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ME 587 HW #1
September 20th, 2011

1. Dispersion and Abbe number
   a. What kind of Abbe V number offers large dispersion
      Ans: A small Abbe V number has a large dispersion

   b. What is the sign of the Abbe V number for a glass with:
      i. Negative dispersion
      ii. Positiive dispersion
      iii. No dispersion

      Ans: Since the formula for Abbe V number is:

$$V = \frac{n_d - 1}{n_F - n_C}$$

Where n_d is always greater than 1, so the numerator is always positive
So the sign of the Abbe number depends on the denominator. The denominator is positive when n_F is greater than n_C and is negative when the reverse is true.
In addition, n_F is at 486.1nm and n_C is at 656.3nm. thus:
i. Negative dispersion: n_F is smaller than n_C, so Abbe number is negative
ii. Positive dispersion: n_F is larger than n_C, so Abbe number is positive
iii. No Dispersion, the Abbe number becomes infinite (could be negative or positive)

   c. Find 3 glasses from a catalogue (mention the source) with
      i. Negative dispersion
      ii. Positive dispersion
      iii. Minimum dispersion

      Ans: Answers are based on Schott's Abbe diagram:
      http://www.us.schott.com/advanced_optics/english/download/schott_abbe_nd_vd_pgf_july_2011_us.pdf?highlighted_text=abbe diagram
      i. Negative dispersion does not exist on the chart
         From Wikipedia: http://en.wikipedia.org/wiki/Dispersion_(optics)

Negative dispersion is usually achieved by prism-pairs, diffratioin gratings, or chirped mirrors with coatings design at certain wavelength(s)

ii. Positive dispersion: all material in the chart has positive dispersion

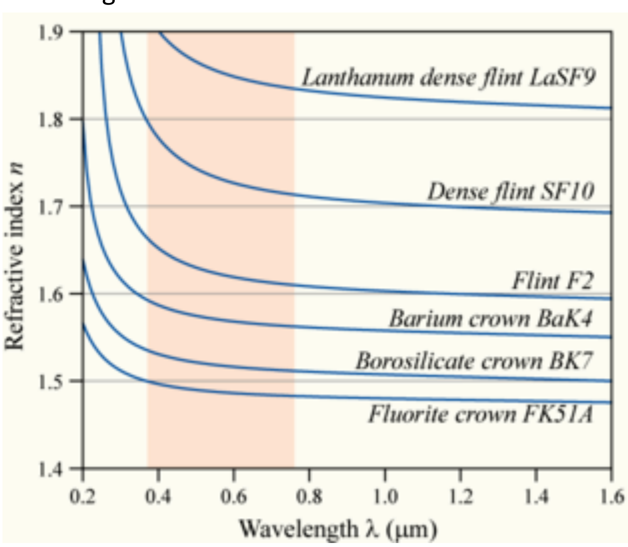
I chose P-SF68 for further study

(source: Schott http://www.us.schott.com/advanced_optics/english/download/schott_abbe_nd_vd_pgf_july_2011_us.pdf?highlighted_text=abbe diagram)

iii. Minimum dispersion: the smallest dispersion, which has the highest Abbe number is N-FK51A (V is about 85)

(source: Schott http://www.us.schott.com/advanced_optics/english/download/schott_abbe_nd_vd_pgf_july_2011_us.pdf?highlighted_text=abbe diagram)

d. What can we say about dispersion of a glass in IR range based on its Abbe V number?
Ans: Since the Abbe number is only useful for wavelength from 486.1nm to 656.3nm and IR is from 10^{-3} to 10^{-6}m or 10^{6} to 10^{3}nm, the behavior must be extrapolated. From Wikipedia (http://en.wikipedia.org/wiki/Dispersion_(optics)) the relationship of Refractive index to wavelength for most glass material tends to flatten out at longer wavelength:



One could predict that the dispersion at IR range will be smaller

e. Find the catalogue information for a low and high dispersion glass and explain meaning of the information listed for them.
Ans: from Schott:

N-SF68 (a material with low V number) is at:

http://www.us.schott.com/advanced_optics/english/our_products/materials/data_tools/index.html

