Chapter 8
Optical Interferometry

Lecture Notes for Modern Optics based on
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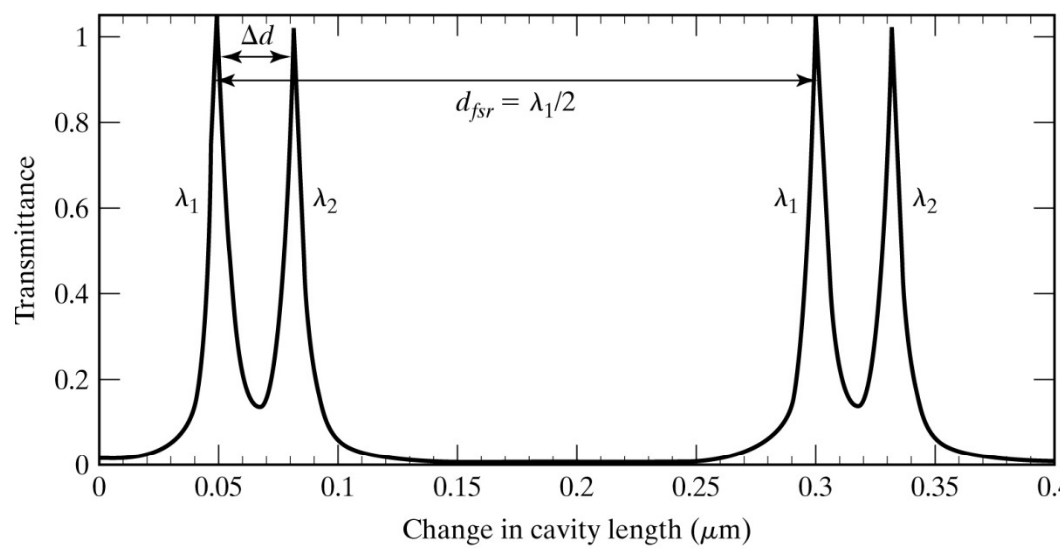
Optical interferometry

Interferometer is an instrument design to exploit the interference of light and fringe patterns that results from the optical path difference .

Interferometers are extended to acoustic and radio waves as well.

Here we will explore two kinds

1) Michelson-Morley Interferometer, a two-beam, amplitude division device.
2) Fabry-Perot interferometer a multiple beam, amplitude division device



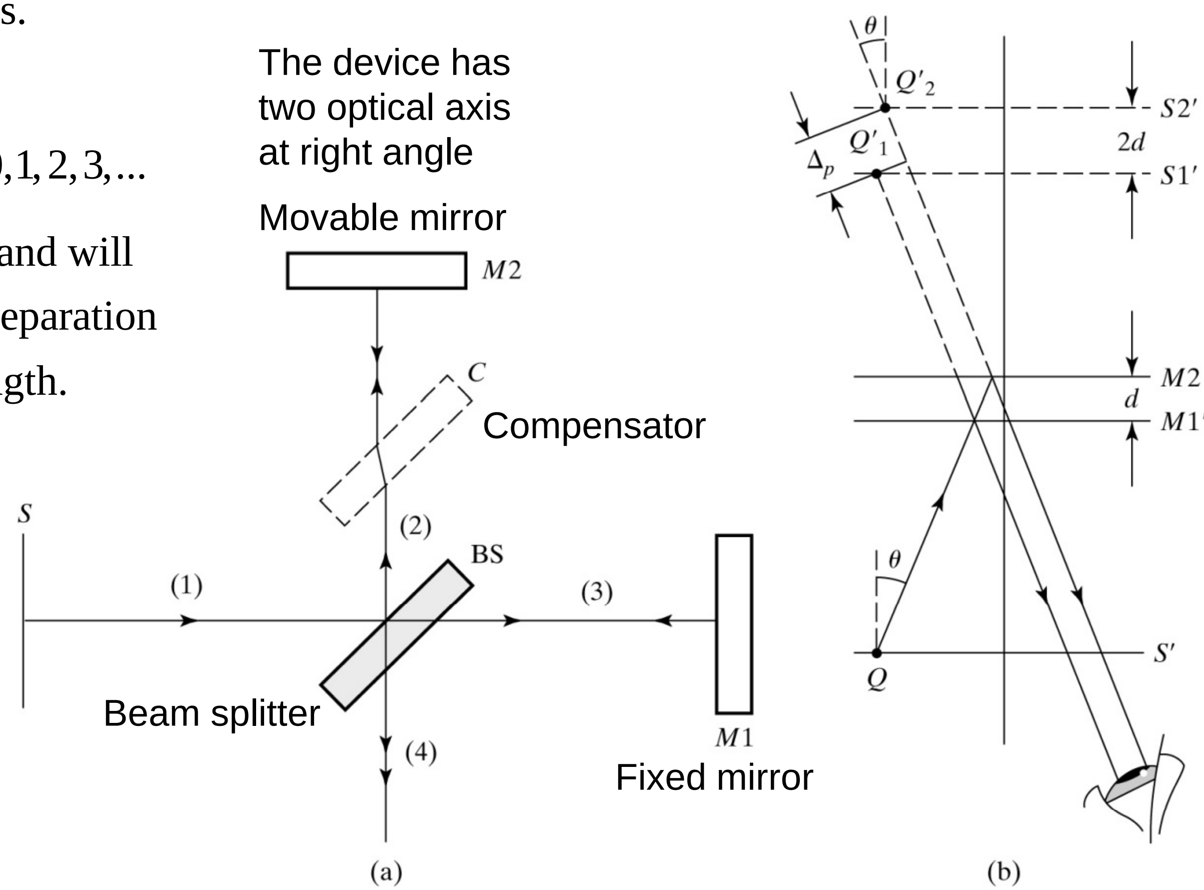
The Michelson interferometer (1881)

Used in applications such as proving special theory of relativity, measuring hyperfine structure in line spectra, tidal effect of the moon on the earth. Principle of operation is shown in the graph. In the graph of the interferometer the path difference between the beams traveling along the two perpendicular paths is: $\boxed{\Delta_{\mathrm{p}} = 2d\cos\theta}$ (see the fig. b) where θ measures the inclination of the beam with respect to the optical axis.

For normal beam $\theta = 0$

$$\Delta_{\mathrm{p}} = 2d = m\lambda \;\rightarrow d = m\frac{\lambda}{2} \text{ for } m = 0,1,2,3,...$$

will form constructive interference and will repeat every $\lambda/2$ so long as $d < L_c$ separation stays smaller than the coherence length.

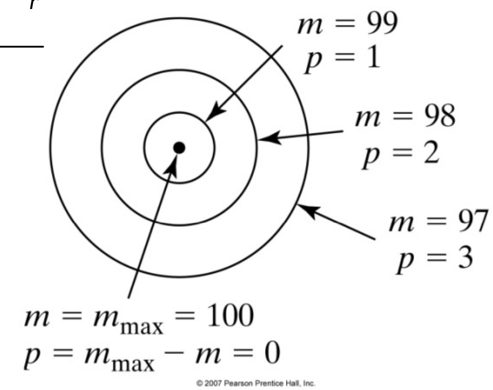


Fringe analysis of Michelson interferometer

Now the optical system is equivalent of the plane air film and the fringes of equal inclination are formed and can be viewed by looking through the path 4 or using a telescope in that path. Fringes are made up of concentric circles centered around the optical axis with intensity of

$I=4I_0 \cos^2\left(\frac{\delta}{2}\right)$ where the phase difference is $\delta = k\Delta = \left(\frac{2\pi}{\lambda}\right)\Delta$ and $\Delta = \Delta_p + \Delta_r$

Here we have a relative π phase shift between the two beams because the reflection coefficient from two sides of the mirror differs by $-1 = e^{i\pi}$



$$\boxed{\begin{array}{l}\Delta = \Delta_p + \Delta_r = 2d\cos\theta + \frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda \\ 2d\cos\theta = m\lambda,\ m = 0,1,2,\ldots \text{ for dark fringes.}\end{array}}$$

The center fringe is dark. For the normal rays at the center $2d = m\lambda$ and $\boxed{m_{max} = \frac{2d}{\lambda}}$ is order of central dark fringe and outer fringes have lower orders. We invert the fringe order for convenience by interoducing another integer $p = m_{max} - m = \frac{2d}{\lambda} - m \rightarrow p\lambda = 2d - m\lambda$

$\boxed{p\lambda = 2d\left(1-\cos\theta\right)}$ where p=0,1,2,3,.. dark fringes

Now the central fring's order is zero.

When optical path difference is decreasing the fringes move inward and disappear at the center.

When optical path difference is increasing the fringes originate at the center and move outward.

