + n_f / n_0} \\ \text{At film-substrate interface } r_{f-s} = \dfrac{1 - n_s / n_f}{1 + n_s / n_f} \end{array}\right\} \rightarrow \underbrace{r_{a-f} = r_{f-s}}_{\text{For maximum extinction}} \rightarrow \frac{n_f}{n_0} = \frac{n_s}{n_f} \rightarrow \boxed{n_f^2 = n_s n_0} \text{ for } \underbrace{\mathrm{n_0} = 1}_{air} \rightarrow \boxed{n_f = \sqrt{n_s}}$$

Example: for the yellow-green $(550\mathrm{nm})$ that is eye's most sensitive range $n_f = 1.22$ the clasest material to this is index is $\mathrm{MgF_2}$ *with* $n = 1.38$.

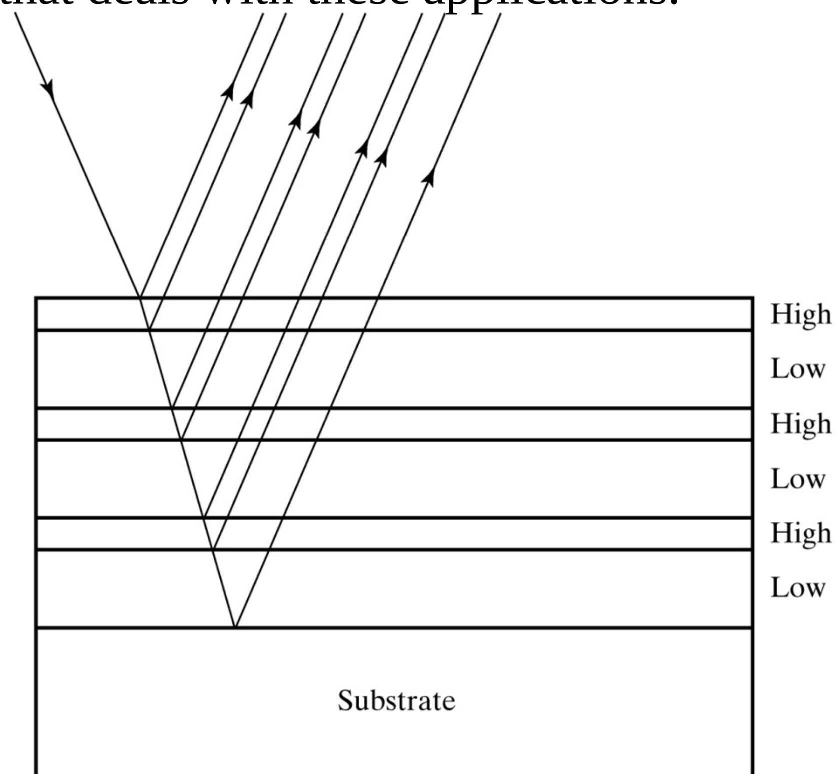
Dielectric mirrors, filters, laser mirrors

We can also design complete reflectors with alternating high-low $\lambda/4$ layers that create constructive interference for the reflected light (here we have phase shift).

Advantage of these mirrors is that they don't heat up (no absorption) and are perfect reflectors for a specific wavelength. Good for making laser resonators.

We can also design low-pass, high-pass, band-pass filters with very high precision.

Optical thin film design is a branch of optics that deals with these applications.

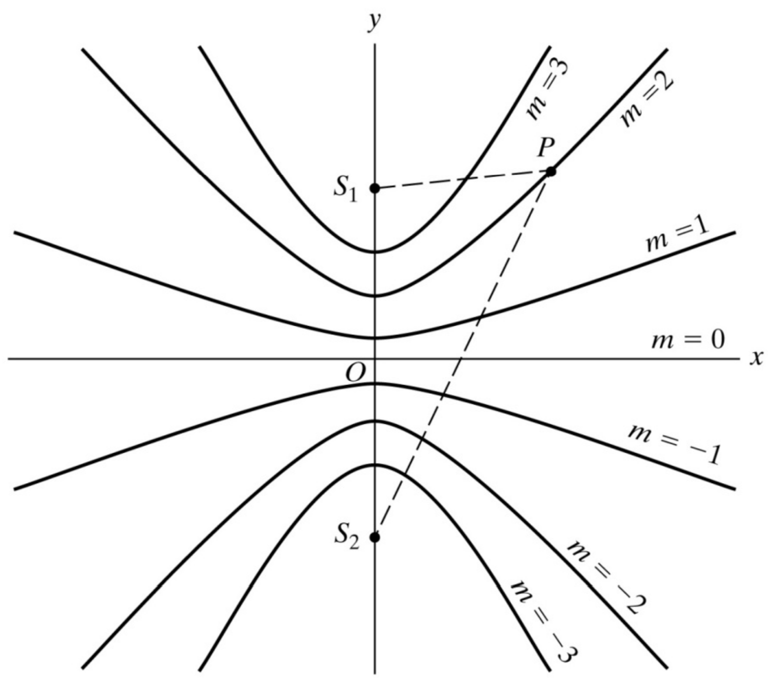


© 2007 Pearson Prentice Hall, Inc.

