Chapter 39
The wave nature of the particles

Wave-particle duality : so far we have seen <u>particle nature of the waves</u> now we want to explore <u>wave nature of the particles</u>

1. De Broglie waves
   - Particles (electrons, protons, etc) can behave like waves
2. Electron Diffraction
   - An evidence for wavelike behavior of the particles
3. Probability and uncertainty
   - Fundamental limits on accuracy of our measurements imposed by Heisenberg's uncertainty principle
4. The electron microscope
   - Higher resolution that optical microscopes due to much smaller wavelengths
5. Wave function and the Schrodinger equation
   - Wave function for particles and the equation that these functions must satisfy

De Broglie waves 1924

- Nature likes symmetry. Light is dualistic in nature, if that is so matter also should have dualistic nature.
- De Broglie assigned a wavelength and frequency to the particles with mass m.
- Wave-particle duality is <u>not a contradiction.</u>
- <u>Principle of complementarity</u> states that we need both particle model and wave model to have a clear picture of the nature

A particle has a wavelngth of $\boxed{\lambda = \dfrac{h}{p} = \dfrac{h}{mv}}$

& $\lambda = \dfrac{h}{p} = \dfrac{h}{\gamma m v_g}$ for relativistic particles,

$h = 6.636.\times 10^{-34}\, J.s$ is the Planck's constant

$\gamma = 1/\sqrt{1 - {v_g}^2 / c^2}$

A particle has an energy of $E = hf$ which is related to its frequency calculated from its DeBroglie wavelength.

<u>Caution</u>: $E = pc$ & $f = c/\lambda$ is only valid for the particles with zero rest mass.

$$\boxed{E = hf} = h\frac{v_\phi}{\lambda} = \frac{hv_\phi}{h/m\gamma v_g} = m\gamma v_g v_\phi = m\gamma c^2$$

$$\boxed{v_g v_\phi = c^2} \begin{cases} \text{for lights } v_g = v_\phi = c \\ \text{for particles } v_g < c \ \&\ v_\phi > c \end{cases}$$

Bohr model and De Broglie waves

Boh'r quantization of angular momentum:

$$mvr = n\frac{h}{2\pi}$$

<u>From de Broglie postulate to Bohr Postulate</u>

Standing waves of a string emit no energy

Electron on a Bohr orbit emeits no energy.

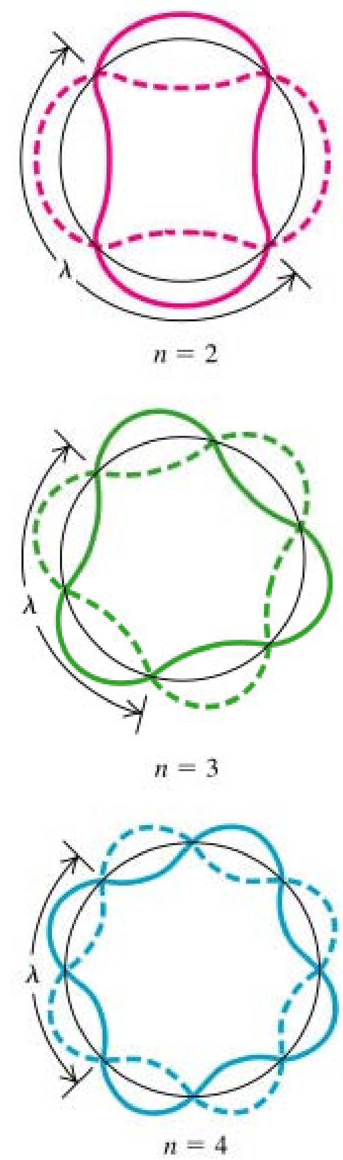
Electron has a wavelike behavior

So electron must form standing waves on its orbit around the nucleus.

Condition for standing waves in circular motion:

$$\begin{cases} 2\pi r = n\lambda \quad n = 1, 2, 3, \ldots \\ \lambda = h / mv \end{cases} \rightarrow 2\pi r = \frac{nh}{mv} \rightarrow \boxed{mvr = n\frac{h}{2\pi}}$$

Angular momentum of the electron on its orbit in an atom is quantized.



De Broglie wavelength of few things around us

- Calculate de Broglie wavelength of
  - an electron with speed of 1.98×10^3m/s
  - Yourself with the same speed
  - Earth moving at 67000mi/h around sun

$$m_e \simeq 10^{-30} kg$$

$$v_e = 2 \times 10^3 m/s$$

$$\lambda_e = \frac{h}{m_e v_e} = \frac{6.6 \times 10^{-34} j.s}{\left(10^{-30} kg\right)\left(2.0 \times 10^3 m/s\right)} = \underline{3.3 \times 10^{-7} m}$$

$$m_{you} \simeq 70\ kg$$

$$v_{you} = 2 \times 10^3 m/s$$

$$\lambda_{you} = \frac{h}{m_{you} v_{you}} = \frac{6.6 \times 10^{-34} j.s}{\left(70kg\right)\left(2.0 \times 10^3 m/s\right)} = \underline{4.7 \times 10^{-33} m}$$