Fringe counting with Michelson interferometer
Wavelength and distance measurement

When optical path difference is decreasing the fringes move inward and disappear at the center.

When optical path difference is increasing the fringes originate at the center and move outward

A smaller mirror spaceing leads to an increase in angular separation $\Delta\theta$ for a given fringe interval.

$2d\cos\theta = m\lambda,\ m = 0,1,2,\ldots$ for dark fringes.

$$\underbrace{2d\sin\theta\Delta\theta = \lambda\Delta m}_{\text{Differenciate } \theta \text{ with respect to m}} \rightarrow \boxed{\left|\Delta\theta\right| = \frac{\lambda\Delta m}{2d\sin\theta}}$$

this means for a smaller optical path difference the fringes are more widely separated.

When $d = \lambda/2$ $\left(\text{standing waves form}\right)$ then $\cos\theta = m$ $\rightarrow$ the entire field encompasses one fringe or $m_{max} = 1$.

For a mirror translation Δd the number of fringes passing the center ($\cos\theta = 1$ and $m = \dfrac{2d}{\lambda}$) is

$\rightarrow \boxed{\Delta m = \dfrac{2\Delta d}{\lambda}}$ in all of these senarios λ is constant and we are playing with finge number and d.

This suggests an experimental method to measure small mirror movements with a known λ or measure λ when Δd is known.

Applications of the Michelson interferometer

1) Measurement of thin film thickness

2) Measurement of index of rfraction of a gass filled in a cell of length L placed in one arm of the interferometer. The fringe count Δm is done as the gas is evacuated fromt the cell. The optical path difference is: $\Delta d = nL - L = L\left(n-1\right) = \frac{\lambda}{2}\Delta m \rightarrow \boxed{n = \frac{\lambda}{2L}\Delta m + 1}$

3) Measurement of temperature of a gas or index change as a function of temperature. You need to know $T = f\left(n\right)$ for the material to use it.

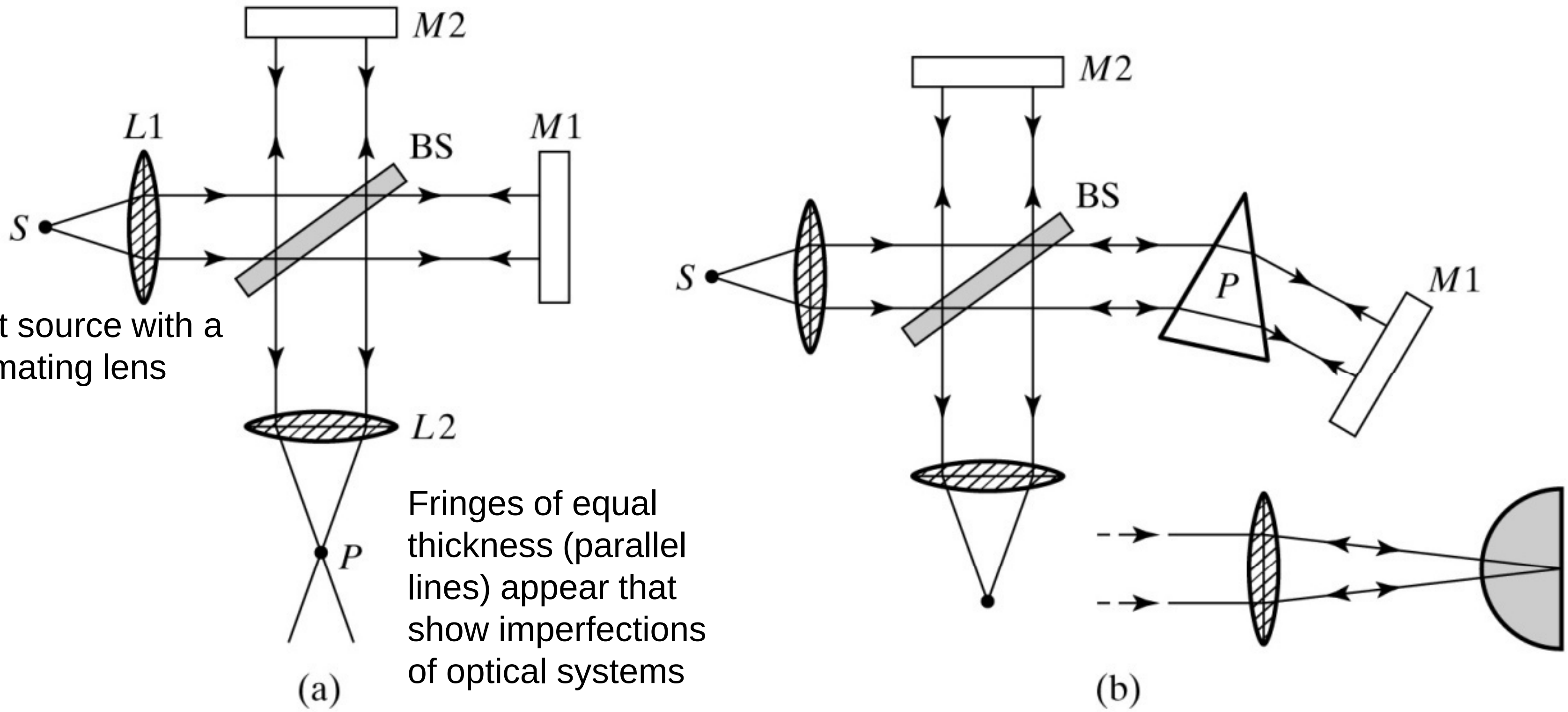
4) Determination of the wavelength difference between two closely spaced spectral lines λ and λ' or ν and ν' used in the lab to measure the difference between the wavelengths of the sodium yellow lines. Two kinds of fringes can be observed with the Michelson interferometer.

1) when the mirrors are absolutely perpendicular to each other virtual fringes of equal inclination are observed.

2) When the mirors are not quite perpendicular (small deviation), an air wedge appears between the mirrors and fringes of equal thickness may form localized at the mirrors that appear as straight lines parallel to the intersection of the two mirrors. For large deviations the fringes appear as hyperbolic curves.

Variations of the Michelson InterferometerTwyman-Green

Testing of a prism and lens for variations in their index of refraction or surface imperfections.

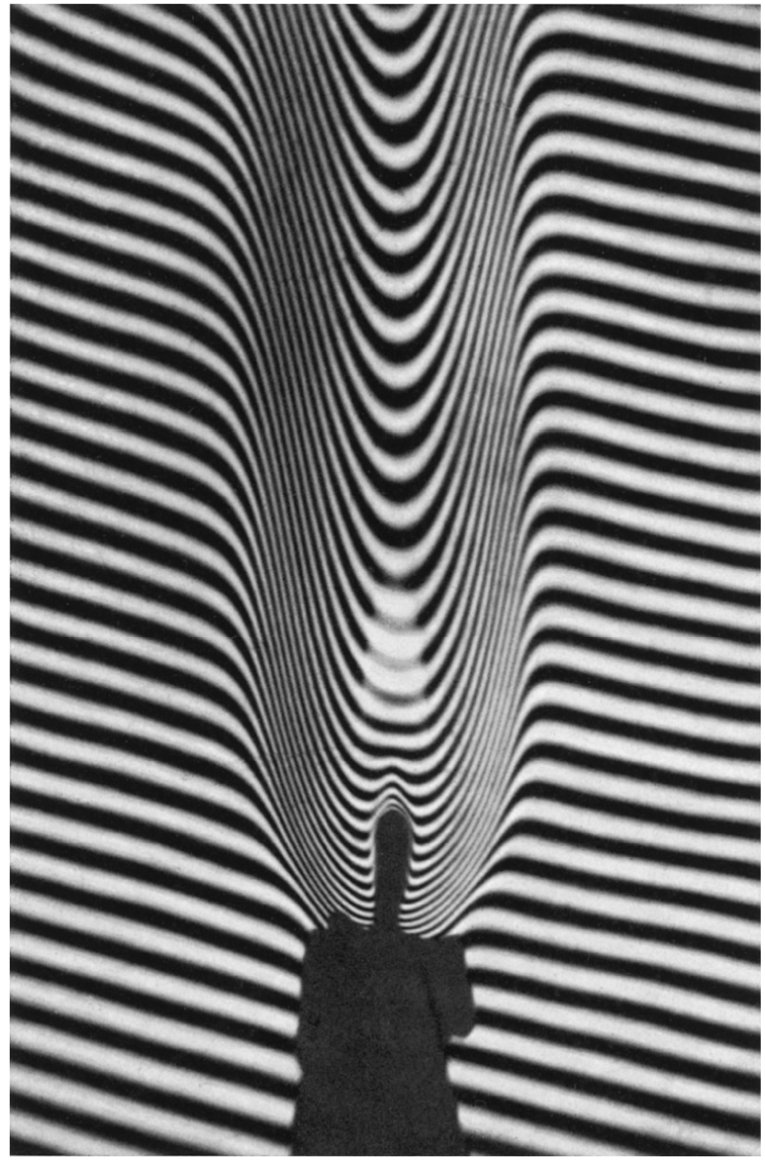


Point source with a collimating lens

Fringes of equal thickness (parallel lines) appear that show imperfections of optical systems