Key Information presented are:

Refractive indices at selected wavelengths

Internal transmittance at selected wavelengths for 10mm and 25mm

Relative Partial Dispersion: this is similar to the Abbe number but defined at different wavelength

▷ Data Sheet (PDF) ▷ Data Sheet-Catalog (PDF) ▷ Back to diagram overview

SCHOTT | OPTICAL GLASS | SF

P-SF68
005210.619

$n_d = 2.00520$ $\nu_d = 21.00$ $n_F - n_C = 0.047867$
$n_e = 2.01643$ $\nu_e = 20.82$ $n_{F'} - n_{C'} = 0.048826$

Refractive Indices

	λ [nm]	
$n_{2325,4}$	2325,4	1.93381
$n_{1970,1}$	1970,1	1.93968
$n_{1529,6}$	1529,6	1.94732
$n_{1060,0}$	1060,0	1.95970
n_t	1014,0	1.96160
n_s	852,1	1.97063
n_r	706,5	1.98449
n_C	656,3	1.99171
$n_{C'}$	643,8	1.99380
$n_{632,8}$	632,8	1.99576
n_D	589,3	2.00479
n_d	587,6	2.00520
n_e	546,1	2.01643
n_F	486,1	2.03958
$n_{F'}$	480,0	2.04262
n_g	435,8	2.07018
n_h	404,7	
n_i	365,0	
$n_{334,1}$	334,1	
$n_{312,6}$	312,6	
$n_{296,7}$	296,7	
$n_{280,4}$	280,4	
$n_{248,3}$	248,3	

Internal Transmittance τ_i

λ [nm]	τ_i (10mm)	τ_i (25mm)
2500	0.793	0.560
2325	0.905	0.780
1970	0.976	0.940
1530	0.996	0.990
1060	0.999	0.998
700	0.997	0.993
660	0.996	0.989
620	0.994	0.985
580	0.989	0.973
546	0.976	0.940
500	0.905	0.780
460	0.758	0.500
436	0.574	0.250
420	0.302	0.050
405	0.036	
400	0.007	
390		
380		
370		
365		
350		
334		
320		
310		
300		
290		
280		
270		
260		
250		

Relative Partial Dispersion

$P_{s,t}$	0.1885
$P_{C,s}$	0.4406
$P_{d,C}$	0.2817
$P_{e,d}$	0.2346
$P_{g,F}$	0.6392
$P_{i,h}$	
$P'_{s,t}$	0.1848
$P'_{C',s}$	0.4746
$P'_{d,C'}$	0.2336
$P'_{e,d}$	0.2300
$P'_{g,F'}$	0.5644
$P'_{i,h}$	

Deviation of Relative Partial Dispersion ΔP from the "Normal Line"

$\Delta P_{C,t}$	-0.0156
$\Delta P_{C,s}$	-0.0113
$\Delta P_{F,e}$	0.0063
$\Delta P_{g,F}$	0.0308
$\Delta P_{i,g}$	

Constants of Dispersion Formula

B_1	2.3330067
B_2	0.452961396
B_3	1.25172339
C_1	0.0168838419
C_2	0.0716086325
C_3	118.707479

Constants of Formula for dn/dT

D_0	$1.55 \cdot 10^{-5}$
D_1	$2.30 \cdot 10^{-8}$
D_2	$-3.46 \cdot 10^{-11}$
E_0	$2.76 \cdot 10^{-8}$
E_1	$2.93 \cdot 10^{-9}$
λ_{TK} [µm]	0.297

Color Code

λ_{80}/λ_5	49/41*

Remarks

suitable for precision molding

Other Properties

$\alpha_{-30/+70°C}$ [10^{-6}/K]	8.4
$\alpha_{+20/+300°C}$ [10^{-6}/K]	9.7
T_g [°C]	428
$T_{10^{13.0}}$ [°C]	430
$T_{10^{7.6}}$ [°C]	504
c_p [J/(g·K)]	0.370
λ [W/(m·K)]	0.650
AT [°C]	468
ρ [g/cm³]	6.19
E [10^3N/mm²]	79
µ	0.275
K [10^{-6}mm²/N]	1.61
$HK_{0,1/20}$	404
HG	
HG-J	298
B	
CR	1
FR	5
SR	53.3
SR-J	4
AR	2.3
PR	2.3