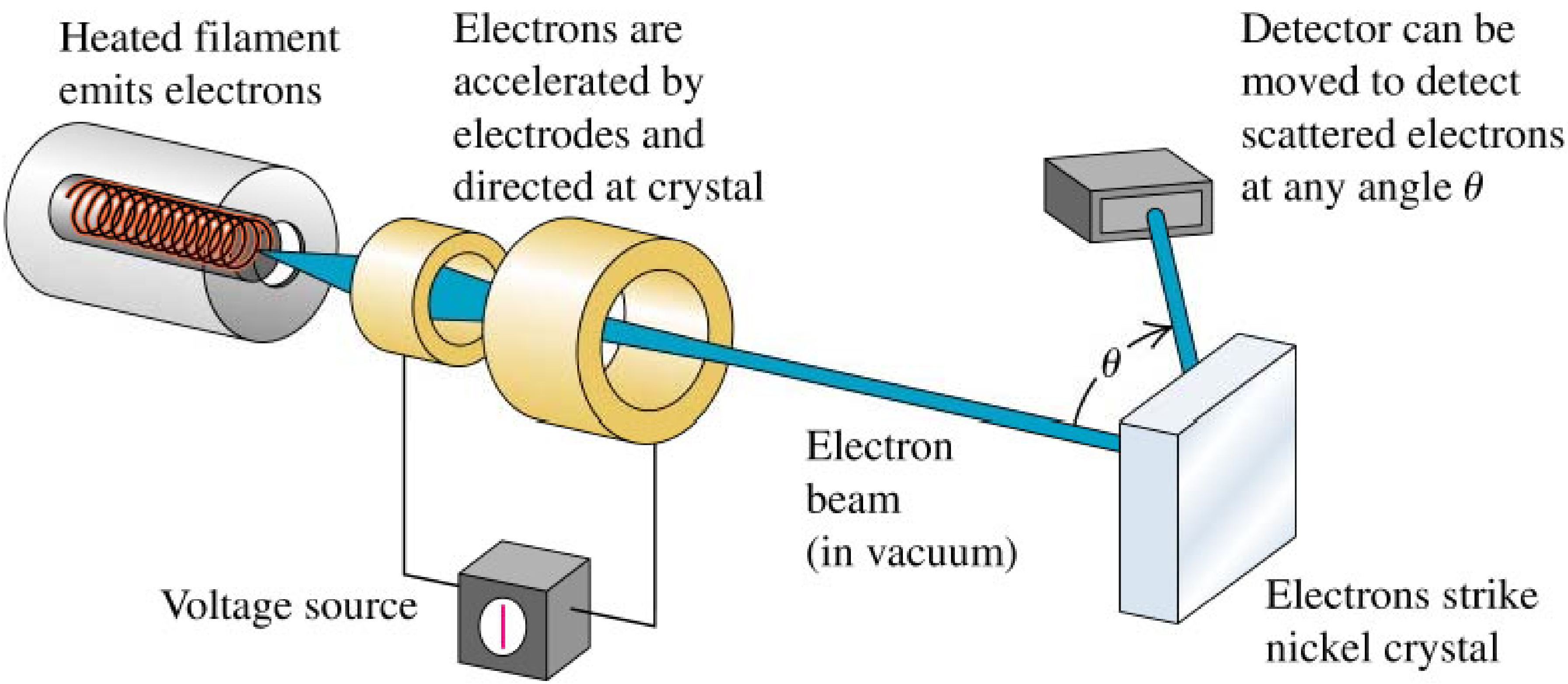
$$m_{earth} \simeq 6.0 \times 10^{24}\ kg$$

$$v_{earth} = 67000 mi/h \simeq 67000\left(\frac{1600m}{mi}\right)\left(\frac{h}{3600s}\right) = 29777 m/s$$

$$\lambda_{earth} = \frac{h}{m_{earth} v_{earth}} = \frac{6.6 \times 10^{-34} j.s}{\left(6.0 \times 10^{24}\ kg\right)\left(3 \times 10^4 m/s\right)} = \underline{0.37 \times 10^{-62} m}$$

Electron Diffraction (Davison Germer experiment 1927)

If an electron has wavelike properties shouldn’t it exhibit diffraction like any wave? If find a system that its structure size is comparable to the wavelength of the electron, We might be able to see a diffraction pattern.



Electron scattering by a lattice

The Bragg condition for constructive interference:

$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

d lattice spacing

λ wavelength of the electron

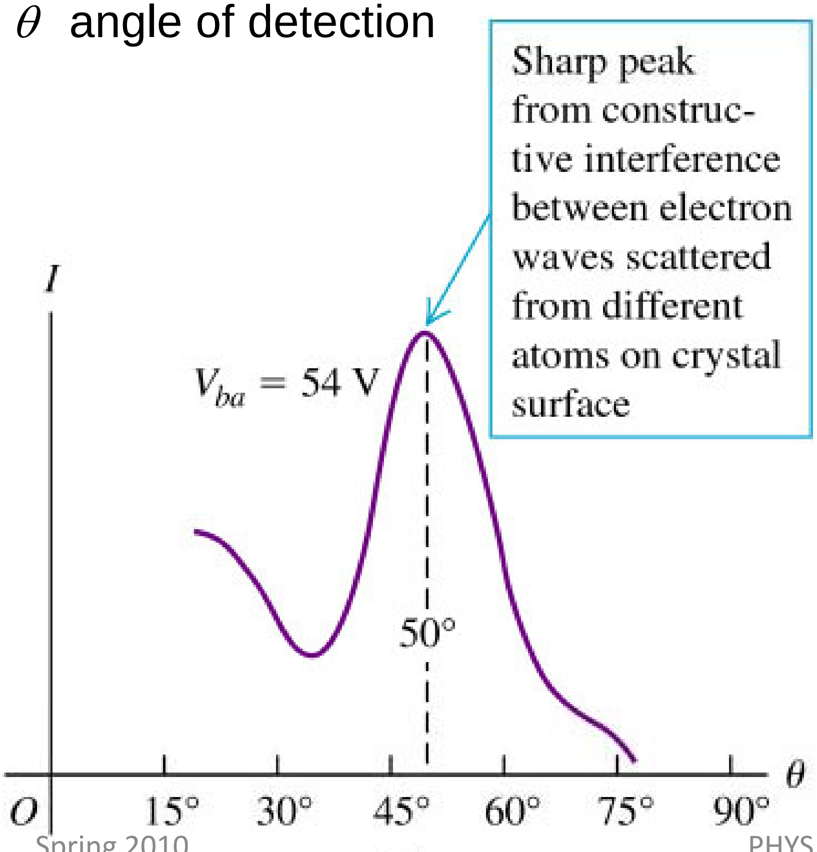
θ angle of detection

Energy of the electron accelerated in potential of V from point a to b

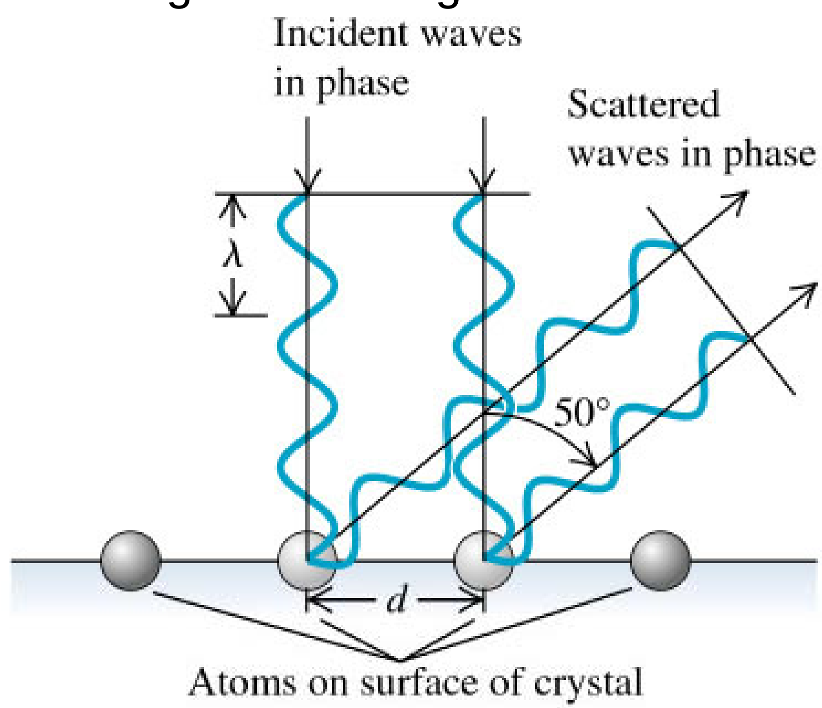
$$eV_{ab} = \frac{1}{2}mv^2 = \frac{p^2}{2m} \rightarrow p = \sqrt{2meV_{ab}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ab}}}$$

de Broglie wavelength of the electron.



(a)



(b)

Electron diffraction experiment

- In diffraction experiment the accelerating voltage is 54V. An scattered electron intensity maximum is detected at 40^0 from the surface. Neglect the kinetic energy of the electron. What is the atomic spacing of the row scattered the electron waves?

Solution:

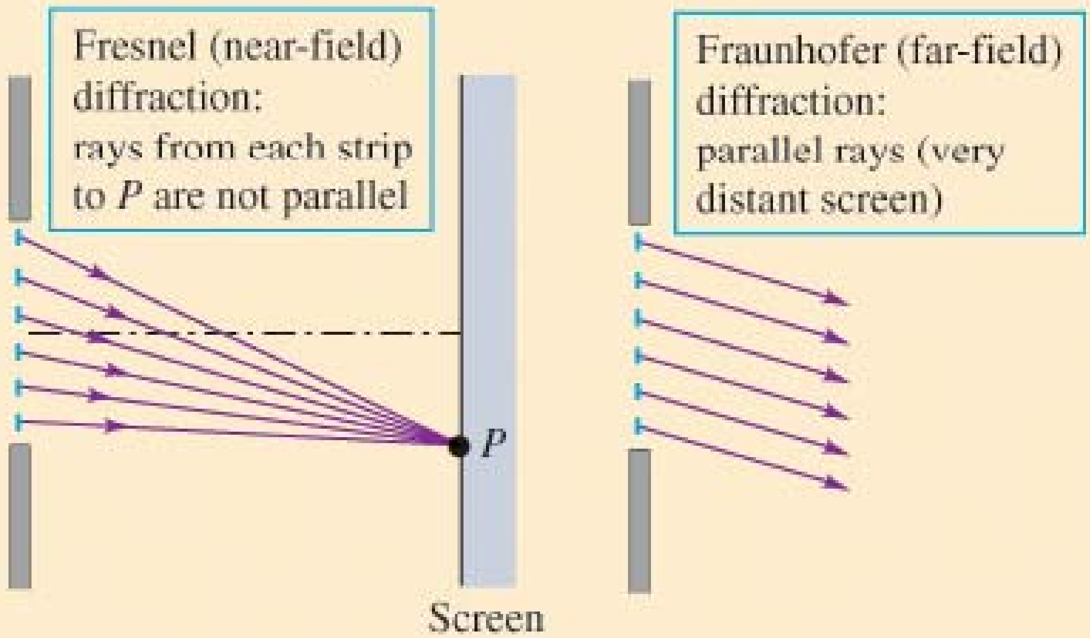
We can use $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