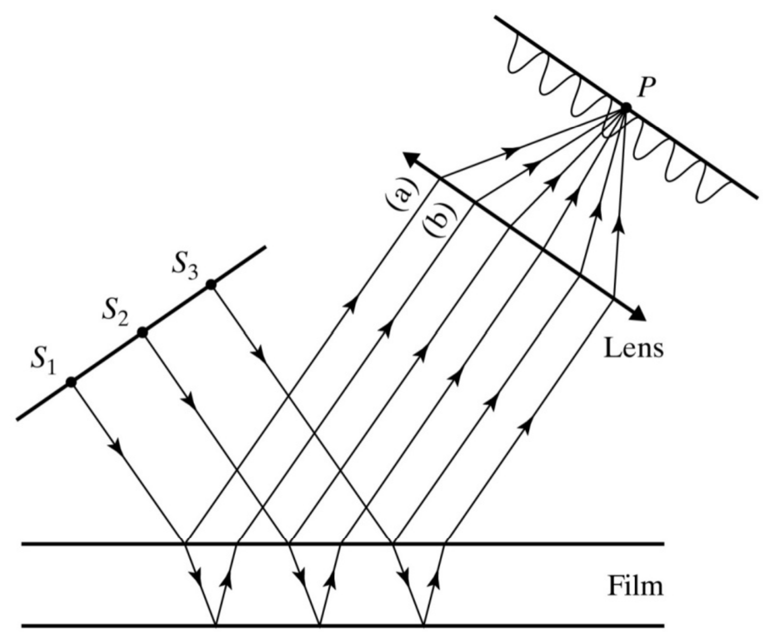
Localized and non-localized finger

Non-localized fringes form everywhere and we can place a screen on their path to see them. For example fringes from a double slit are observable at every distance from the sources.

Localized fringes form at a specific location in space and the screen has to be placed there to view them. For example fringes from a thin film are localized at infinity and we need a converging lens to project them on a screen. The fringes in the picture (on the right side) are fringes of equal inclination of Heidinger fringes formed by parallel rays not possible with point sources.

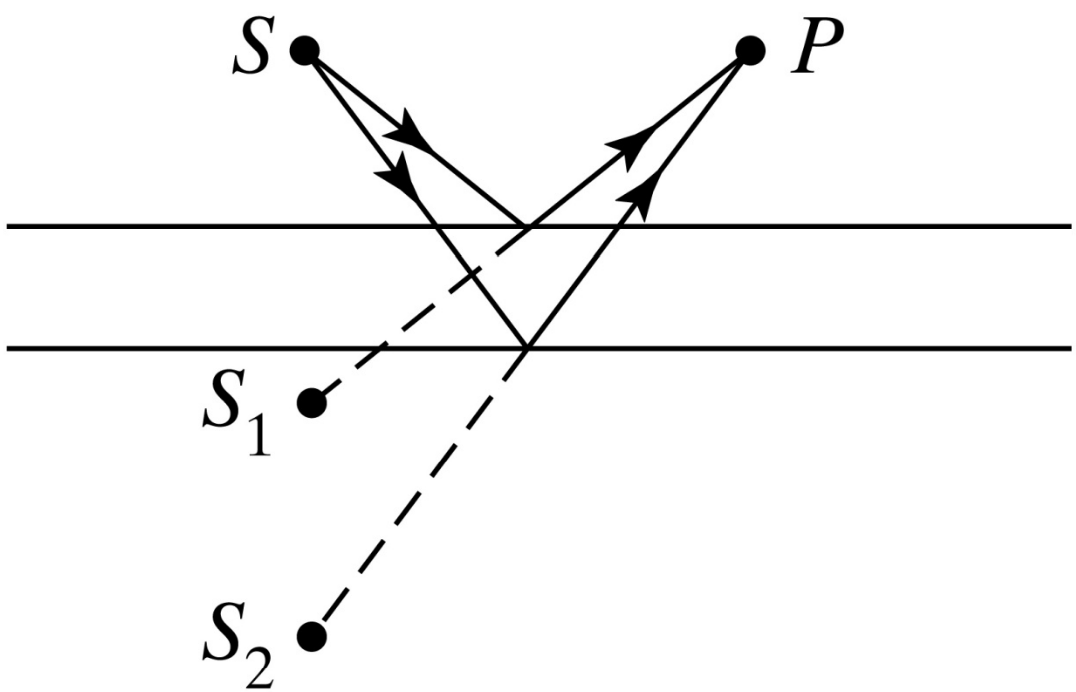


© 2007 Pearson Prentice Hall, Inc.



© 2007 Pearson Prentice Hall, Inc.