Fringes boost interferometer resolution http://optics.org/cws/article/research/38395
A recent article on increasing resolution of the measurements by interferometry

Fringes of equal thickness in neighborhood of a candle by Michelson Interferometer

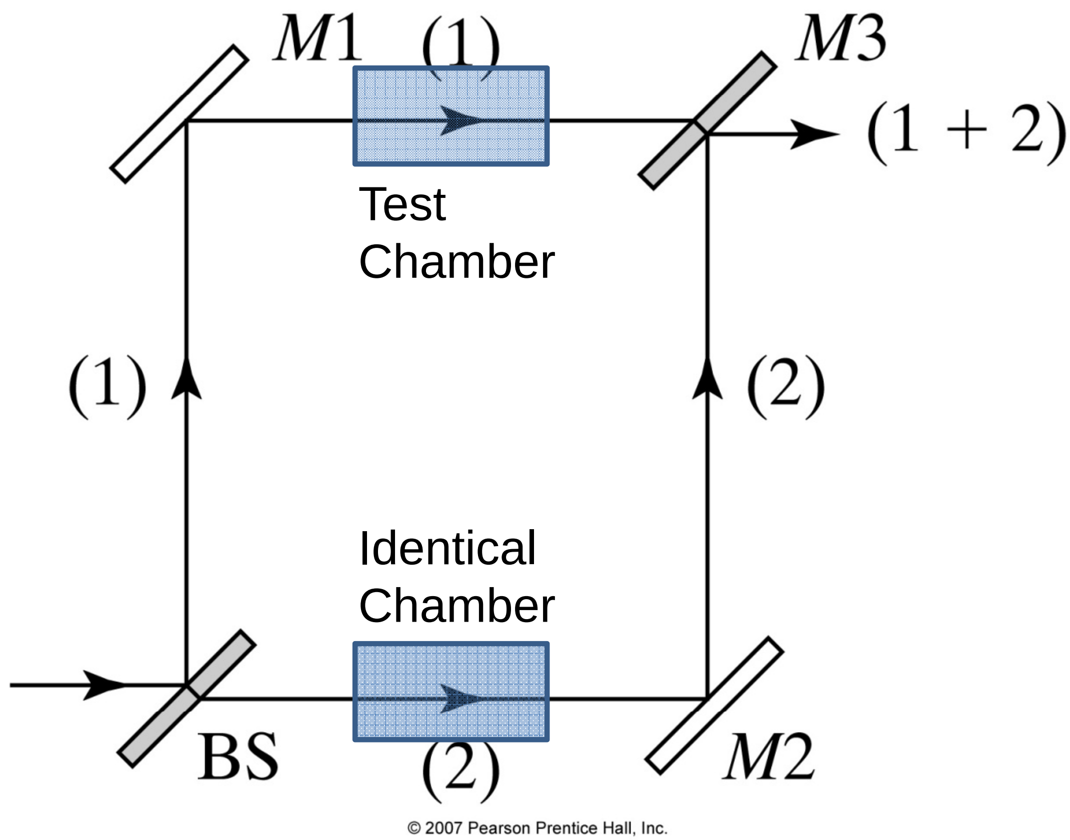


Mach-Zehnder Interferometer

Used in aerodynamic research.

It uses two beam and amplitude splitting.

Advantage over Michelson interferometer is possibility of making fringes at the object of study so that both can be photographed together.



The Fabry-Perot Interferometer

The fabry-Perot interferometer is composed of two semi-transparent parallel plates. Very simple in structure but a very precise measurement tool. The interference pattern is composed of superposition of the multiple beams of transmitted and reflected light.

Applications: precision wavelength measurements, hyperfine spectral structures, .measuring refractive indices of gasses, calibration of the standard meter in terms of wavelength.

The interferometer is composed of the inner surfaces of the cavity mirrors that usually polished to $\lambda/50$ and coated with silver or aluminium layer ~50nm. Outer surfaces are cut at small angle so that reflection from them does not interfere. When the mirror spacing is fixed the cavity is called etalon.

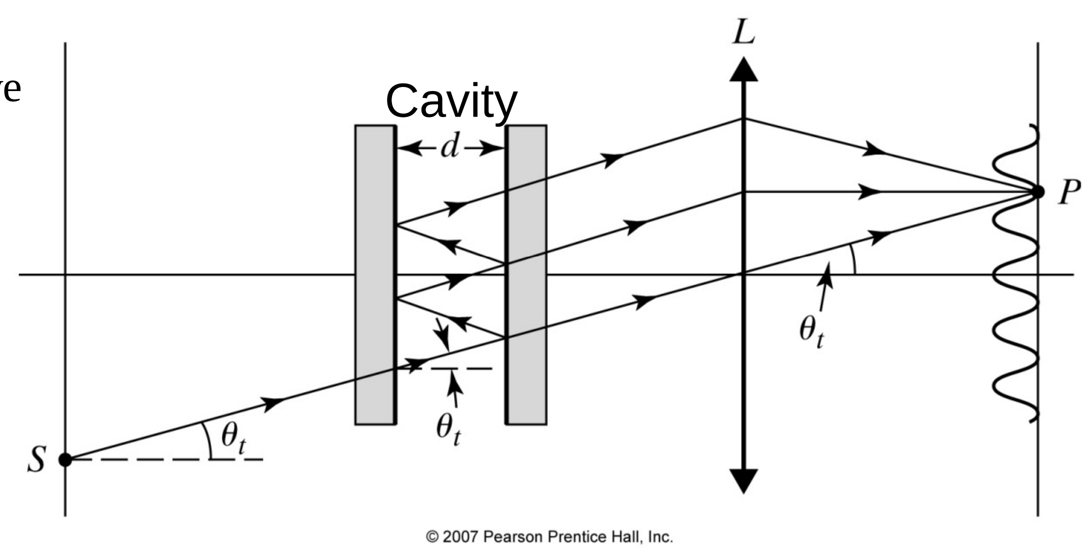
The beam from source S generates multiple cohernt beams in the interferometer the emerging rays are brought together at P. The path difference between the succesive beams is $\boxed{\Delta_p = 2n_f d\cos\theta_i, \text{ for air } n_f = 1}$

$\Delta_r = 2\pi$ or no effect on the interference.

The condition for bright fringes is

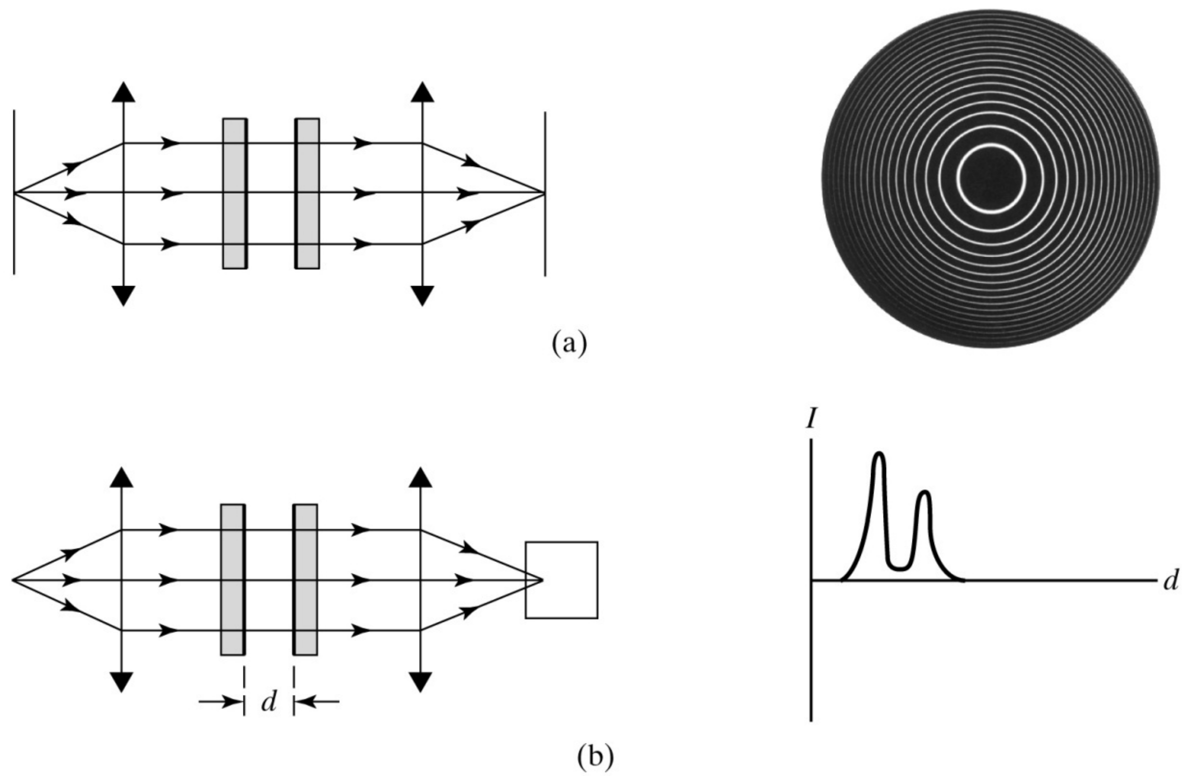
$\boxed{2d\cos\theta_i = m\lambda}$

The fringe pattern, concentric rings, are the fringes of equal inclination.