to find the electron's de Broglie wavelength then use the Bragg relation for electron scattering to determine the atomic spacing of the crystal.

$m\lambda = d \sin\theta$

With $\theta = 90^0 - 40^0 = 50^0$ and $m = 1$

Diffraction from a single slit I (review)

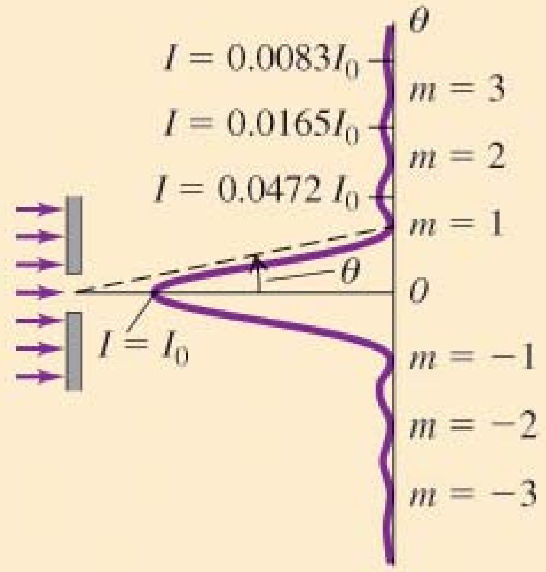


Diffraction occurs when light passes through an aperture or around an edge. When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction. When the source or the observer is relatively close to the obstructing surface, it is Fresnel diffraction.

Diffraction from a single slit II (Review)

$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \dots) \qquad (36.2)$$

$$I = I_0 \left\{ \frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda} \right\}^2 \qquad (36.7)$$

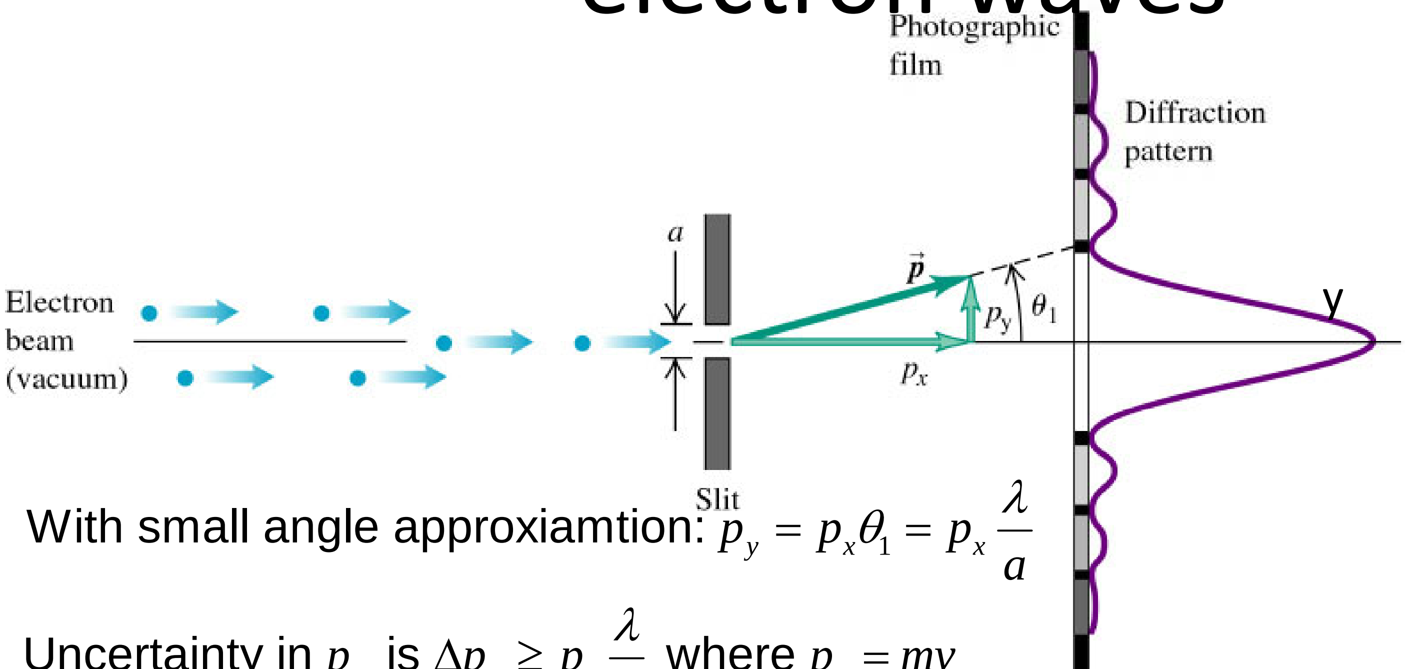


Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen. Equation (36.2) gives the condition for destructive interference (a dark fringe) at a point P in the pattern at angle θ. Equation (36.7) gives the intensity in the pattern as a function of θ. (See Examples 36.1 through 36.3)