Generating non-localized real fringes with dielectric films using point sources

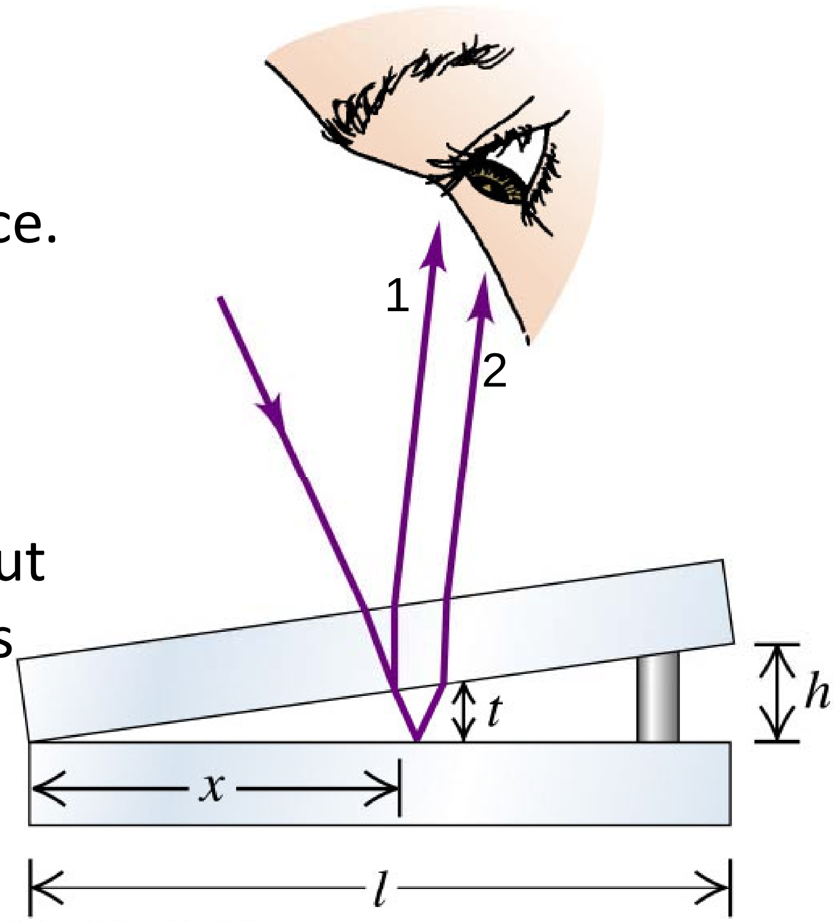


© 2007 Pearson Prentice Hall, Inc.

Interference at a wedge

- Path difference between two rays is almost equal to 2t
- Wave 1 has zero phase shift due to reflection from the glass-air interface.
- Wave 2 has 180^0 phase shift due to reflection from air-glass interface.
- That is why the left corner appears dark. Light path difference is zero but phase difference due to reflection is 180^0 or half a cycle.

$$\left|E_r\right| = \frac{n_a - n_b}{n_a + n_b}\left|E_i\right|$$



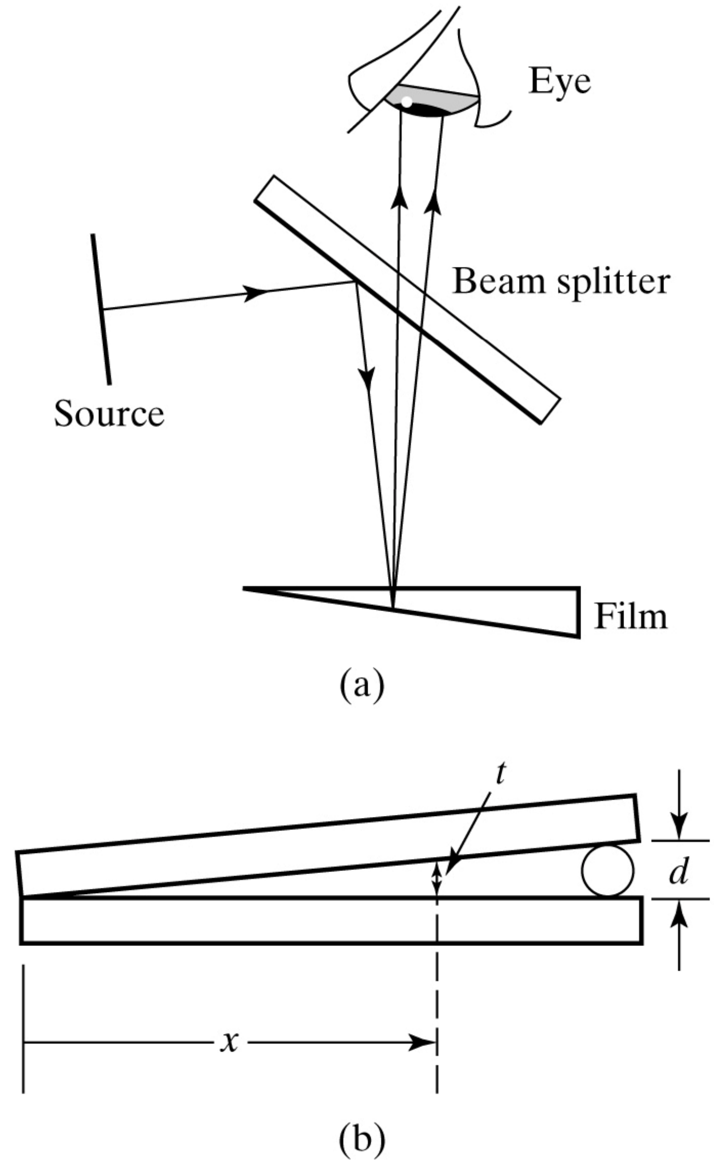
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley.