Different arrangements of the Fabry-Perot interferometer

a) Extended source and a fixed plate spacing. Circular fringe pattern.
b) Point source and a variable plate spacing. A detector records intensity as a function of the plate spacing. With a laser sources the lenses are not needed.



Fabry-Perot transmission: the Airy function

Goal: calculating the irradiance transmitted through a Fabry-Perot interferometer. Two identical mirrors separated by a distance d that have real reflection and transmission coefficients $r,\ t$ and no absorption so $\boxed{r^2+t^2=1}$. Cavity roundtrip-time $\boxed{\tau = 2d/V = 2nd/c}$. We want to express the transmitted electric field E_T in terms of incident electric field E_I.

Propagation factor is the ratio of a taveling monochromatic plane wave with electric field $E(z,t)$ to $E(z_0,t_0)$

$P_F = \dfrac{E(z_0+\Delta z, t_0+\Delta t)}{E(z_0,t_0)}$ For the traveling plane monochromatic wave with no change in optical media

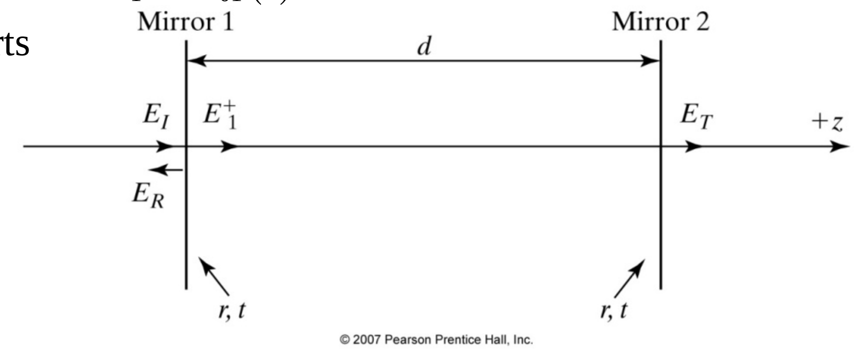
$$P_F(\Delta z,\Delta t) = \frac{E(z_0+\Delta z, t_0+\Delta t)}{E(z_0,t_0)} = \frac{E_0 e^{i[\omega(t_0+\Delta t)-k(z_0+\Delta z)]}}{E_0 e^{i(\omega t_0 - kz_0)}} = e^{i(\omega\Delta t\text{-}k\Delta z)}$$

Right-going electric field incident on cavity from the left $E_I = E_{0I}e^{i\omega t}$

Amplitude (time-dependent) of intercavity right-going electric field $E_1^+ = E_{01}^+(t)e^{i\omega t}$

At time $t+\tau$ the right-going intercavity field is sum of two parts

$$E_1^+(t+\tau) = \underbrace{tE_I(t+\tau)}_{\text{Incident field transmitted through mirror 1}} + \underbrace{r^2 P_F(\Delta z = 2d, \Delta t = \tau)E_1^+(t)}_{\text{The right-going intercavity field that existed at mirror one cavity round-trip time } \tau \text{ earlier}}$$



$$E_{01}^+(t+\tau)e^{i\omega(t+\tau)} = tE_{0I}e^{i\omega(t+\tau)} + r^2E_{01}^+(t)e^{i\omega t}e^{i(\omega\tau\text{-}2kd)}$$

After some time the intercavity field will settle to a steady-state value then $E_{01}^+(t+\tau) = E_{01}^+(t) = E_{01}^+$ and $E_{01}^+e^{i\omega(t+\tau)} = tE_{0I}e^{i\omega(t+\tau)} + r^2E_{01}^+e^{i\omega t}e^{i(\omega\tau\text{-}2kd)}$

$\boxed{E_{01}^+ = \dfrac{t}{1-r^2e^{-i\delta}}E_{0I}}$ where $\boxed{\delta = 2kd = 2\dfrac{2\pi}{\lambda}d}$ is the round-trip phase shift.

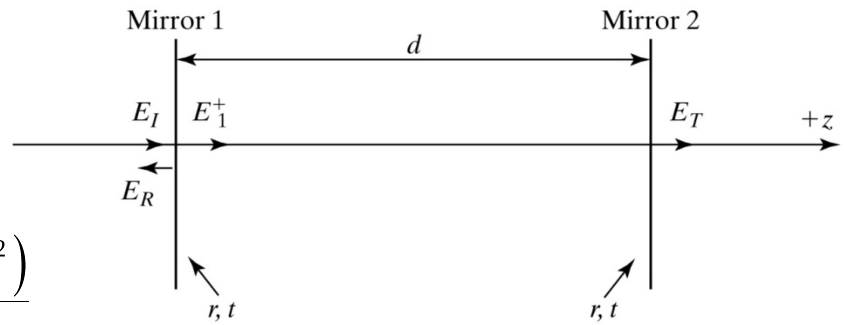
Fabry-Perot transmission: the Airy function

Goal: calculating the irradiance transmitted through a Fabry-Perot interferometer.

$\boxed{E_{01}^{+} = \frac{t}{1-r^2e^{-i\delta}}E_{0I}}$ where $\boxed{\delta = 2kd}$ is the round-trip phase shift.

$$E_T\left(t+\tau/2\right) = E_{0T}e^{i\omega(t+\tau/2)} = \underbrace{tP_F\left(\Delta z = d, \Delta t = \tau/2\right)E_1^{+}\left(t\right)}_{\substack{\text{Portion of } E_{01}^{+}(t) \text{ that is transmitted through} \\ \text{the mirror 2 after propagating the length of the} \\ \text{cavity in half round-trip time or } \tau/2}} = te^{i\left(\omega\tau/2-\delta/2\right)}E_{01}^{+}e^{i\omega t}$$

$$E_T\left(t+\tau/2\right) = \frac{t^2e^{-i\delta/2}}{1-r^2e^{-i\delta}}e^{i\omega t}E_{0I} \rightarrow E_{0T} = \frac{t^2e^{-i\delta/2}}{1-r^2e^{-i\delta}}E_{0I}$$



$\boxed{\text{Irradiance } I_T \propto E_{0T}E_{0T}^{*}}$

$$\boxed{\text{Transmitance } T = \frac{I_T}{I_I}} = \frac{E_{0T}E_{0T}^{*}}{E_{0I}E_{0I}^{*}} = \frac{\dfrac{\left(E_{0I}t^2e^{-i\delta/2}\right)\left(E_{0I}^{*}t^2e^{+i\delta/2}\right)}{\left(1-r^2e^{-i\delta}\right)\left(1-r^2e^{+i\delta}\right)}}{E_{0I}E_{0I}^{*}}$$

$$\boxed{T = \frac{t^4}{1+r^4-2r^2\cos\delta} = \frac{\left(1-r^2\right)^2}{1+r^4-2r^2\cos\delta} = I_T\left(\text{Transmittance of a parallel plate from chapter 7}\right)}$$

The Airy functoin $\boxed{T = \frac{1}{1+\frac{4r^2}{\left(1-r^2\right)^2}\sin^2\frac{\delta}{2}}}$ where we used $\cos\delta = 1-2\sin^2\frac{\delta}{2}$

Coefficient of Finesse

Fabry called $\boxed{F(r) = \frac{4r^2}{(1-r^2)^2}}$ the coefficient of finesse. whith that the Airy function can be expresses as

The Airy functoin: $\boxed{T = \frac{1}{1 + F\sin^2\frac{\delta}{2}}}$ $\begin{cases} T_{\max} = 1 \text{ for } \sin(\delta/2) = 0 \\ T_{\min} = 1/(1+F) \text{ for } \sin(\delta/2) = \pm 1 \end{cases}$

Coefficient of finesse is a very sensitive function of the reflection coefficient r. For $0 < r < 1 \rightarrow \ 0 < F < \infty$

Fringe contrast $= \frac{I_{T,\max} - I_{T,\min}}{I_{T,\min}} = \frac{T_{\max} - T_{\min}}{T_{\min}} = \frac{1 - 1/(1+F)}{1/(1+F)} = F$ the larger the fringe contrast the better the cavity.