For small angles: $\theta \simeq \dfrac{\lambda}{a}$

the wave is mostly confined within the angular spread of 2θ

Single slit diffraction pattern of the electron waves



Where is the electron exactly? Probability of finding electron in central lobe 85%

With small angle approxiamtion: $p_y = p_x\theta_1 = p_x \dfrac{\lambda}{a}$

Uncertainty in p_y is $\Delta p_y \geq p_x \dfrac{\lambda}{a}$ where $p_x = mv_x$

The de Broglie wavelength of the electron is: $\lambda = \dfrac{h}{p_x}$

$\Delta p_y \geq p_x \dfrac{1}{a}\dfrac{h}{p_x} \rightarrow \boxed{\Delta p_y a \geq h}$ or $\boxed{\Delta p_y \Delta y \geq h}$

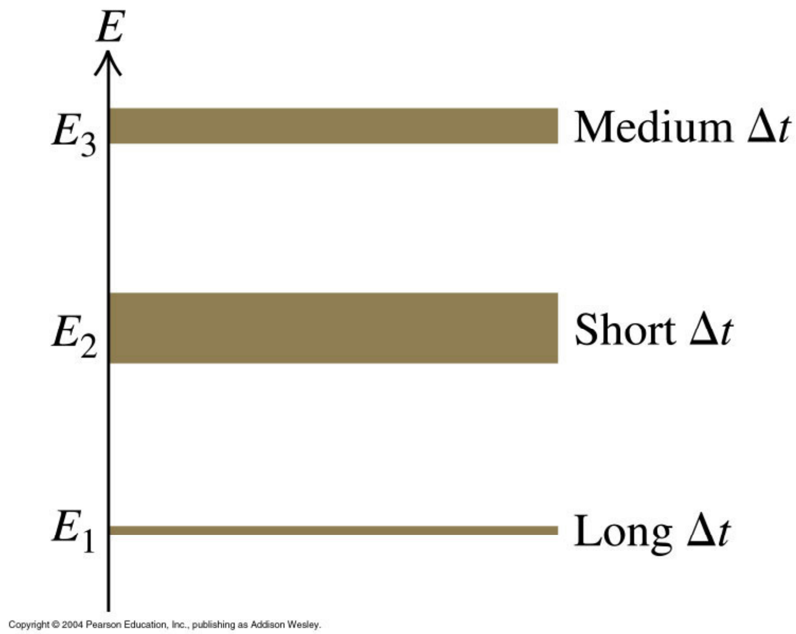
Uncertainty in location times uncertainty in momentum is greater than h

If we use standard deviations then $\boxed{\Delta p_y \Delta x \geq \hbar}$ where $\hbar = h/2\pi = 1.055\times 10^{-34}\ j.s$

The uncertainty in energy

The longer the lifetime of a state the smaller the spread of its energy

$\Delta E \Delta t \geq \hbar$

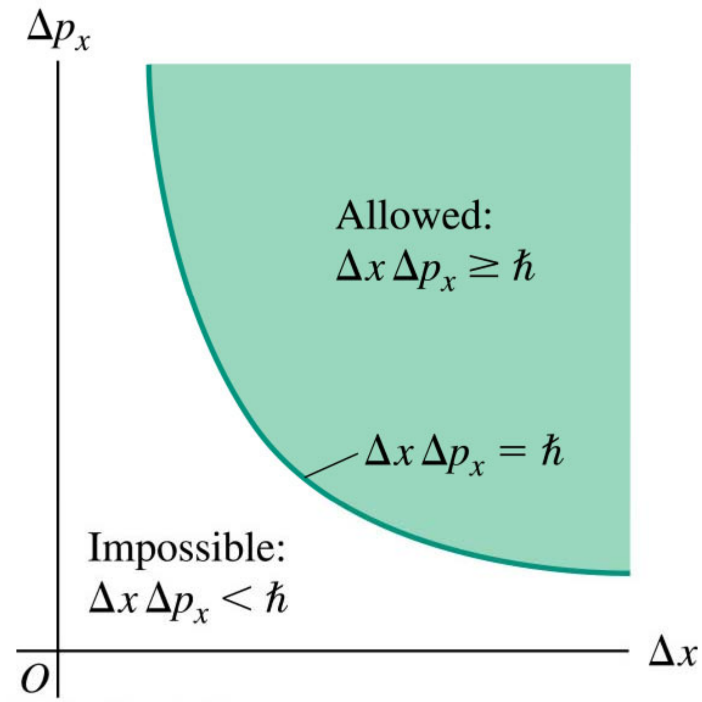


The Heisenberg (1901-1976) Uncertainty Principle

- It is impossible to make precise determination of particle's coordinate and corresponding componer of its angular momentum at the same time