Temperature Coefficients of Refractive Index

	$\Delta n_{rel}/\Delta T$ [10^{-6}/K]			$\Delta n_{abs}/\Delta T$ [10^{-6}/K]		
[°C]	1060,0	e	g	1060,0	e	g
-40/-20	13.7	21.5	32.3	11.1	18.8	29.5
+20/+40	15.2	24.1	36.5	13.5	22.3	34.6

Similar information can be found for N-FK51A

SCHOTT	OPTICAL GLASS	FK

N-FK51A
487845.368

n_d = 1.48656	ν_d = 84.47	$n_F - n_C$ = 0.005760
n_e = 1.48794	ν_e = 84.07	$n_{F'} - n_{C'}$ = 0.005804

Refractive Indices

	λ [nm]	
$n_{2325,4}$	2325,4	1.46958
$n_{1970,1}$	1970,1	1.47271
$n_{1529,6}$	1529,6	1.47608
$n_{1060,0}$	1060,0	1.47959
n_t	1014,0	1.47999
n_s	852,1	1.48165
n_r	706,5	1.48379
n_C	656,3	1.48480
$n_{C'}$	643,8	1.48508
$n_{632,8}$	632,8	1.48534
n_D	589,3	1.48651
n_d	587,6	1.48656
n_e	546,1	1.48794
n_F	486,1	1.49056
$n_{F'}$	480,0	1.49088
n_g	435,8	1.49364
n_h	404,7	1.49618
n_i	365,0	1.50046
$n_{334,1}$	334,1	1.50501
$n_{312,6}$	312,6	1.50911
$n_{296,7}$	296,7	
$n_{280,4}$	280,4	
$n_{248,3}$	248,3	

Internal Transmittance τ_i

λ [nm]	τ_i (10mm)	τ_i (25mm)
2500	0.891	0.750
2325	0.933	0.840
1970	0.976	0.940
1530	0.992	0.980
1060	0.998	0.994
700	0.998	0.995
660	0.998	0.995
620	0.998	0.996
580	0.999	0.997
546	0.999	0.997
500	0.998	0.996
460	0.997	0.993
436	0.997	0.992
420	0.997	0.992
405	0.997	0.993
400	0.997	0.993
390	0.997	0.992
380	0.995	0.988
370	0.990	0.976
365	0.985	0.963
350	0.948	0.875
334	0.831	0.630
320	0.618	0.300
310	0.428	0.120
300	0.262	0.035
290	0.137	0.010
280	0.058	
270		
260		
250		

Relative Partial Dispersion

$P_{s,t}$	0.2879
$P_{C,s}$	0.5465
$P_{d,C}$	0.3062
$P_{e,d}$	0.2388
$P_{g,F}$	0.5359
$P_{i,h}$	0.7429
$P'_{s,t}$	0.2858
$P'_{C',s}$	0.5909
$P'_{d,C'}$	0.2554
$P'_{e,d}$	0.2370
$P'_{g,F'}$	0.4759
$P'_{i,h}$	0.7373

Deviation of Relative Partial Dispersion ΔP from the "Normal Line"

$\Delta P_{C,t}$	-0.1112
$\Delta P_{C,s}$	-0.0533
$\Delta P_{F,e}$	0.0110
$\Delta P_{g,F}$	0.0342
$\Delta P_{i,g}$	0.1675

Constants of Dispersion Formula

B_1	0.971247817
B_2	0.216901417
B_3	0.904651666
C_1	0.00472301995
C_2	0.0153575612
C_3	168.68133

Constants of Formula for dn/dT

D_0	$-1.83 \cdot 10^{-5}$
D_1	$-7.89 \cdot 10^{-9}$
D_2	$-1.63 \cdot 10^{-12}$
E_0	$3.74 \cdot 10^{-7}$
E_1	$3.46 \cdot 10^{-10}$
λ_{TK} [μm]	0.15