Fringes of equal thickness

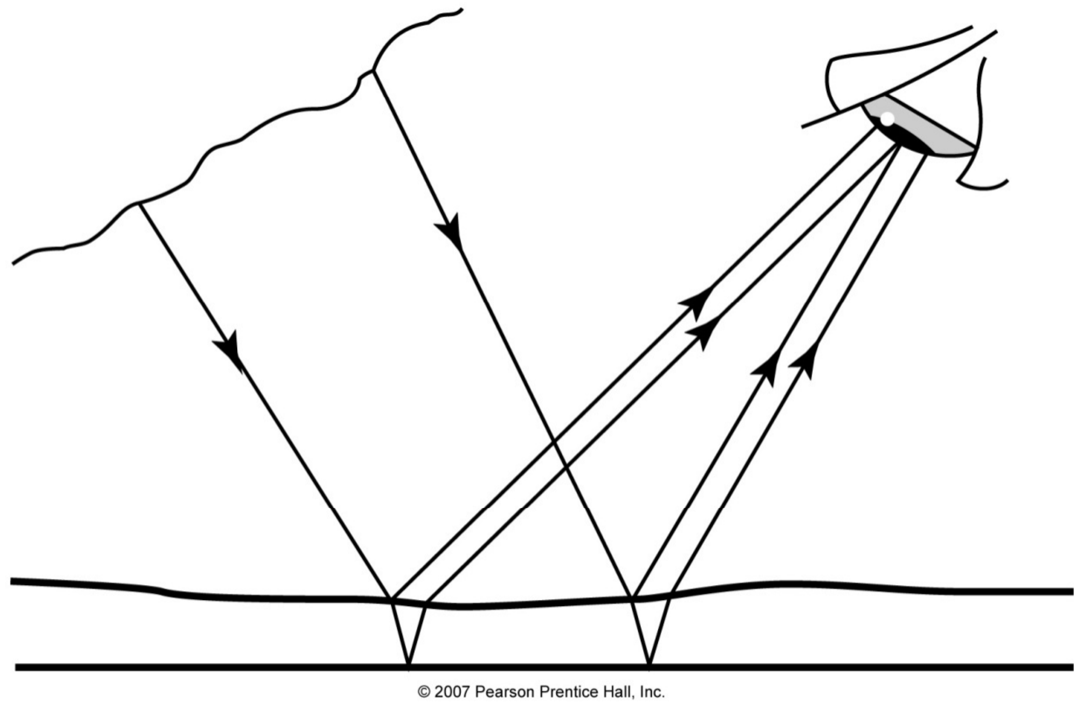
For a wedge with verying thickness the path difference $\Delta = 2n_f t\cos_t$ varies even if the angle of incidence is constant. For a fixed direction of incident light bright or dark fringes will be associated with a particulat thickness (fringes of equal thickness).

Fringes can be viewd with the arrangement in the figure and are called Fizeu Fringes.
At normal incidence $\cos\theta = 1$
and the light path difference is $\Delta_p = 2n_f t$

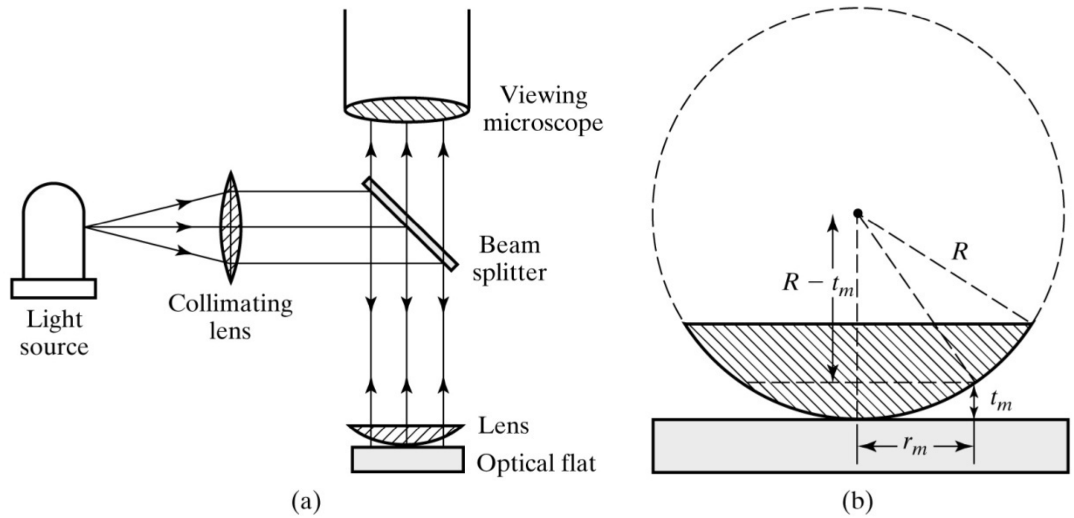
$$2n_f t + \Delta_r = \begin{cases} m\lambda & \text{bright} \\ \left(m + \frac{1}{2}\right)\lambda & \text{dark} \end{cases}$$