  $\Delta x \Delta p_x \geq \hbar$ where $\hbar = h/2\pi$

- Energy ΔE of a particle th has occupied a state for duration of Δt is uncertain by: $\Delta E \Delta t \geq \hbar$ where $\hbar = h/2\pi$



Example

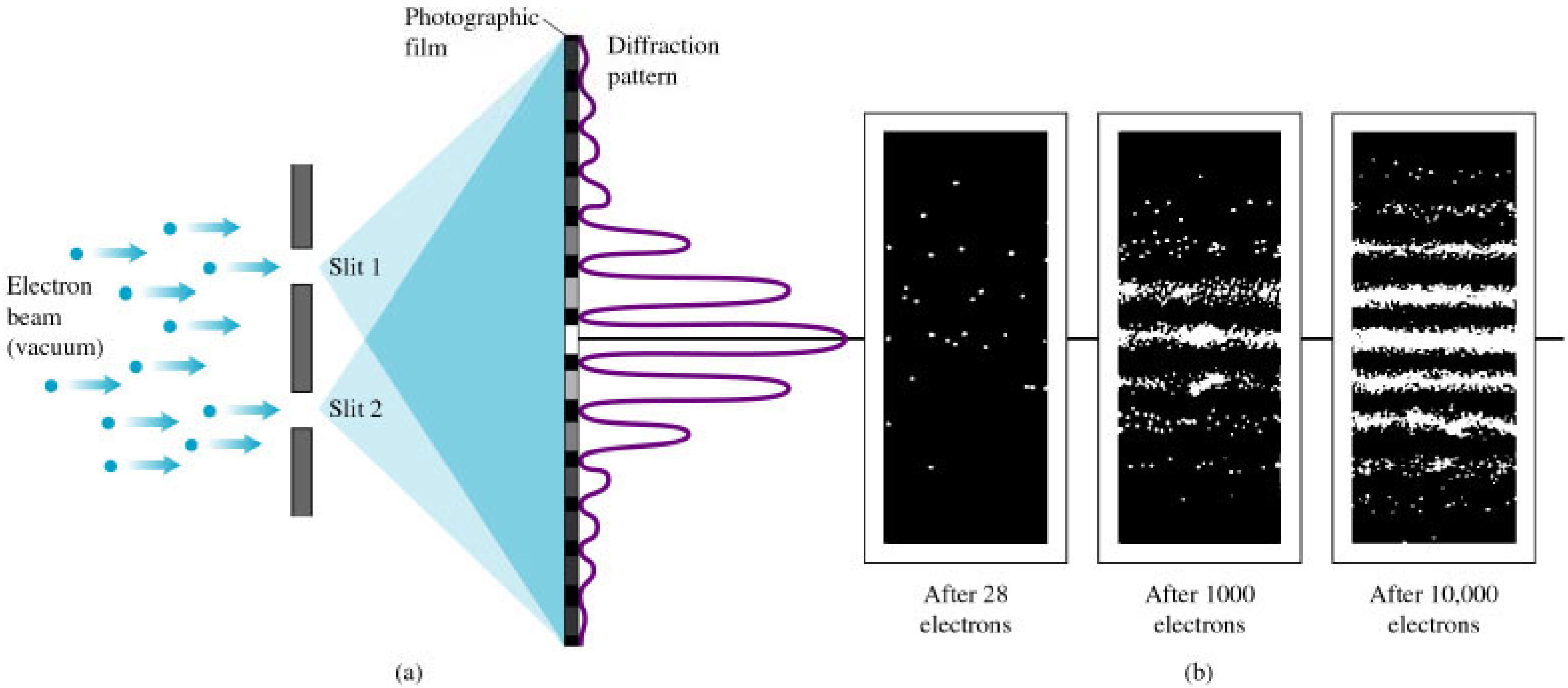
- Approximate kinetic energy of an electron confined to a region of 1.0×10^{-10}m if magnitude of the momentum of the electron is equal to uncertainty in its momentum.