$\boxed{\text{Fringe contrast} = F = \text{Coefficient of finesse}}$

Note the difference between fringe visibility,

and fringe contrast $\frac{I_{T,\max} - I_{T,\min}}{I_{T,\min}}$

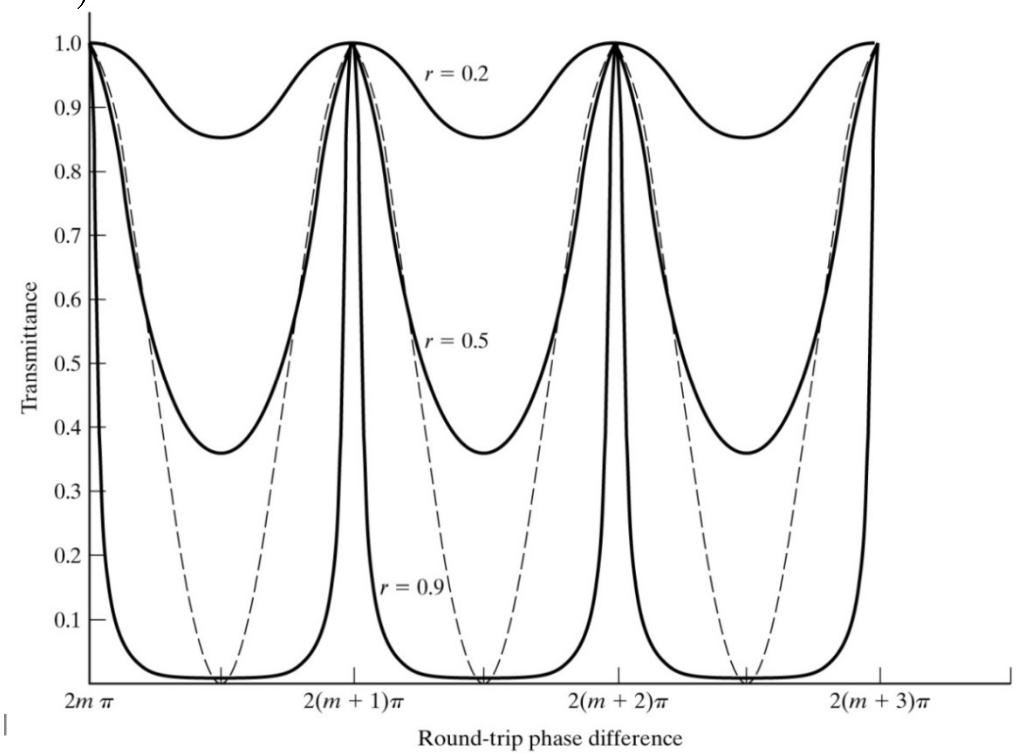
Regardless of r, $T_{\max} = 1$ *is at* $\delta = m(2\pi)$

and $T_{\min} = 1/(1+F)$ at $\delta = (m + 1/2)2\pi$

and $\lim_{r\to 1} T_{\min} = 0$ is never zero but appoaches it

as r $\rightarrow 1$. The fringes become very sharp compared

to the Michelson interferometer.



Finesse

Finesse of a cavity: $\mathscr{F} = \dfrac{\pi\sqrt{F}}{2} = \dfrac{\pi r}{1-r^2}$ is different than the coerfficient of finesse: $F = \dfrac{4r^2}{\left(1-r^2\right)^2} = \dfrac{4\mathscr{F}^2}{\pi^2}$

Goal: showing that the finesse $\mathscr{F} = \dfrac{\text{separation between the transmitance peaks}}{\text{Full width at half maximum of the peaks}} = \dfrac{FSR}{FWHM}$

Transmittance of a cavity $T = \dfrac{1}{1+F\sin^2\left(\delta/2\right)} = \dfrac{1}{1+\left(4\mathscr{F}^2/\pi^2\right)\sin^2\left(\delta/2\right)}$

Free spectral range $\left(\text{FSR}\right)$ of a cavity: the phase separation between the adjacent transmittance peaks.

$\delta_{\text{fsr}} = \delta_{m+1} - \delta_m = \left(m+1\right)2\pi - 2\pi m = 2\pi$

Half width at half maximum $\left(\text{HWHM}\right)$ $\delta_{1/2}$ of the transmittance peak can be found by setting:

$$T = \frac{1}{1+\left(4\mathscr{F}^2/\pi^2\right)\sin^2\left(\delta/2\right)} = \frac{1}{2} \rightarrow \left(\frac{4\mathscr{F}^2}{\pi^2}\right)\sin^2\left(\frac{\delta}{2}\right) = 1 \rightarrow \sin^2\left(\frac{\delta}{2}\right) = \left(\frac{\pi^2}{4\mathscr{F}^2}\right)$$

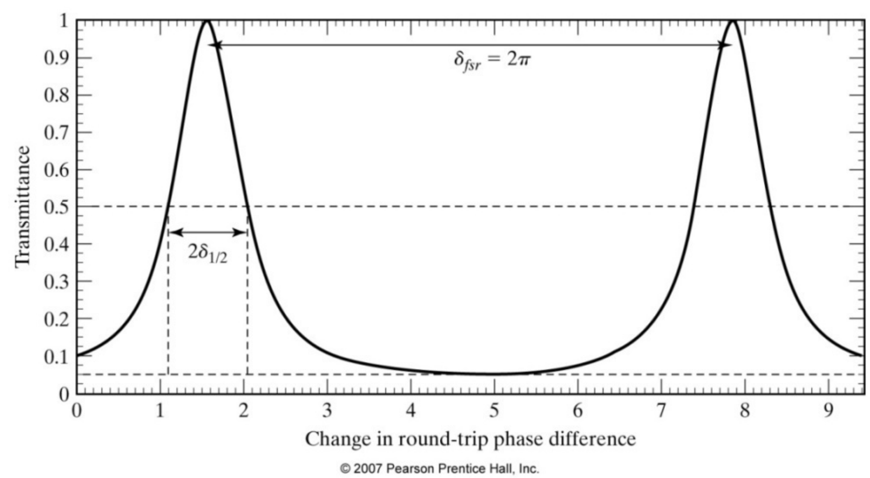
Phase of a maximum is $2m\pi$ and half-maximum just after of a maximum has a phase of $\delta = 2m\pi + \delta_{1/2}$

$$\sin^2\left(m\pi + \frac{\delta_{1/2}}{2}\right) = \sin^2\left(\frac{\delta_{1/2}}{2}\right) \simeq \left(\frac{\delta_{1/2}}{2}\right)^2 = \left(\frac{\pi}{2\mathscr{F}}\right)^2$$

For the narrow transmission peaks $\delta_{1/2}$ is small.

This approximation is good for cavities with high reflectivity mirrors the finesse is high and the transmittance peaks are narrow.

$$\boxed{\delta_{1/2} = \pi/\mathscr{F}} \rightarrow \boxed{\mathscr{F} = \frac{\pi}{\delta_{1/2}} = \frac{2\pi}{2\delta_{1/2}} = \frac{\delta_{fsr}}{FWHM}}$$



Thus finesse of a cavity is the ratio of FSR and FWHM of the cavity transmittance peaks.

Finesse as a figure of merit

Coerfficient of finesse: $F = \dfrac{4r^2}{\left(1-r^2\right)^2} = \dfrac{4\mathscr{F}^2}{\pi^2}$

The figure of merit finesse: $\boxed{\mathscr{F} = \dfrac{\pi\sqrt{F}}{2} = \dfrac{\pi r}{1-r^2} = \dfrac{\delta_{fsr}}{FWHM} = \dfrac{2\pi}{2\delta_{1/2}}}$

The transmittance is function of round-trip phase shift δ which in-turn is a function of

$$\left.\begin{array}{l} d,\ \text{Mirror spacing (cavity length)} \\ \nu,\ \text{Frequency} \\ n,\ \text{Index of refraction of the inter-mirror material} \end{array}\right\}\text{One of these are varied in different modes of operation.}$$

As a result FWHM and FSR change by varying the independent variable. But their ratio, finesse of the cavity, stays constant. That is why finesse is a good figure of merrit to identify a cavity.