Color Code

λ_{80}/λ_5	34/28

Remarks

suitable for precision molding

Other Properties

$\alpha_{-30/+70°C}$ [10^{-6}/K]	12.7
$\alpha_{+20/+300°C}$ [10^{-6}/K]	14.8
T_g [°C]	464
$T_{10^{13.0}}$ [°C]	463
$T_{10^{7.6}}$ [°C]	527
c_p [J/(g·K)]	0.690
λ [W/(m·K)]	0.760
AT [°C]	503
ρ [g/cm³]	3.68
E [10^3N/mm²]	73
μ	0.302
K [10^{-6}mm²/N]	0.70
$HK_{0.1/20}$	345
HG	6
HG-J	528
B	1
CR	1
FR	0
SR	52.3
SR-J	3

Temperature Coefficients of Refractive Index

	$\Delta n_{rel}/\Delta T$ [10^{-6}/K]			$\Delta n_{abs}/\Delta T$ [10^{-6}/K]		
[°C]	1060.0	e	g	1060.0	e	g

2. Rule of thumb

Name for Rule	Speed of Light
The rule of Thumb	c = 3*10^8 m/s
When is this used	Most equations involving speed of light
Limitations	c is really 299,792,458 m/s; this rule of thumb gives 0.06% error, which is reasonable for most calculations; this should be avoided if more than 4-decminal precision is needed

Name for Rule	Thin lens analysis (a)
The rule of Thumb	wavefronts arriving at a thin lens are curved and stay in phase
When is this used	Ray tracing
Limitations	At different distances from the optical axis, the ray will travel through different amount of lens, thus exiting the lens with a slightly different shift in phase

Name for Rule	Thin lens analysis (b)
The rule of Thumb	the bending of the ray through a lens occurs at a single point
When is this used	Ray tracing
Limitations	This will not count the bending of the ray within a lens. This is sufficient with the width of the lens is much smaller than the object and image distance (about 10 times smaller)

Name for Rule	Fraunhofer Diffraction
The rule of Thumb	when a source or the observer is very far, the outgoing rays can be considered parallel
When is this used	Ray tracing
Limitations	This cannot be used when the object or image is question is relatively close to the lens (about 10 times the focal length)

Name for Rule	Small Angle Approximation (other version)
The rule of Thumb	Cos θ = 1 (in radians)
When is this used	For small angles (<0.2 radians) Application of this approximation greatly simplifies analysis and calculation
Limitations	cos (0.2) = 0.98 which gives 2% error. The error will drastically increase as the angle increases

3. A camera focused between 1 meter and infinity

   A point and shoot camera company claims their digital camera can take sharp images of the objects located between 1m and infinity. The pixel size is 3x3 (micron squared), lens aperture is 5mm in diameter, lens focal length is 10mm.

   a. Evaluate validity of the claim numerically for the green light (550nm). Draw diagrams if necessary.

      <u>Ans:</u> first we calculate the diffraction limit of the system:

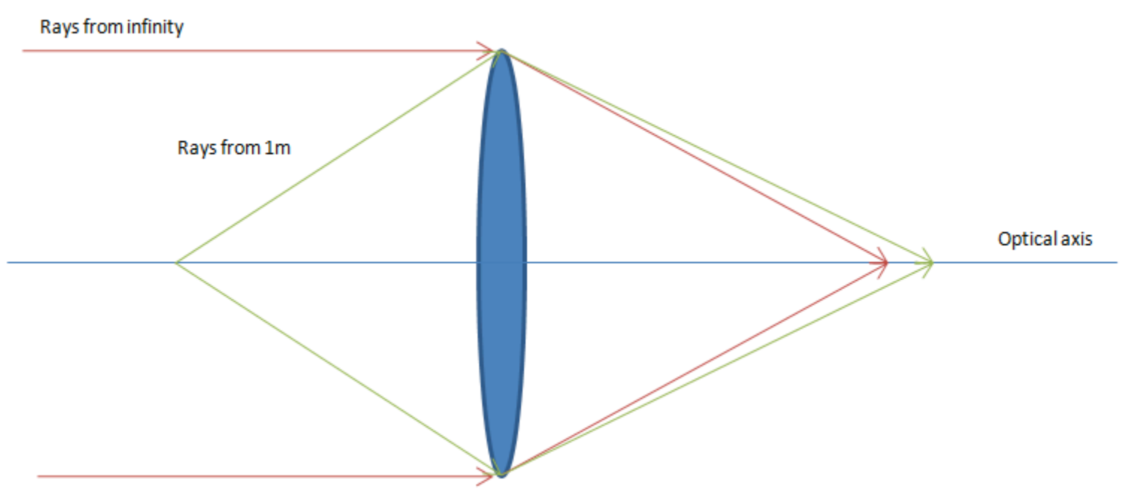
      $$\Delta l = 1.220\frac{f\lambda}{D}$$