© 2007 Pearson Prentice Hall, Inc.

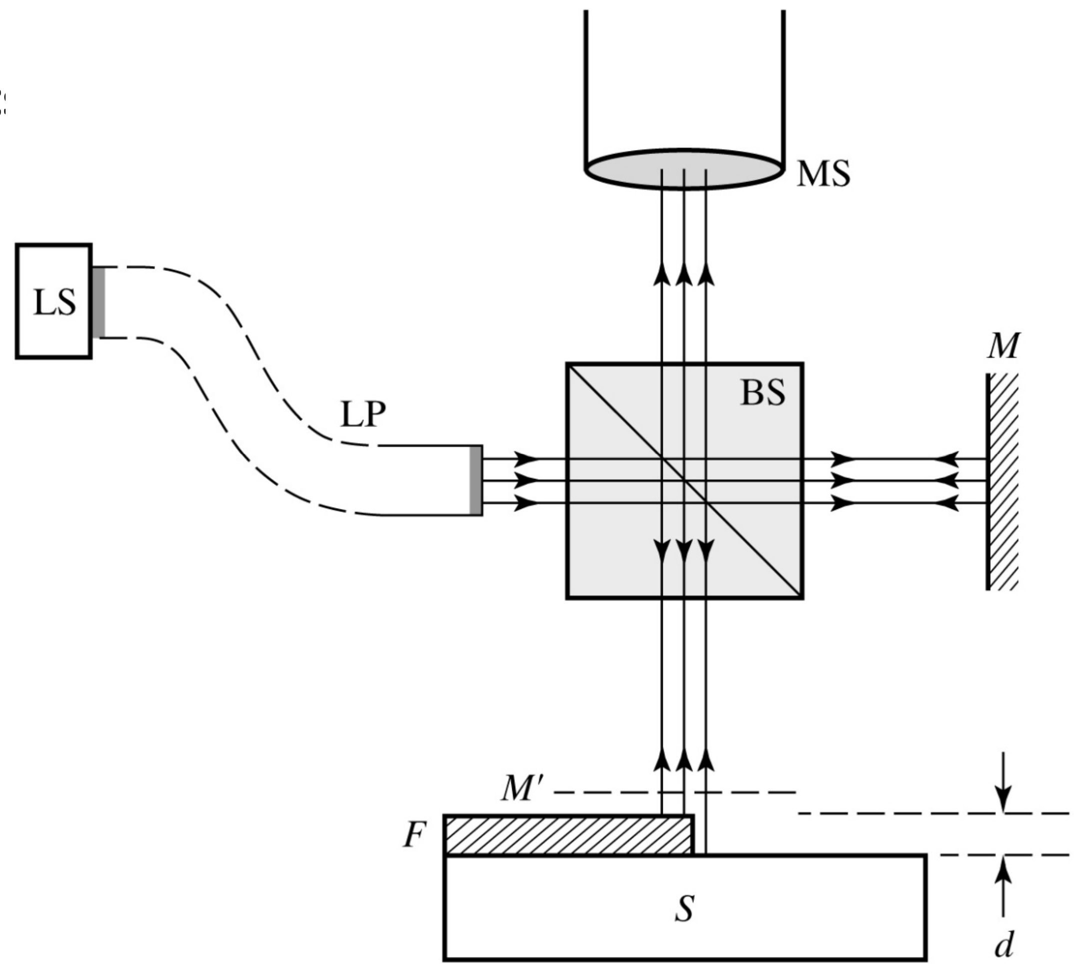

© 2007 Pearson Prentice Hall, Inc.