Finesse only depends on the reflectivity of the mirrors $\begin{cases} \lim\limits_{r\to 0} \mathscr{F} = 0 \\ \lim\limits_{r\to 1} \mathscr{F} = \infty \end{cases}$

FSR $\left(\delta = 2\pi\right)$ for different modes of operation corresponds to variations in different parameters:

d_{fsr} is the free spectral range of a variable length Fabry-Perot interferometer.

ν_{fsr} is the free spectral range of a variable input frequency Fabry-Perot interferometer.

$\left|\lambda_{\mathrm{fsr}}\right| = \dfrac{c}{\lambda^2}\nu_{\mathrm{fsr}}$ is the free spectral range of a variable input wavelength Fabry-Perot interferometer.

Scanning Fabry-Perot Interferometer

The cavity length can be changed by moving one of the mirrors of a Fabry-Perot interferometer.

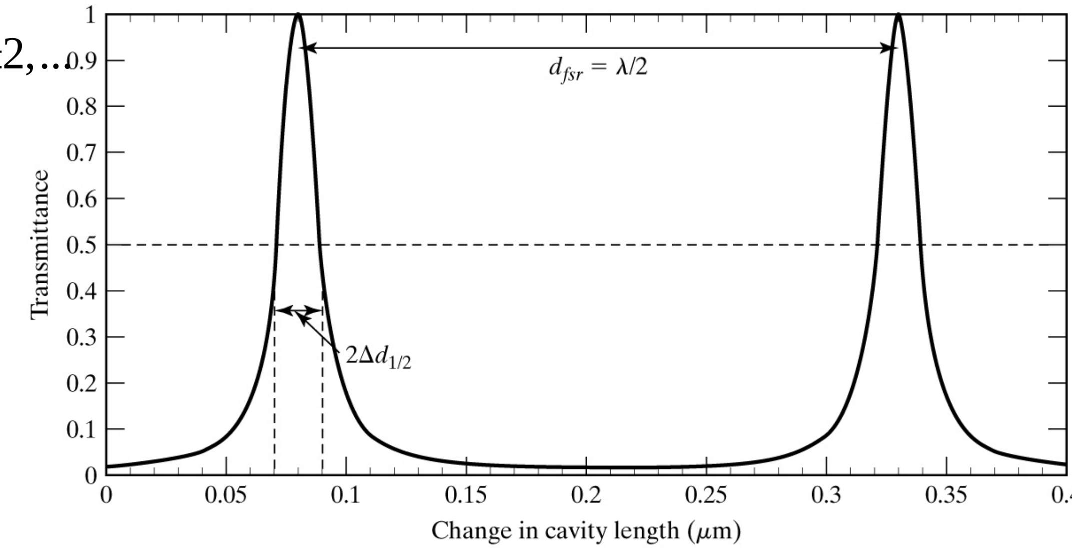
This can be used to measure wavelength of a monochromatic source.

At maximum transmission T_{max} the phase difference between the interfering waves is:

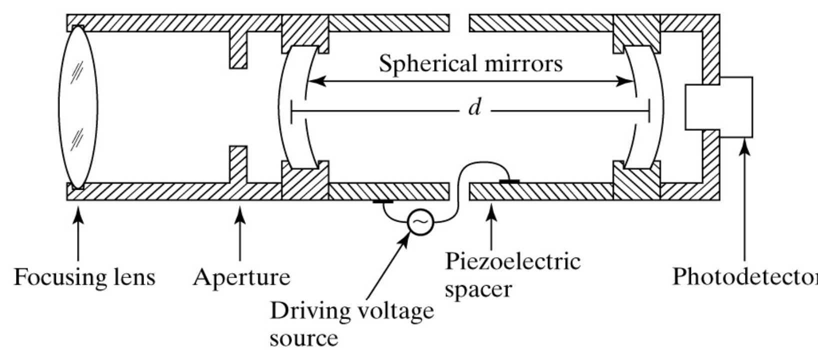
$$\delta = \underbrace{2kd}_{\text{Round-trip phase gain}} = 2\frac{2\pi}{\lambda}d = 2m\pi \quad m = 0, \pm 1, \pm 2, \ldots$$

$$d_{\text{m}} = m\frac{\lambda}{2} \quad \text{and} \quad \boxed{d_{fsr} = d_{m+1} - d_m = \frac{\lambda}{2}}$$

For more accurate measurements this is not good because the best actuators (piezoelectric) have have $\sim 10nm$ accuracy.



(a)



(b)

Scanning Fabry-Perot Interferometer

We can send two wery closely spaced wavelengths λ_1 and λ_2 simultaneously into a Fabry-Perot cavity.

Nominal length of the cavity $d = 5cm$

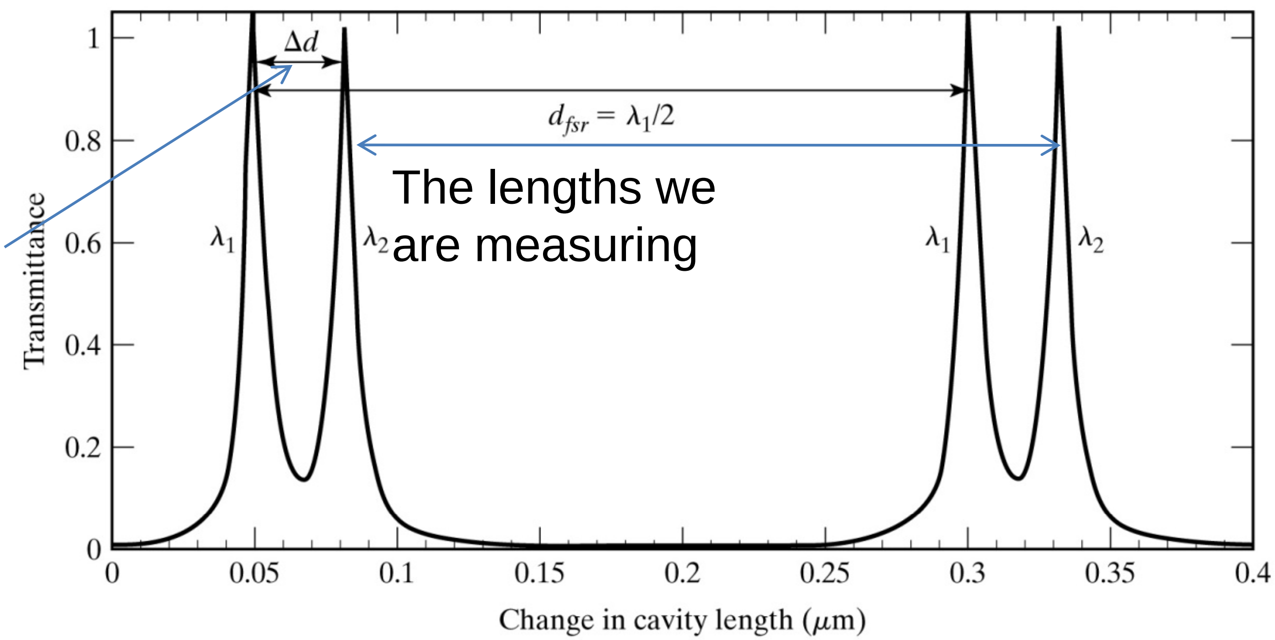
Nominal wavelength of the cavity λ=500 nm is very close to the λ_1 and λ_2 and by slight adjustment of the cavity length we get the FSR of both wavelengths. We have to make sure both transmission peaks correspond to the same mode order m then we can write:

$$\left.\begin{aligned}\lambda_1 &= 2d_1/m\\ \lambda_2 &= 2d_2/m\end{aligned}\right\}\lambda_2-\lambda_1=\Delta\lambda=\frac{2}{m}\left(d_2-d_1\right)=\frac{2}{\left(2d_1/\lambda_1\right)}\Delta d\rightarrow\frac{\Delta\lambda}{\lambda_1}=\frac{\Delta d}{d_2}$$