      so in this case the diffraction limit is 1.220*10*550nm/5 = 1342nm or 1.342 micron (this is the best focus the system can have for green light)

      Assuming the primary subject of the photograph is at 1m so the correct image distance is given by:

      $$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

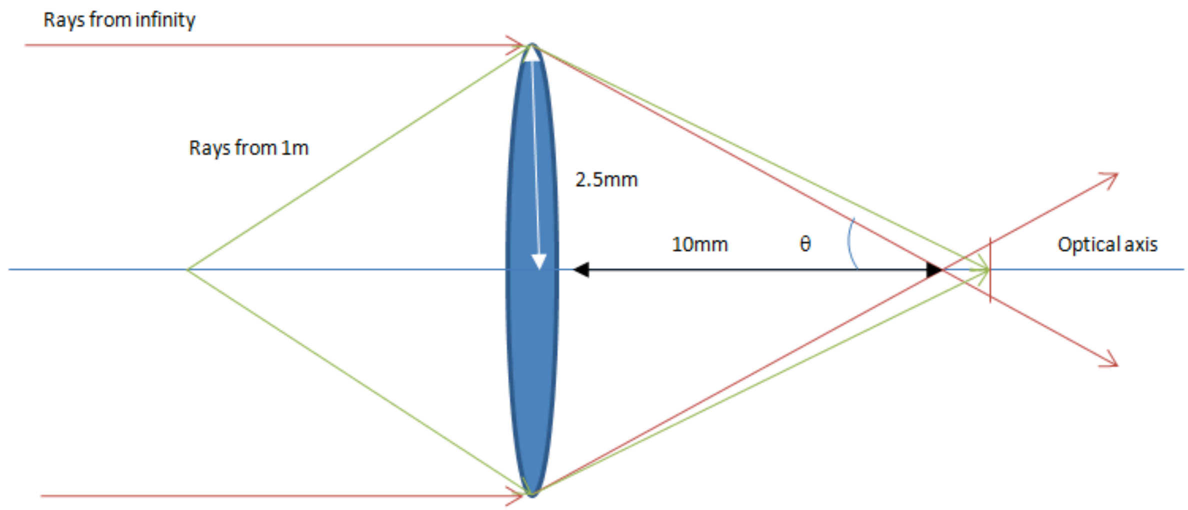
      where S_1 = 1m and f = 10mm so S_2 is 10.1 mm and the object from infinity is of course focused at 10mm. So the difference between the two is 0.1 mm.