$$\Delta p_x \Delta x \geq \hbar$$

$$\left(\Delta p_x\right)_{\min} = h / \Delta x \simeq p_x$$

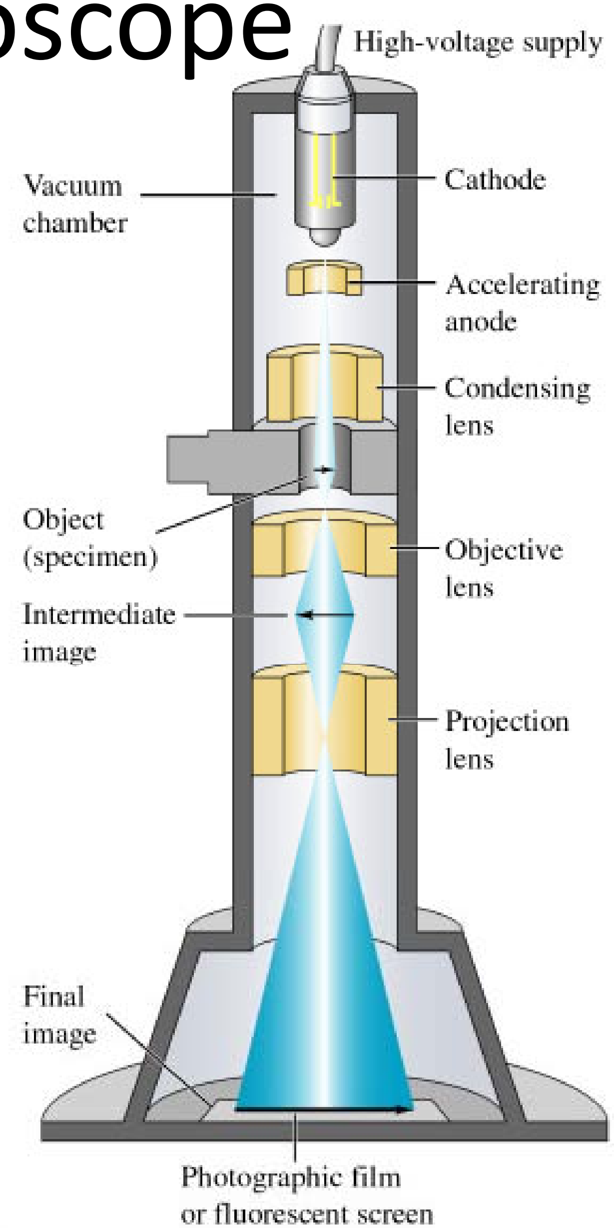
$$K = \frac{p^2}{2m}$$

Two slit interference



Electron Microscope

- Now if electrons can behave like wave, why not use them in microscopy?
- What kind of resolution do you expect from electron waves?
- How can we focus the electron beams?
- What constitutes an electron lens?



Diffraction pattern formed by a circular aperture (Review)

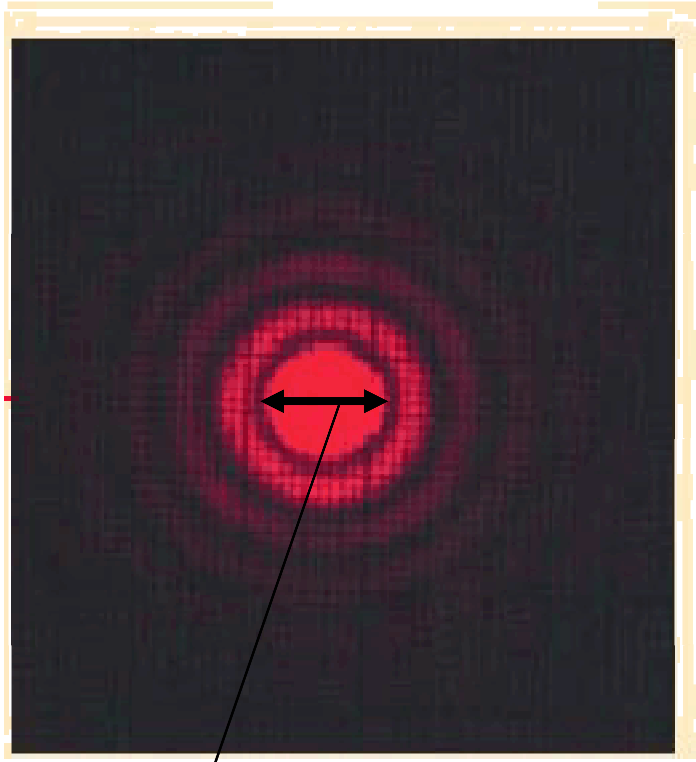
Angle of Airy disk: $\sin\theta_1 = 1.22\dfrac{\lambda}{D}$

Second dark ring: $\sin\theta_2 = 2.23\dfrac{\lambda}{D}$

Third dark ring: $\sin\theta_3 = 3.24\dfrac{\lambda}{D}$

Tree bright rings at

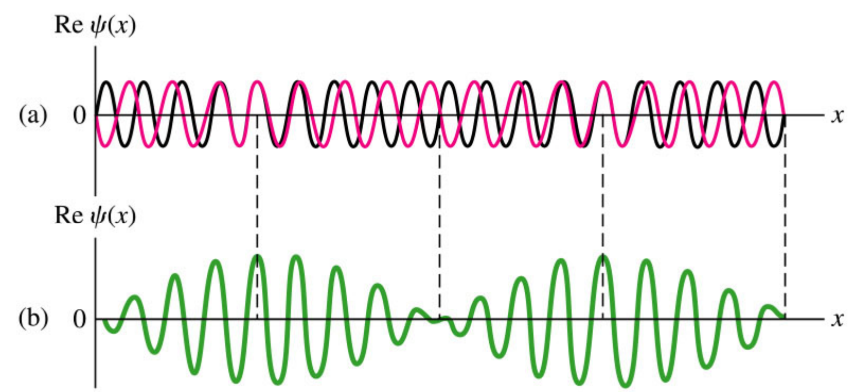
$$\sin\theta = 1.63\frac{\lambda}{D} \qquad 2.68\frac{\lambda}{D} \qquad 3.70\frac{\lambda}{D}$$



Airy disk 85% of the light falls within this disk

An Electron microscope

- What acceleration voltage is needed to produce electrons with wavelength of 10pm? Neglect the initial kinetic energy of the electrons.
- How does this compare to the wavelength of the visible light?