It is hard to know absoloute wavelength and length of the cavity but we can replace the λ_1 and d_1 with the nominal values λ and d and conclude:

$$\boxed{\frac{\Delta\lambda}{\lambda}=\frac{\Delta d}{d}}$$

The lengths we are detecting



Resolving power of a Fabry-Perot cavity

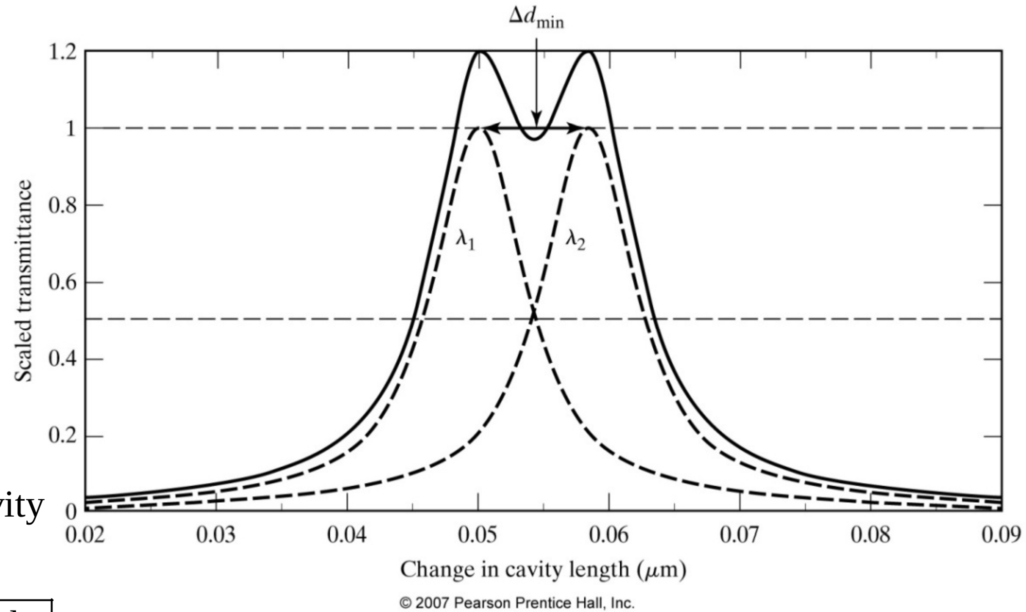
The minimum wavelength difference that can be determined by a cavity $\Delta\lambda_{\min}$ is limitted by the width of the transmittance peaks of the two λs. Minimum resolvable between the cavity lengths, $\Delta d_{\min} = FWHM$ of the peaks.

Resolving criteria is $\Delta d \geq 2\Delta d_{1/2} \equiv \Delta d_{\min}$

$$\mathscr{F} = \frac{\delta_{fsr}}{2\delta_{1/2}} = \frac{kd_{fsr}}{2k\Delta d_{1/2}} = \frac{d_{fsr}}{2\Delta d_{1/2}} \rightarrow 2\Delta d_{1/2} = \frac{d_{fsr}}{\mathscr{F}} = \frac{\lambda}{2\mathscr{F}}$$

$$\left.\begin{aligned} \frac{\Delta\lambda}{\lambda} &= \frac{\Delta d}{d} \\ \Delta d_{\min} &= 2\Delta d_{1/2} \end{aligned}\right\} \frac{\Delta\lambda_{\min}}{\lambda} = \frac{\Delta d_{\min}}{d} = \frac{2\Delta d_{1/2}}{d}$$

$$\frac{\Delta\lambda_{\min}}{\lambda} = \frac{\lambda}{2d\mathscr{F}}$$



$$\boxed{\mathscr{R} = \frac{\lambda}{\Delta\lambda_{\min}} = \frac{2d\mathscr{F}}{\lambda} = m\mathscr{F}}$$ Resolving power of a FP cavity

Where $m = \frac{2d}{\lambda}$ is the $\boxed{\text{mode number associated with the}}$ nominal λ and d of the cavity. Large resolving power corrsponds to:

1) higher modes that means large mirror spacing $d = m\frac{\lambda}{2}$ (can't change λ much)

2) large finesse that is achieved by r as close to 1 as possible $\mathscr{F} = \frac{\pi r}{1-r^2}$

Good Fabry -Perot interferometers have resolving powers as high as $\sim 10^6$ about 2 order of magnitude better than prisms and gratings.

Resolving power of a Fabry-Perot cavity (maximum)

The maximum wavelength difference that can be determined by a cavity $\Delta\lambda_{\max}$ is limitted by the overlap of the mth order transmittance peak of λ_1 and $m+1$th order transmittance peak of λ_2.

$$\Delta d = m\frac{\lambda_2}{2} - m\frac{\lambda_1}{2} = m\frac{\Delta\lambda}{2} \quad \text{between } \lambda_1 \text{ and } \lambda_2$$

$$d_{fsr} = (m+1)\frac{\lambda_1}{2} - m\frac{\lambda_1}{2} = \frac{\lambda_1}{2} \quad \text{between } \lambda_1 \text{ and } \lambda_1$$

Difference in cavity length associated with adjacent paeks has to be equal to the FSR of the variable length interferometer

$$\Delta d = d_{fsr} \rightarrow m\frac{\Delta\lambda}{2} = \frac{\lambda_1}{2} \rightarrow \boxed{\Delta\lambda_{\max} \approx \lambda_1 / m} \rightarrow \frac{\Delta\lambda_{\max}}{\Delta\lambda_{\min}} = \frac{\lambda / m}{\lambda / (m\mathcal{F})} = \mathcal{F}$$

$\boxed{\mathcal{F} = \frac{\Delta\lambda_{\max}}{\Delta\lambda_{\min}}}$ Another good feature of the finesse: high finesse cavities have ability to resolve very small wavelength differncescan over a large wavelength range.

The $\Delta\lambda_{\max}$ is also known as wavelength FSR:

$$\boxed{\Delta\lambda_{\max} = \lambda_{fsr}}$$

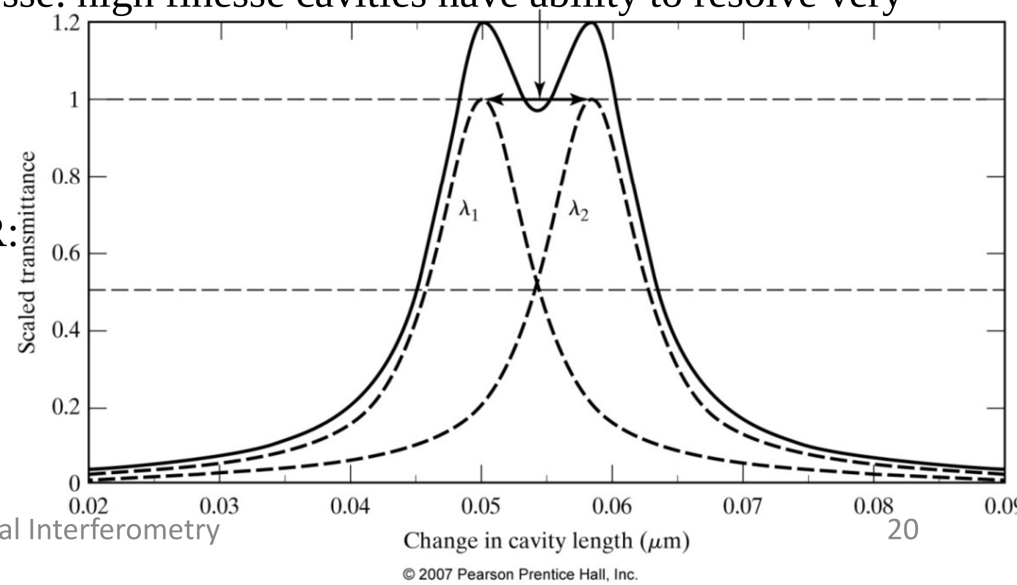


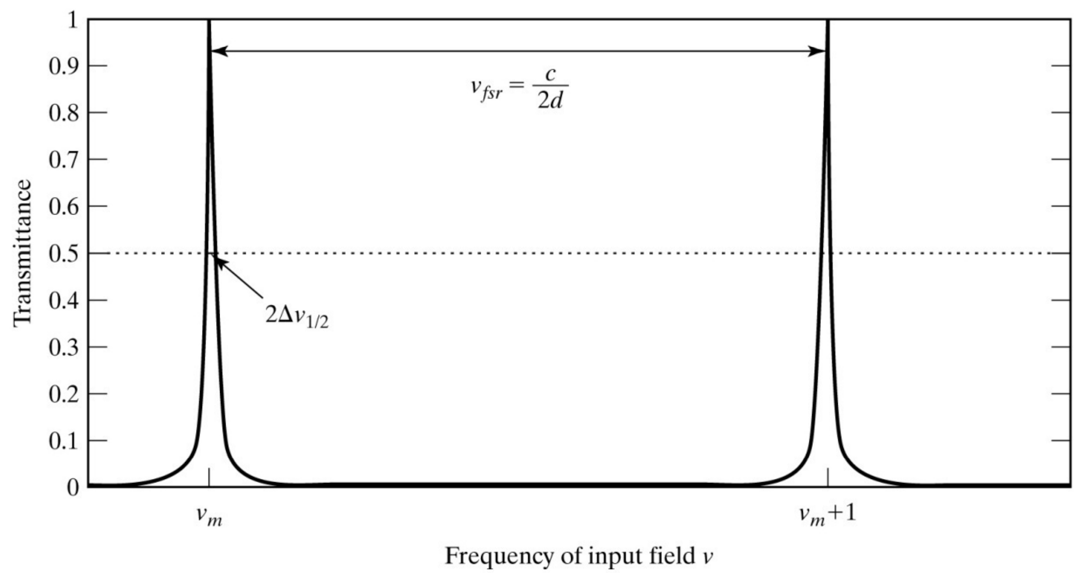
TABLE 8-1 FABRY-PEROT FIGURES OF MERIT.