Newton rings

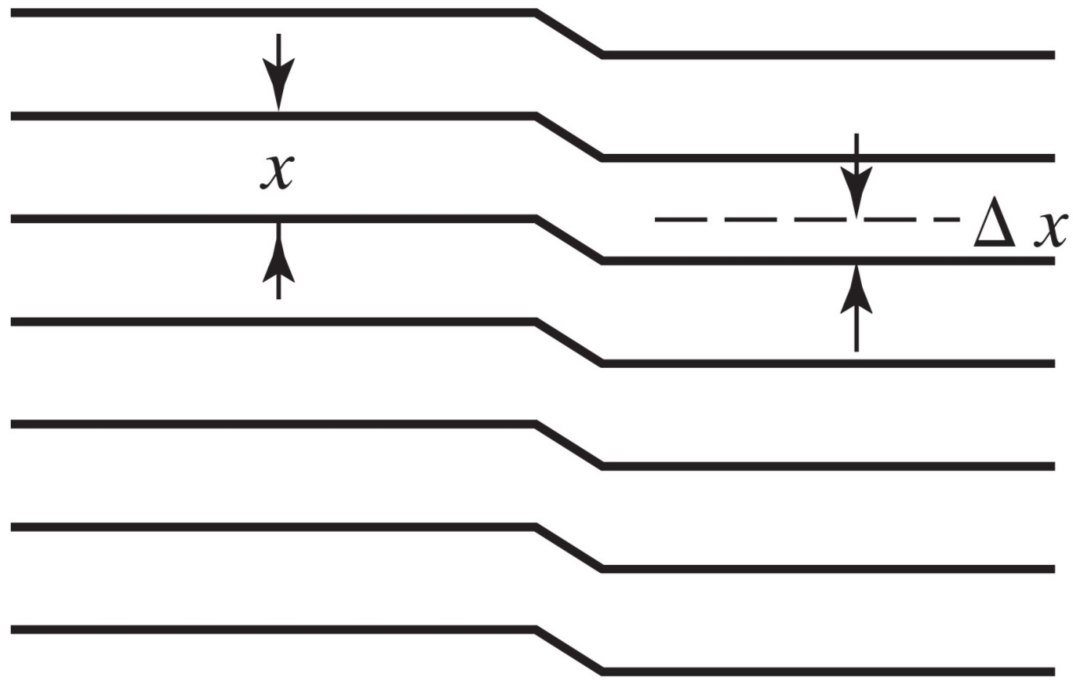


© 2007 Pearson Prentice Hall, Inc.

Film thickness measurements



© 2007 Pearson Prentice Hall, Inc.



(b)

© 2007 Pearson Prentice Hall, Inc.

Stokes relations

In reality many internal reflections happen in the films. The Stokes relations offer a formalism to track the rflection and transmssion coefficients for the electric fields arriving at an interface. Let

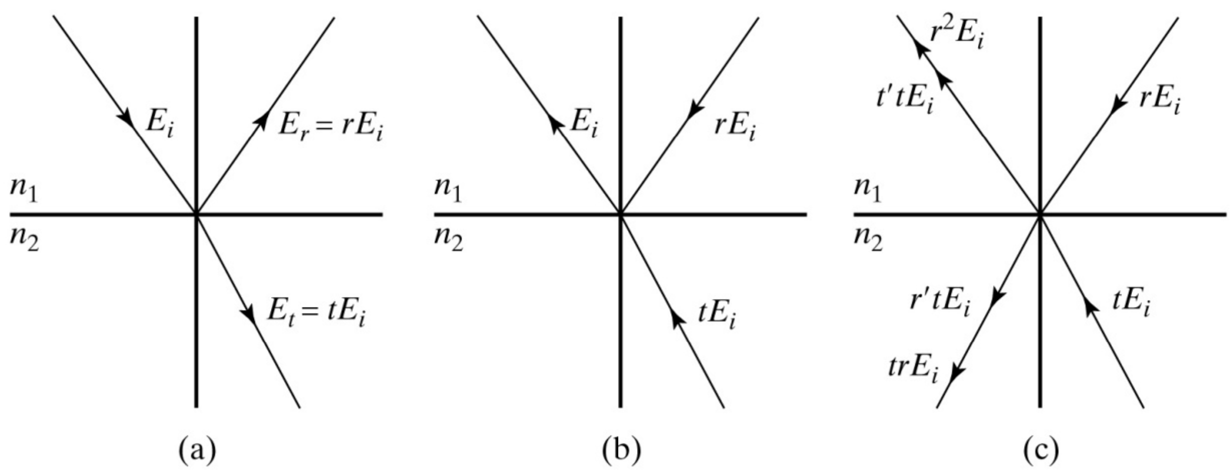
$\mathrm{E_i}$ = amplitude of the incident light, $\mathrm{E_t}$ = amplitude of the tranmitted light, $\mathrm{E_r}$ = amplitude of the reflected light

We define the reflection coefficient r as: $r = \dfrac{E_r}{E_i}$ and the transmission coefficient t as: $\mathrm{t} = \dfrac{E_t}{E_i}$

At the interface $\mathrm{E_i}$ is devided th to parts $\begin{cases} \mathrm{E_r} = rE_i \\ \mathrm{E_t} = tE_i \end{cases}$