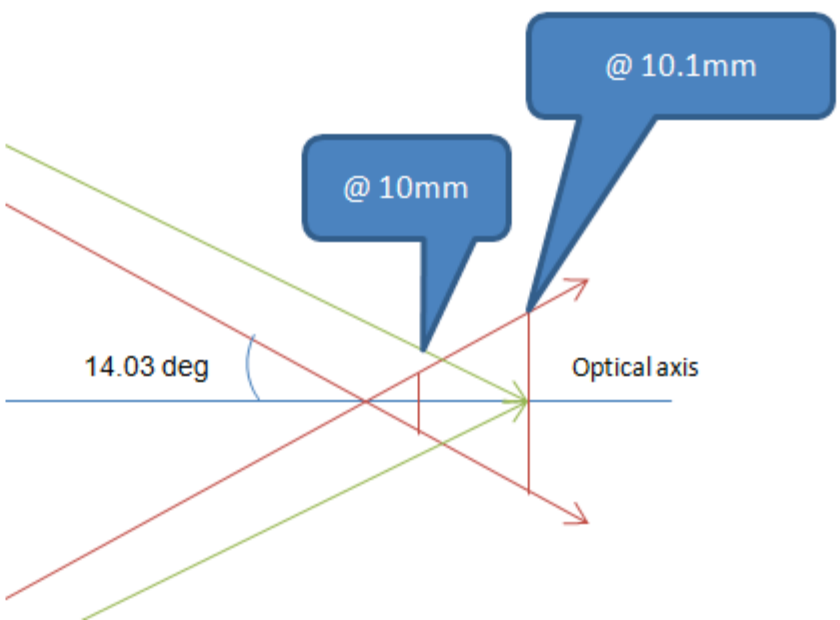
      The question becomes what happens to the image from infinity at 10.1mm instead of 10mm. Using small angle the spot size becomes:

      the angle between the optical axis and the ray is θ so that:

      tan θ = 2.5/10   θ = 14.03 degrees



so at 10.1mm the resolution becomes 1.342 + tan(14.03)*100 = 26.33 microns, which is still much larger than 3 microns, so it cannot be in focus!



So trying some reverse calculation (since the aperture on a camera can be adjusted):

D = 1.22*(10mm)*(0.55um)/(3um)

D = 2.2367 mm

Theta = 6.38 deg

So the answers become (still not in focus):

D = 2.2367mm f = 10mm	wave length [nm]	resolution limit [nm]	@ 10.1 mm defocus [um] theta = 6.38 deg
blue light	450	2454.51	13.64
green light	550	2999.96	14.18
red light	750	4090.85	15.27

Ok, trying again with the primary object at 3 meters:

the following equation will yield:

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

S1 = 3000mm and f = 10mm so S2 = 10.033mm

and the table becomes:

D = 5mm f = 10mm	wave length [nm]	resolution limit [nm]	@ 10.033 mm defocus [um] theta = 14.03 deg
blue light	450	1098.00	9.34
green light	550	1342.00	9.59
red light	750	1830.00	10.08

These results still proves that the object from infinity is not in focus!

They are much closer.

There is the table again with the aperture closed down to allow green light to resolve at 3um:

D = 2.2367mm f = 10mm	wave length [nm]	resolution limit [nm]	@ 10.033 mm defocus [um] theta = 6.38 deg
blue light	450	2454.51	6.14
green light	550	2999.96	6.69
red light	750	4090.85	7.78

This is the smallest resolution I can get for objects at infinity and they are still not in focus.

b. What happens when you consider red (750nm) and blue (450nm) light? For which wavelength the focus is better.

Ans: following the equations above:

the 750nm light has a diffraction limit of 1.830 microns and thus the de-focus of 10.1mm will be 26.82 microns

The 450nm light has a diffraction limit of 1.098 microns and at the de-focus of 10.1mm the size of resolution is 26.086 microns

Thus the blue light is easier to focus but is still not in focus

c. What if the diameter of the lens is 2.5mm and f is 5mm

Ans: everything is the same because the ratio between the diameter and the focus length is the same

d. What if the diameter of the lens is 10mm and the focal length is 10mm

Ans:

D = f = 10 mm	wave length [nm]	resolution limit [nm]	@ 10.1 mm defocus [um] theta = 26.56 deg
blue light	450	549	50.54
green light	550	671	50.66
red light	750	915	50.90

e. Organize your findings as a function of f# (f/D) and wavelength in the form of an easy rule to remember

Ans: as f# increases, image at infinity becomes clear (more in focus)
as wavelength decreases the image becomes more in focus
but the impact is much smaller than f#

I have recently purchased a SLR camera and this finding is consistent with introduction to photography. This also explains the lenses designed for portrait have very small f#.