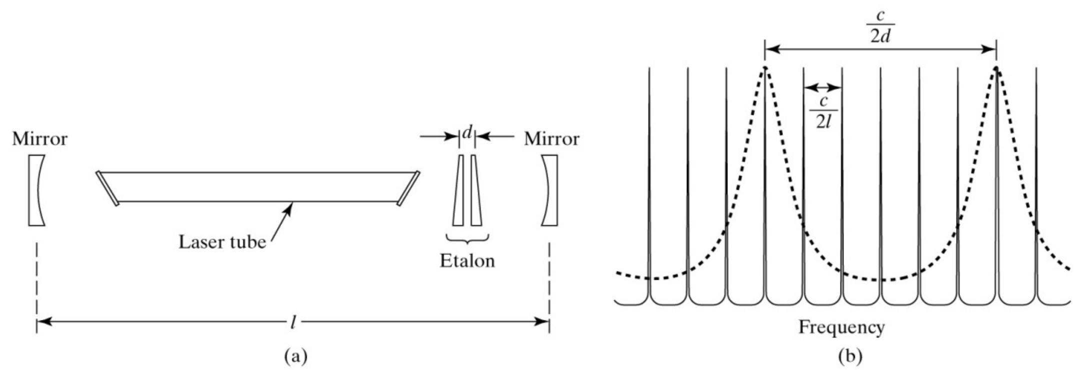
Here r is the end mirror reflection coefficient, T is the Fabry-Perot transmittance, R is the resolving power of the Fabry-Perot with an input field of nominal wavelength λ whose mirror spacing d is varied, $2\,\Delta\nu_{1/2}$ is the FWHM of a transmittance peak when the frequency of the input is varied around frequency ν, Γ is the decay rate of the light within the Fabry-Perot cavity, τ_p is the photon lifetime of the cavity, and FS12 stands for free spectral range.

Coefficient of Finesse	$F = \dfrac{4r^2}{(1-r^2)^2}$	$T = \dfrac{1}{1 + F\sin^2(\delta/2)}$	$F = \dfrac{T_{\max} - T_{\min}}{T_{\min}}$
Finesse	$\mathcal{F} = \dfrac{\pi\sqrt{F}}{2} = \dfrac{\pi r}{1-r^2}$	$\mathcal{F} = \dfrac{\text{FSR}}{\text{FWHM}}$	$\mathcal{R} \equiv \dfrac{\lambda}{\Delta\lambda_{\min}} = \dfrac{2d\mathcal{F}}{\lambda}$
Quality Factor	$Q = \dfrac{\nu}{\nu_{fsr}}\mathcal{F}$	$Q = \dfrac{\nu}{2\Delta\nu_{1/2}}$	$Q \approx \dfrac{\omega}{\Gamma} = \omega\tau_p$

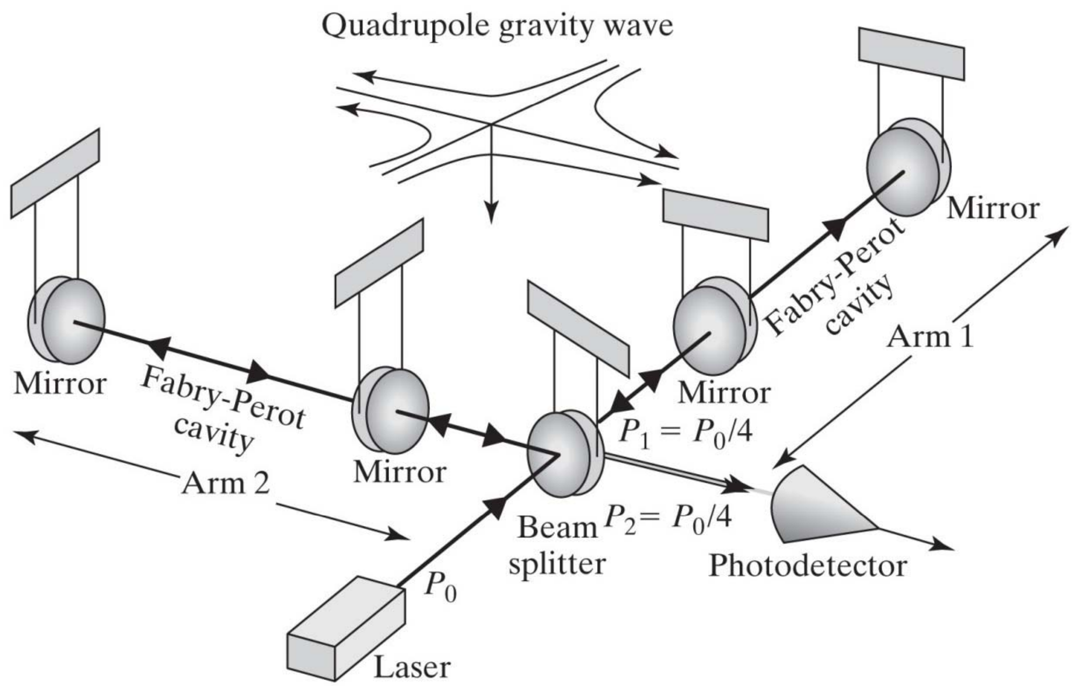
TABLE 8-2 Fabry-Perot parameters for a cavity with a nominal spacing of $d = 5$ cm, a nominal input wavelength of $\lambda = 500$ nm, and a nominal frequency of $\nu = 6 \cdot 10^{14}$ Hz. Photon lifetime and FWHM are quantities that are not applicable (NA) if the reflection coefficient is too low.

Mirror Reflection Coefficient	r	0.2	0.5	0.8	0.9	0.97	0.99
Coefficient of Finesse, F	$\frac{4r^2}{(1-r^2)^2}$	0.174	1.78	19.8	89.8	1080	9900
Finesse, $\mathcal{F}$	$\frac{\pi r}{1-r^2}$	0.655	2.09	6.98	14.9	51.6	156
Quality Factor, Q	$\frac{\nu}{(c/2d)}\mathcal{F}$	$1.31 \cdot 10^5$	$4.19 \cdot 10^5$	$1.40 \cdot 10^6$	$2.98 \cdot 10^6$	$1.03 \cdot 10^7$	$3.13 \cdot 10^7$
Photon Lifetime, τ_p (s)	$\frac{d}{c(1-r^2)}$	NA	NA	$4.63 \cdot 10^{-10}$	$8.77 \cdot 10^{-10}$	$2.82 \cdot 10^{-9}$	$8.38 \cdot 10^{-9}$
Resolving Power, $\mathcal{R}$	$\frac{2d\mathcal{F}}{\lambda}$	$1.31 \cdot 10^5$	$4.19 \cdot 10^5$	$1.40 \cdot 10^6$	$2.98 \cdot 10^6$	$1.03 \cdot 10^7$	$3.13 \cdot 10^7$
$\Delta\lambda_{min}$ (nm)	$\frac{\lambda^2}{2d\mathcal{F}}$	$3.82 \cdot 10^{-3}$	$1.19 \cdot 10^{-3}$	$3.58 \cdot 10^{-4}$	$1.68 \cdot 10^{-4}$	$4.85 \cdot 10^{-5}$	$1.60 \cdot 10^{-5}$
FSR (Variable Spacing) (nm)	$\lambda/2$	250	250	250	250	250	250
FWHM (Variable Spacing) (nm)	$\frac{\lambda}{2\mathcal{F}}$	NA	NA	35.8	16.8	4.85	1.6
FSR (Variable Frequency) (GHz)	$\frac{c}{2d}$	3	3	3	3	3	3
FWHM (Variable Frequency) (GHz)	$\frac{c}{2d\mathcal{F}}$	NA	NA	0.43	0.202	0.0582	0.0192


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Transmittance
$\nu_{fsr} = \frac{c}{2d}$
$2\Delta\nu_{1/2}$
ν_m
ν_m+1
Frequency of input field ν



Mirror
Laser tube
d
Mirror
Etalon
l
(a)
$\frac{c}{2d}$
$\frac{c}{2l}$
Frequency
(b)


Quadrupole gravity wave
Mirror
Fabry-Perot cavity
Mirror
Mirror
Fabry-Perot cavity
Mirror
Arm 1
Arm 2
$P_1 = P_0/4$
$P_2 = P_0/4$
Beam splitter
Photodetector
P_0
Laser