Re $\psi(x)$
(a)
0
x
Re $\psi(x)$
(b)
0
x

Wave Functions

The wavefunction of a particle $\Psi\left(x,y,z,t\right)$ contains all of the information about the particle.

The probability distribution function of a particle's wavefunction $\left|\Psi\left(x,y,z,t\right)\right|^2$ is probability of finding a particle near a given position at a stationary state.

$$\underbrace{\Psi\left(x,y,z,t\right)}_{\substack{\text{Wavefunction}\\\text{of a particle for}\\\text{an stationary state}}}=\underbrace{\psi\left(x,y,z\right)}_{\substack{\text{spatial coordinates}\\\text{dependent portion}\\\text{of the wavefunction}}}\underbrace{e^{-iEt/\hbar}}_{\substack{\text{Time-dependent}\\\text{portion of the}\\\text{wavefunction}}}$$

For a <u>stationary state</u> the probability distribution function is independent of time i.e.

$$\left|\Psi\left(x,y,z,t\right)\right|^2=\left(\psi\left(x,y,z\right)e^{-iEt/\hbar}\right)\ \left(\psi\left(x,y,z\right)e^{-iEt/\hbar}\right)^*$$

$$\left|\Psi\left(x,y,z,t\right)\right|^2=\left(\psi\left(x,y,z\right)e^{-iEt/\hbar}\right)\ \left(\psi^*\left(x,y,z\right)e^{+iEt/\hbar}\right)$$

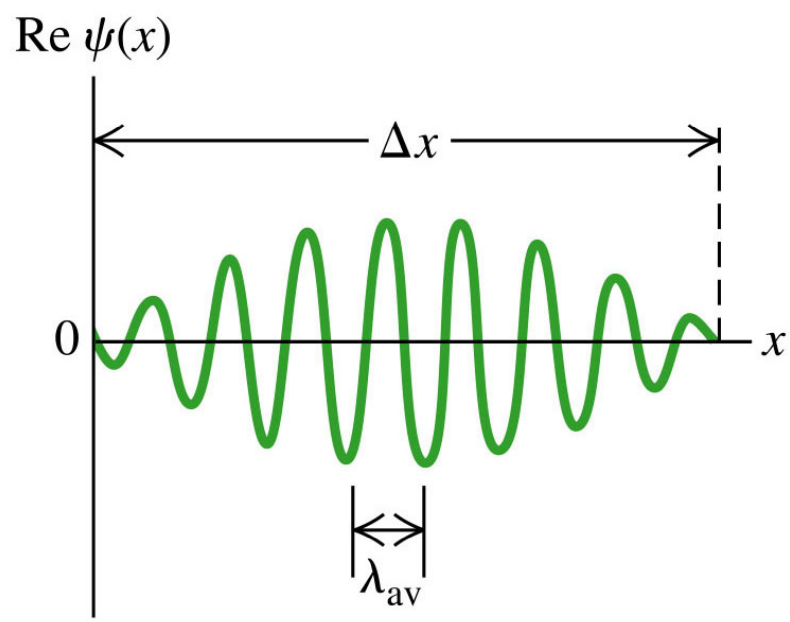
$$\left|\Psi\left(x,y,z,t\right)\right|^2=\psi\left(x,y,z\right)\psi^*\left(x,y,z\right)=\underbrace{\left|\psi\left(x,y,z\right)\right|^2}_{\text{Independent of time}}$$

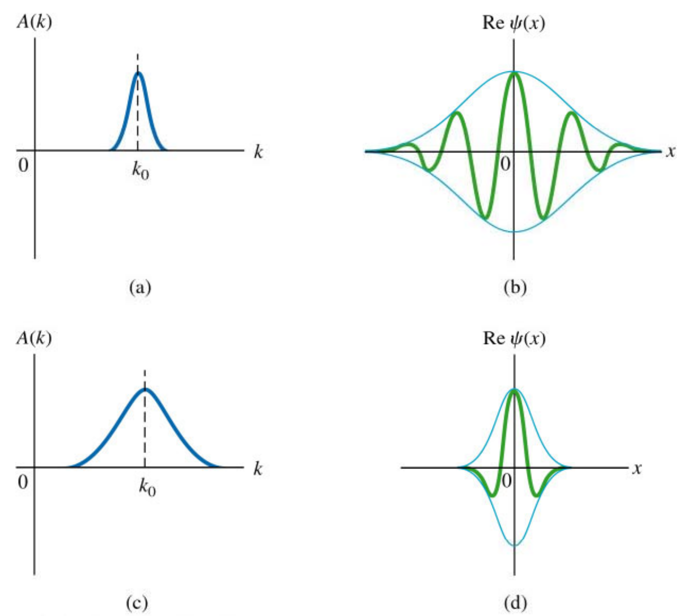
The Schrodinger equation

- For a particle that moves in one dimension in the presence of a potential energy function *U(x)* the wavefunction for a stationary state of energy *E* satisfies the Schrodinger equation.
- More complicated wavefunctions can be constructed by superposition of the stationary-state wavefunctions.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$$

Time-independent one-dimensional Schrodinger Equation




A(k)
0
k₀
k
Re ψ(x)
x
0
(a)
(b)
(c)
(d)