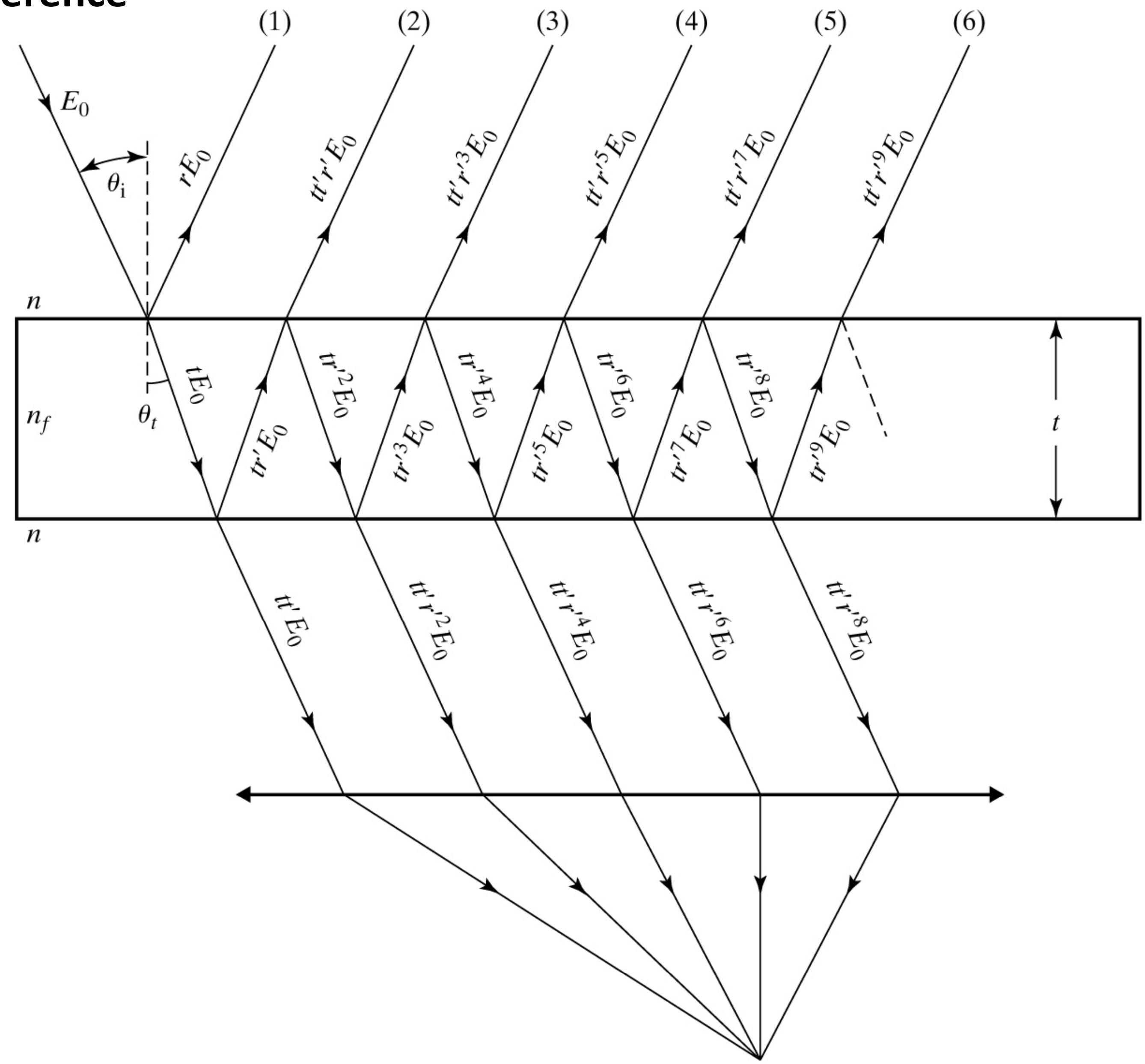
According to the principle of ray reversibility both cases in fig a and b are valid. When they happen together (figure c) we mark the reverse senario by primed parameters. But b and c are physically equivalent. So $\left.\begin{cases} \mathrm{E_i} = \left(\mathrm{r}^2 + tt'\right)E_i \rightarrow \boxed{tt' = 1 - r^2} \\ 0 = \left(r't + tr\right)E_i \rightarrow \boxed{r = -r'} \end{cases}\right\}$ Stokes relations between the amplitude coefficients for angles of incidence related through the Snell's law.

There is a π phase difference between the rays incident from each side.



© 2007 Pearson Prentice Hall, Inc.

Multiple beam interference



© 2007 Pearson Prentice Hall, Inc.

Multiple beam interference I

We now consider interference between a narrow beam of amplitude E_0 and angle of incidence θ_i and the multiple reflections from inside of a film of thickness t, index n_f surrounded by air.

We want to find superposition of the reflected waves from the top of the plate.

The phase difference between the successive reflected beams is: $\boxed{\delta = k\Delta = k2n_f t\cos\theta_t}$

Incident beam: $E_0 e^{i\omega t}$

$E_1 = (rE_0)e^{i\omega t}$

$E_2 = (tt'r'E_0)e^{i(\omega t-\delta)}$

$E_3 = (tt'r'^3 E_0)e^{i(\omega t-2\delta)}$

$E_4 = (tt'r'^5 E_0)e^{i(\omega t-3\delta)}$

$...E_N = (tt'r'^{(2N+1)} E_0)e^{i(\omega t-(N-1)\delta)}$ for N=2,3,...

this form does not hold for E_1 that never passes through the film.

$$E_R = \sum_{N=1}^{\infty} E_N = rE_0 e^{i\omega t} + \sum_{N=2}^{\infty} tt'E_0 r^{(2N-3)} e^{i[\omega t-(N-1)\delta]} = E_0 e^{i\omega t}\left(r + tt'e^{-i\delta}\sum_{N=2}^{\infty} r^{(2N-4)} e^{i(N-2)\delta}\right)$$

$\sum_{N=2}^{\infty} x^{N-2} = 1 + x + x^2 + ... = \frac{1}{1-x}$ where $x = r'^2 e^{-i\delta}$ and $|x| < 1$ the series converges

Multiple beam interference II

$$E_R = E_0 e^{i\omega t}\left(r + \frac{tt'r'e^{-i\delta}}{1-r'^2\, e^{-i\delta}} \right) \text{ using the Stokes relationships}$$

$$E_R = E_0 e^{i\omega t}\left(r - \frac{\left(1-r^2\right) r e^{-i\delta}}{1-r^2 e^{-i\delta}} \right) = E_0 e^{i\omega t}\left(\frac{r\left(1-e^{-i\delta}\right)}{1-r^2 e^{-i\delta}} \right)$$

$$I_R = \left|E_R\right|^2 = E_0^2 r^2 \left[\frac{e^{i\omega t}\left(1-e^{-i\delta}\right)}{1-r^2 e^{-i\delta}} \right]\left[\frac{e^{-i\omega t}\left(1-e^{i\delta}\right)}{1-r^2 e^{i\delta}} \right] \text{ using } 2\cos\delta = \mathrm{e}^{\mathrm{i}\delta} + e^{-i\delta} \text{ and } \frac{I_R}{I_i} = \frac{\left|E_R\right|^2}{\left|E_0\right|^2}$$

$$I_R = \left|E_R\right|^2 = E_0^2\, 2r^2 \left(\frac{\left(1-\cos\delta\right)}{1+r^4-2r^2\cos\delta} \right) \rightarrow \boxed{I_R = \left(\frac{2r^2\left(1-\cos\delta\right)}{1+r^4-2r^2\cos\delta} \right) I_i}$$

Using the conservation of energy equation $I_i = I_R + I_T$ we find

$$I_T = \left|E_T\right|^2 = I_i - I_R = I_i - \left(\frac{2r^2\left(1-\cos\delta\right)}{1+r^4-2r^2\cos\delta} \right) I_i = \left(\frac{1+r^4-2r^2\cos\delta - 2r^2\left(1-\cos\delta\right)}{1+r^4-2r^2\cos\delta} \right) I_i$$

$$\boxed{I_T = \left(\frac{\left(1-r^2\right)^2}{1+r^4-2r^2\cos\delta} \right) I_i}$$

Multiple beam interference III

$$I_R = \left(\frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta} \right) I_i \text{ and } I_T = \left(\frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta} \right) I_i$$

A minimum for reflected irradiance occurs when $(1-\cos\delta)=0 \to \cos\delta = 1 \to \delta = 2\pi m$

$\boxed{\Delta = m\lambda = 2n_f t\cos\theta_t}$ condition for minimum reflectance

This also provides the condition for maximum transmission $I_T = \left(\frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta} \right) I_i \xrightarrow{\cos\delta=1} I_T = I_i$

All the secondary and higher order reflections are in-phase with each other and they all have π phase difference with the first reflection. So they all cancel out with the first reflection.

$\left|\frac{E_2}{E_1}\right| = \left|\frac{tt'r'E_0}{rE_0}\right| = 1-r^2$ for interface of air and glass $n_1 = 1,\ n_2 = 1.5,\ r^2 = 0.04$ and 96% of the light is cancelled in the first reflection so ignoring the higher order reflections is perfectly justified.

When $\cos\delta = -1$ the reflection maximum occurs.

$\delta = \left(m+\frac{1}{2}\right)\pi \to \boxed{\Delta = \left(m+\frac{1}{2}\right)\lambda = 2n_f t\cos\theta_t}$ condition for maximum reflectance

and in that case $I_R = \left(\frac{4r^2}{(1+r^2)^2} \right) I_i$ and $I_T = \left(\frac{1-r^2}{1+r^2} \right)^2 I_i$